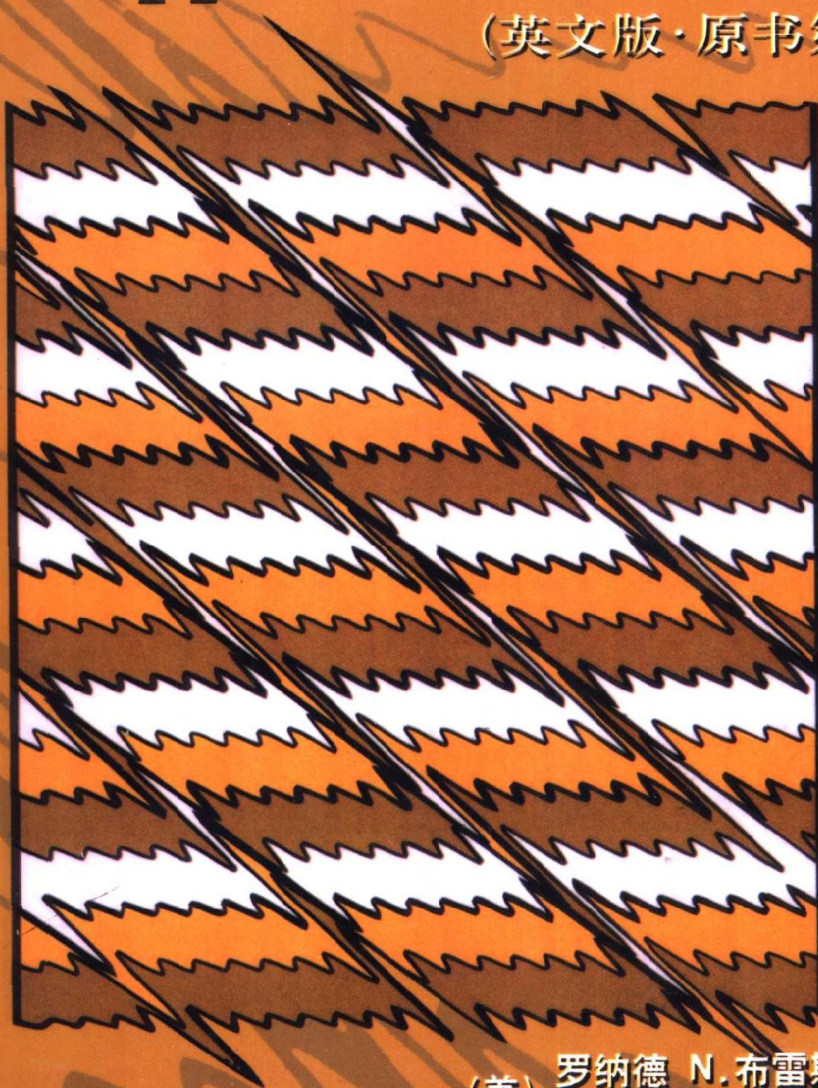


时代教育·国外高校优秀教材精选

傅里叶变换 及其应用

The Fourier Transform and Its Applications

(英文版·原书第3版)



(美) 罗纳德 N. 布雷斯韦尔 著
(Ronald N. Bracewell)



机械工业出版社
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序

R. N. Bracewell 教授的力作《THE FOURIER TRANSFORM AND ITS APPLICATIONS》一书初版于 1965 年，本书是它的第 3 版，2000 年由著名的 McGraw-Hill 教育出版公司出版。本书是 McGraw-Hill 出版社《电子和计算机工程系列丛书》之一，被国外多所著名大学选作教材，具有很大的影响。

在介绍本书特点之前，我们先对 Fourier 在科学发展史上的地位和贡献进行简要的说明。

Jean Baptiste Joseph Fourier 虽已去世 100 多年了，但其卓越的工作却因对热的传播和扩散的研究，在现代谱分析中的两个重要的方面产生了极其深远的影响。如果我们了解 Fourier 级数展开中的两个要素是正弦谐波项和系数项，那么我们可以发现这样的事实：

