

21 世纪 .....  
信息与通信技术教程

英文版

**SIGNAL PROCESSING ADVANCES IN  
WIRELESS AND MOBILE COMMUNICATIONS**

**VOLUME 1: TRENDS IN CHANNEL  
ESTIMATION AND EQUALIZATION**

**无线与移动通信中的信号处理新技术  
第1册：信道估计与均衡**

GEORGIOS B. GIANNAKIS YINGBO HUA PETRE STOICA LANG TONG

· 编 著 ·

PH  
PTR



人民邮电出版社  
POSTS & TELECOMMUNICATIONS PRESS

TN928.5  
35

人民邮电出版社

21 世纪信息与通信技术教程

# 无线与移动通信中的信号处理新技术

## 第 1 册：信道估计与均衡

(英文版)

**Signal Processing Advances in Wireless and Mobile  
Communications**

**Volume 1: Trends in Channel Estimation and  
Equalization**

Georgios B. Giannakis    Yingbo Hu    德莘  
Petre Stoica    Lang Tong

人民邮电出版社

## 图书在版编目 (CIP) 数据

无线与移动通信中的信号处理新技术. 第1册,  
信道估计与均衡 / (美) 贾纳科斯 (Giannakis, G.B.) 编著.  
—北京: 人民邮电出版社, 2002.11

21 世纪信息与通信技术教程

ISBN 7-115-10828-5

I. 无... II. 贾 III. ①移动通信—信号处理—新技术应用  
—高等学校—教材—英文②无线电信道—高等学校—教材—英文  
IV. TP929.5

中国版本图书馆 CIP 数据核字 (2002) 第 084067 号

21 世纪信息与通信技术教程

无线与移动通信中的信号处理新技术

第 1 册: 信道估计与均衡 (英文版)

---

◆ 编 著 Georgios B. Giannakis Yingbo Hua

Petre Stoica Lang Tong

责任编辑 陈万寿

◆ 人民邮电出版社出版发行 北京市崇文区夕照寺街 14 号

邮编 100061 电子函件 315@ptpress.com.cn

网址 <http://www.ptpress.com.cn>

读者热线 010-67129258

北京汉魂图文设计有限公司制作

北京朝阳展望印刷厂印刷

新华书店总店北京发行所经销

◆ 开本: 850×1168 1/32

印张: 14

2002 年 11 月第 1 版

印数: 1-3 000 册

2002 年 11 月北京第 1 次印刷

著作权合同登记 图字: 01-2002-3520 号

ISBN 7-115-10828-5/TN · 1961

---

定价: 29.00 元

本书如有印装质量问题, 请与本社联系 电话: (010) 67129223

## 内 容 提 要

《无线与移动通信中的信号处理新技术》丛书，介绍了近年来无线与移动通信中使用的信号处理（SP）工具的最新的重要进展，以及世界范围内该领域的领先者的贡献。本书是两本书中的第 1 册。本丛书的内容涵盖了范围广泛的技术和方法论，包括噪声与干扰消除、调制解调器设计、移动互联网业务、下一代音频/视频广播、蜂窝移动电话和无线多媒体网络等。

本书（第 1 册）重点阐述单用户点对点链路的信道识别与均衡的关键技术。由于信息承载信号是在衰落介质中传播的，所以现代的均衡器必须充分考虑移动无线信道的可变性，减小符号间干扰和同（共）信道干扰，并抑制在单个或多个传感器的接收机中的噪声。本书介绍了最近提出的带宽节省（半）盲算法与性能分析，以及线性预编码技术，这些技术利用发射冗余使基于训练序列的系统获得明显的改善。本书内容包括：

- 盲识别与反卷积的子空间方法
- 有色信号驱动的信道的盲识别与均衡
- 最优子空间方法；多信道均衡的线性预测算法
- FIR 多信道估计的半盲方法
- 盲判决反馈均衡等

本书还介绍了在世界范围内各种期刊中的研究成果，全面汇集了用于优化单用户点对点链路的先进信号处理技术。本书对于通信工程师、研究人员、管理人员、通信系统设计人员和参与最新通信系统设计或构造的同行将是极其有价值的。

## 版 权 声 明

English Reprint Edition Copyright ©2002 by PEARSON EDUCATION NORTH ASIA LIMITED and POSTS & TELECOMMUNICATIONS PRESS.

Signal Processing Advances in Wireless and Mobile Communications

Volume 1: Trends in Channel Estimation and Equalization

By Georgios B.Giannakis, Yingbo Hua, Petre Stoica, Lang Tong

Copyright ©2002

All Rights Reserved.

Published by arrangement with the original publisher, Pearson Education, Inc., publishing as Prentice-Hall PTR.

This edition is authorized for sale only in the People's Republic of China (excluding the Special Administrative Region of Hong Kong and Macau).

本书封面贴有 **Pearson Education** (培生教育出版集团) 激光防伪标签, 无标签者不得销售。

---

---

# PREFACE

Signal processing has always played a critical role in the research and development of wireless communication systems. As the demand for high capacity and high reliability systems increases, signal processing has an even more important role to play. This two-volume book, comprising chapters written by leading experts, provides an easy access to recent and important research findings in this area.