1. 1807 年，Fourier 发现在表示一个物体的温度分布时，正弦函数及其谐波的级数是非常有用的。

2. Fourier 进而断言，“任何”周期信号都可用具有谐波关系的正弦函数级数表示。

3. 但在 Fourier 的后续研究中，有关 Fourier 级数部分的数学结论并不严密。

4. 在 Fourier 级数的系数公式中，Fourier 本人可能还没有意识到正交特性，尽管在他之前已经存在函数正交的概念了。

5. 在 Fourier 之前，至少 D. Bernoulli 就曾提出过一根弦的运动完全可以用标准振荡模（正弦谐波）的线性组合来描述。

6. 只有在 P. L. Dirichlet 给出他的著名的三个充分条件之后，在这些条件约束下，我们知道一个周期信号才可以用一个 Fourier 级数表示。

由此可见，Fourier 实际上并没有对 Fourier 级数的数学理论做出实质性的贡献。那么科学上为什么仍然以 Fourier 的名字来命名周期函数的级数展开和非周期函数的积分变换呢？这是因为 Fourier 敏锐地洞察出这个级数表示法的潜在应用价值，而且正是由于他的努力才真正推动了 Fourier 级数问题的深入研究。另外，Fourier 在这一问题上的研究成果比他的前辈们都大大前进了一步，主要体现在他得出了非周期信号的描述形式不是正弦信号高次谐波的加权和，而是不全成谐波关系的正弦信号的加权积分，这就是众所周知的从傅里叶

级数到傅里叶积分（或变换）的推广。与信号的傅里叶（级数）分析一样，傅里叶变换仍然是今天分析 LTI 系统最有力的工具之一。

Fourier 的贡献就在于他将前人的这些思想巧妙地综合到一起，抛弃了繁杂的数学求证过程，直接了当地申明任何函数均可用正弦谐波函数的无穷和来表示。的确，正弦信号（进而傅里叶级数和傅里叶变换）在科学研究的和工程应用的大量问题中起着重要的作用。例如，交流电产生的电压和电流，行星运动的周期，气候的周期性变化等都很自然地会用到正弦信号；海浪也可认为是由不同波长的正弦函数的线性组合形成；无线电台和电视台发射的信号基本上也是正弦函数；甚至今天最新的短时 FFT、小波变换等也是以 Fourier 变换为其基础的。

本书作者 R. N. Bracewell 教授是著名的 Fourier 分析专家，曾在 Stanford 大学射电天文研究所主持设计和建造了具有革新意义的射电天文望远镜，包括具有人眼分辨率的天线。他还参与了 19 世纪 60 年代初发现宇宙背景辐射的工作。Bracewell 教授在本书中用清晰的图解化数学思想讲授变换方法以及怎样将其应用于电子、电力系统，电路，天线，光学，统计信号处理等领域；讨论的中心议题是连续时间信号、离散时间信号和脉冲序列。本书的特点是：

1. 本书的选材以信号处理领域的核心内容——Fourier 变换为主题，用主要篇幅讨论了 Fourier 变换的基本概念及其变换域分析方法。这是信号处理中的经典内容，也是进一步学习和掌握高级信号处理理论及技术的基础。此外，本书用五章篇幅讨论了 Fourier 变换的具体应用，天线和光学、统计信号应用、随机噪声和波形、热传导和热扩散以及动态功率谱的概念。故从选材内容考虑，本书不仅非常适合电类专业，而且同样适合机电信号、物探信号、生物医学信号处理以及物理等类专业的高年级本科生用作教材，当然也可作为研究生的参考教材和科技人员的自学参考书。

2. 本书叙述清晰，几乎所有的数学公式都有图形解释，疑难之处处理得体。书中关于信号、谱、滤波和线性化的讨论，关于离散 Hartley 变换的讨论以及 Fourier 变换关系的讨论等都很有特色。这样，当读者自学书中内容时，一般不会遇到太大的困难。

3. 本书给出许多经典而又内容充实的实例来讨论所涉及的理论问题，这些例子大多与作者本人的研究活动直接有关，无疑使本书更具有可读性和启发性。这一方面有利于读者掌握书中的理论，另一方面则有利于读者将这些理论应用于实际。

4. 本书另外一个重要的特点是把传统理论研究和先进的计算工具——Matlab 相结合。我们知道，Matlab 是当前最优秀的科学计算软件，其中的 Fourier 分析工具以其内容丰富和功能强大而著称。本书作者特意安排了五章

内容进行二者的整合，这样有助于读者在学习 Fourier 分析理论的同时，借助 Matlab 加深对问题的理解。这非常有利于读者用计算机实践 Fourier 分析的众多理论和算法，同时也为把这些理论和算法应用于工程实际打下了很好的基础。

张延华

北京工业大学电子信息与控制工程学院

2002 年 4 月

出版说明

随着我国加入 WTO, 国际间的竞争越来越激烈, 而国际间的竞争实际上也就是人才的竞争、教育的竞争。为了加快培养具有国际竞争力的高水平技术人才, 加快我国教育改革的步伐, 教育部近来出台了一系列倡导高校开展双语教学、引进原版教材的政策。以此为契机, 机械工业出版社拟于近期推出一系列国外影印版教材, 其内容涉及高等学校公共基础课, 以及机、电、信息领域的专业基础课和专业课。

引进国外优秀原版教材, 在有条件的学校推动开展英语授课或双语教学, 自然也引进了先进的教学思想和教学方法, 这对提高我国自编教材的水平, 加强学生的英语实际应用能力, 使我国的高等教育尽快与国际接轨, 必将起到积极的推动作用。

为了做好教材的引进工作, 机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究, 对引进原版教材提出许多建设性意见, 并慎重地对每一本将要引进的原版教材一审再审, 精选再精选, 确认教材本身的质量水平, 以及权威性和先进性, 以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中, 审定委员会还结合我国高校教学课程体系的设置和要求, 对原版教材的教学思想和方法的先进性、科学性严格把关, 同时尽量考虑原版教材的系统性和经济性。

这套教材出版后, 我们将根据各高校的双语教学计划, 举办原版教材的教师培训, 及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议, 使我们更好地为教学改革服务。

机械工业出版社

2002 年 3 月

Transform methods provide a unifying mathematical approach to the study of electrical networks, devices for energy conversion and control, antennas, and other components of electrical systems, as well as to complete linear systems and to many other physical systems and devices, whether electrical or not. These same methods apply equally to the subjects of electrical communication by wire or optical fiber, to wireless radio propagation, and to ionized media—which are all concerned with the interconnection of electrical systems—and to information theory which, among other things, relates to the acquisition, processing, and presentation of data. Other theoretical techniques are used in handling these basic fields of electrical engineering, but transform methods are virtually indispensable in all of them. Fourier analysis as applied to electrical engineering is sufficiently important to have earned a permanent place in the curriculum—indeed much of the mathematical development took place in connection with alternating current theory, signal analysis, and information theory as formulated in connection with electrical communication.

This is why much of the literature dealing with technical applications has appeared in electrical and electronic journals. Despite the strong bonds with electrical engineering, Fourier analysis nevertheless has become indispensable in biomedicine and remote sensing (geophysics, oceanography, planetary surfaces, civil engineering), where practitioners now outnumber those electrical engineers who regularly use Fourier analysis. But the teaching of Fourier analysis and its applications still finds its home in electrical engineering.

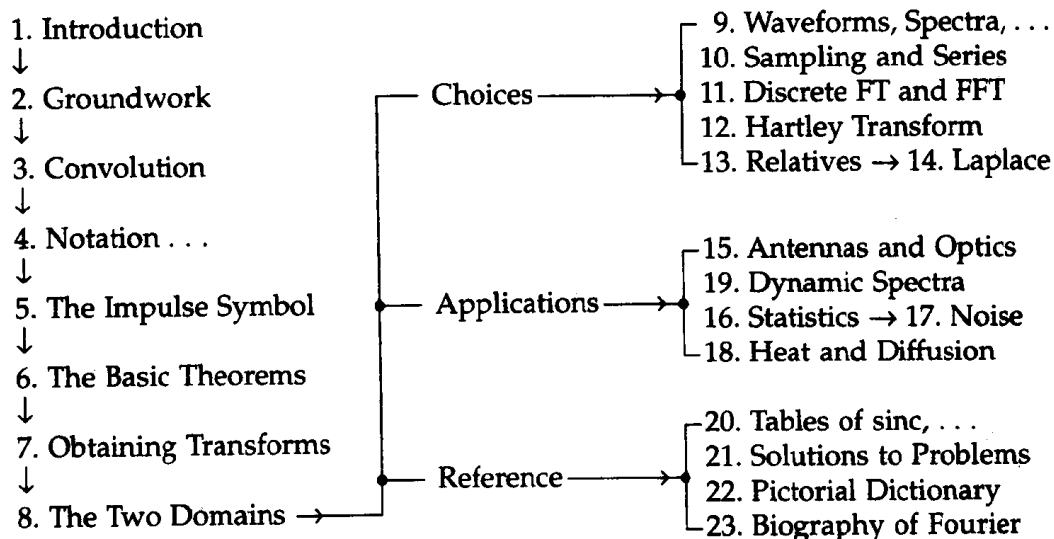
A course on transforms and their applications has formed part of the electrical engineering curriculum at Stanford University for many years and has been given with no prerequisites beyond those that the holder of a bachelor's degree normally possesses. One objective has been to develop a pivotal course to be taken at an early stage by all graduates, so that in later, more specialized courses, the student would be spared encountering the same material over and over again; later instructors can then proceed more directly to their special subject matter.

It is clearly not feasible to give the whole of linear mathematics in a single course; the choice of core material must necessarily remain a matter of local judgment. The choice will, however, be of most help to later instructors if sharply defined.

An early-level course should be simple, but not trivial; the objective of this book is to simplify the presentation of many key topics that are ordinarily dealt

with in advanced contexts, by making use of suitable notation and an approach through convolution.

One way of working from the book is to begin by taking the chapters in numerical order. This sequence is feasible for students who could read the first half unassisted or who could be taken through it rapidly in a few lectures; but if the material is approached at a more normal pace, then, as a practical matter, it is a good idea to interpret each theorem and concept in terms of a physical example. Waveforms and their spectra and elementary diffraction theory are suitable. After that, the chapters on applications can then be selected in whatever sequence is desired. The organization of chapters is as follows:



The amount of material is suitable for one semester, or for one quarter, according to how many of the later chapters on applications are included. A practical plan is to leave the choice of chapters on applications to the current instructor.

Many fine mathematical texts on the Fourier transform have been published. This book differs in that it is intended for those who are concerned with applying Fourier transforms to physical situations rather than with pursuing the mathematical subject as such. The connections of the Fourier transform with other transforms are also explored, and the text has been purposely enriched with condensed information that will suit it for use as a reference source for transform pairs and theorems pertaining to transforms.

My interest in the subject was fired while studying analysis from H. S. Carslaw's "Fourier Series and Integrals" at the University of Sydney in 1939, I learned about physical applications as a colleague of J. C. Jaeger at C.S.I.R. Radiophysics Laboratory and inherited the physical wisdom of the crystallographers of the Cavendish Laboratory, Cambridge, as transmitted by J. A. Ratcliffe. Transform methods are at the heart of the electrical engineering curriculum. Digital

computing and data processing, which have emerged as large curricular segments, though rather different in content from the rigorous study of circuits, electronics, and waves, nevertheless do share a common bond through the Fourier transform. The diffusion equation, which long ago had a connection with submarine cable telegraphy, has reemerged as an essential consideration in solid state physics and devices, both through the practice of doping, by which semiconductor devices are fabricated, and as a controlling influence in electrical conduction by holes and electrons. Needless to say, a grasp of Fourier fundamentals is an asset in the solid-state laboratory.

The explosion of image engineering, much of which can be interpreted via two-dimensional generalization, has reinforced the value of a core course. Consequently, the subject matter of this book has easily moved into the pivotal role foreseen for it, and faculty members from various specialties have found themselves comfortable teaching it. The course is taken by first-year graduate students, especially students arriving from other universities, and by students from other departments, notably applied physics and earth sciences. The course is accessible to students in the last year of their bachelor's degree.

Introduction of the fast Fourier transform (FFT) algorithm has greatly broadened the scope of application of the Fourier transform to data handling and to digital formulation in general and has brought prominence to the discrete Fourier transform (DFT). The technological revolution associated with discrete mathematics as treated in Chapter 11 has made an understanding of Fourier notions (such as aliasing, which only aficionados used to guard against) indispensable to any professional who handles masses of data, not only engineers but experts in many subfields of medicine, biology, and remote sensing. Developments based on Ralph V. L. Hartley's equations (Chapter 12) have made it possible to dispense with imaginaries in computed Fourier analysis and to proceed elegantly and simply using the real Hartley formalism.

Hartley's equations, which quietly received honorable mention in the first edition of this book, gained major relevance to signal processing as computers flourished. In 1983 I gave them new life in a time-series context with modern notation under the title of discrete Hartley transform, a name that is now universally recognized, while Z. Wang (*Appl. Math. and Comput.*, vol. 9, pp. 53–73, 153–163, 245–255, 1983) independently stimulated mathematicians. Hartley's cas (cosine and sine) function is now widely recognized.

For those who like to do their own computer programming some segments of pseudocode have been supplied. Translating into your language of choice may give some insights that complement the algebraic and graphical viewpoints. Furthermore, executing numerical examples develops a useful sort of intuition which, while not as powerful as physical intuition, adds a further dimension to one's experience. Pseudocode is suited to readers who cannot be expected to know several popular languages. The aim is to provide the simplest intelligible instructions that are susceptible to simultaneous transcription into the language of fluency of the reader, who provides the necessary protocol, array declarations, and other distinctive features.

The code segments in this book are presented to supplement verbal explanation, not to be a substitute for a computational toolbox.

However, it is often more important to be able to use a computer algorithm than to understand in detail how it was constructed, just as when using a table of integrals or an engineering design handbook. To meet this need and to bring the power of Fourier transformation into the hands of a much wider constituency, packages of software tools have been created commercially and have become indispensable. A popular example is MATLAB®, a user-friendly, higher-level, special-purpose application whose use is illustrated in Chapters 7 and 11.

Caution is needed in circumstances where the user is shielded from the algorithmic details; it is handy to know what to expect before being presented with computer output. For this and other reasons, transforms presented graphically in the Pictorial Dictionary have proved to be a useful reference feature. Graphical presentation is a useful adjunct to the published compilations of integral transforms, where it is sometimes frustrating to seek commonly needed entries among the profusion of rare cases and where, in addition, simple functions that are impulsive, discontinuous, or defined piecewise may be hard to recognize or may not be included at all.

A good problem assigned at the right stage can be extremely valuable for the student, but a good problem is hard to compose. Many of the problems here go beyond mathematical exercises by inclusion of technical background or by asking for opinions. Those wishing to mine the good material in the problems will appreciate that many of them are now titled, a practice that should be more widely adopted. Many of the problems are discussed in Chapter 21, but occasionally it is nice to have a new topic followed by an exercise that is in close proximity rather than at the end of the chapter; a sprinkling of these is provided.

Notation is a vital adjunct to thinking, and I am happy to report that the *sinc function*, which we learned from P. M. Woodward, is alive and well, and surviving erosion by occasional authors who do not know that “sine x over x ” is not the sinc function. The *unit rectangle function* (unit height and width) $\Pi(x)$, the transform of the sinc function, has also proved extremely useful, especially for blackboard work. In typescript or other media where the Greek letter is less desirable, $\Pi(x)$ may be written “rect x ,” and it is convenient in any case to pronounce it *rect*. The *jinc function*, the circular analogue of the sinc function, has the corresponding virtues of normalization and the distinction of describing the diffraction field of a telescope or camera. The *shah function* $\text{III}(x)$ has caught on. It is easy to print and is twice as useful as you might think because it is its own transform. The asterisk for convolution, which was in use a long time ago by Volterra and perhaps earlier, is now in wide use and I recommend ****** to denote two-dimensional convolution, which has become common as a result of the explosive growth of image processing.

Early emphasis on digital convolution in a text on the Fourier transform turned out to be exactly the way to start. Convolution has changed in a few years from being presented as a rather advanced concept to one that can be easily explained at an early stage, as is fitting for an operation that applies to all those systems that respond sinusoidally when you shake them sinusoidally.

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