The first volume of this book focuses on channel estimation and equalization. Since the physical environment surrounding the propagation path of a radio signal is generally difficult to control, the channel characteristics in wireless communications are often time varying. This time-varying nature is a major obstacle to increasing the capacity and reliability of wireless communication systems. Fast real-time channel estimation and equalization are essential. The traditional techniques for channel estimation and equalization use training data, which not only consumes a significant portion of available bandwidth but also requires a perfect cooperation between the transmitter and receiver. In recent years, the so-called blind techniques have been explored intensively in the literature. The blind techniques do not use any training data except for certain prior information inherent in the original strings of symbols, which hence saves the bandwidth and relaxes the relationship between the transmitter and receiver. Consequently, the blind techniques have a clear potential to increase the capacity and reliability of wireless systems. As a subject area, the blind techniques have had in recent years a very fast growing rate in the general field of signal processing for communications. For this reason, the first volume of this book is devoted to the blind (or semi-blind) techniques.

The volume begins with a chapter by Tugnait on higher-order statistics methods. The higher-order statistics approach is among the earliest blind approaches and has a relatively long history. The second chapter by Stoica and Ng derives the Cramer-Rao bound for blind channel estimation problems. In the blind context, the estimated channel response is ambiguous up to (at least) a scalar. The traditional Cramer-Rao bound on estimation variance is generalized in this chapter. The third chapter by Loubaton, Moulines and Regalia introduces a versatile approach known as the subspace approach. The subspace approach often provides a good trade-off between computation and accuracy. The fourth chapter by Hua

on blind identification of multiple input–multiple output (MIMO) channels driven by colored signals presents one of the latest advances in the theory of channel estimation. This theory is readily applicable to wireless communications (interfaces) between human and “smart” microphones, where the voice signals are naturally colored. The *fifth* chapter by Kristensson and Ottersten addresses the optimization of subspace based methods. The results shown prove that the statistical performance of this class of methods can be enhanced by careful design. The *sixth* chapter by Ding on linear prediction introduces a number of ways to exploit the whiteness of spectrum spread signals. This exploitation significantly increases the robustness of estimation as long as the whiteness is present. The *seventh* chapter by Carvalho and Slock on semi-blind methods explores the compromise between a fully trained system and a fully blind system. The highest capacity of a wireless system may be achievable via a semi-blind method. The *eighth* chapter by Tong, Gu and Kung on the geometrical approach provides a new look at some of the classical techniques for symbol estimation. The *ninth* chapter by Scaglione, Giannakis and Barbarossa on linear precoding is a tutorial presentation of this relatively new concept. Linear precoding is a versatile coding approach to combat signal distortions by unknown channels. The *tenth* chapter by Manton and Hua on blind channel identifiability with an arbitrary linear precoder explores in depth the implications of linear precoding by making extensive use of abstract algebra. The final chapter by Casas, Endres, Touzni, Johnson and Treichler on decision feedback equalization brings the reader back to a classic equalization approach for which there is a rich body of theory and applications. This “feedback” will ensure continued renewal, cross-fertilization and further development of many useful concepts and techniques for wireless communications.

On a closing note, the co-editors wish to thank all the contributors to this fine collection, as well as their students, mentors and collaborators that introduced them and continue to spark their interest in signal processing and communications-related research. The first author wishes to extend special thanks to his graduate student and co-author Zhengdao Wang for his extra help with the typesetting and compiling of all the chapters in both volumes.

*G. B. Giannakis, Y. Hua, P. Stoica and L. Tong*  
May 2000

---

---

# CONTENTS

<b>PREFACE</b>	<b>xi</b>
<b>1 CHANNEL ESTIMATION AND EQUALIZATION USING HIGHER-ORDER STATISTICS</b>	<b>1</b>
1.1 Introduction	1
1.2 Single-User Systems: Baud Rate Sampling	4
1.2.1 Cumulant Matching	4
1.2.2 Inverse Filter Criteria	8
1.2.3 Equation Error Formulations	8
1.2.4 Simulation Examples	8
1.3 Single-User Systems: Fractional Sampling	12
1.3.1 Cumulant Matching	13
1.3.2 Simulation Example	20
1.4 Multi-user Systems	24
1.4.1 Inverse Filter Criteria	26
1.4.2 Cumulant Matching	28
1.4.3 Simulation Examples	31
1.5 Concluding Remarks	35
Bibliography	37
<b>2 PERFORMANCE BOUNDS FOR BLIND CHANNEL ESTIMATION</b>	<b>41</b>
2.1 Introduction	42
2.2 Problem Statement and Preliminaries	42
2.2.1 The Blind Channel Identification Problem	42
2.2.2 Ambiguity Elimination	44
2.2.3 The Unconstrained FIM	46
2.2.4 Achievability of the CRB	47
2.3 CRB for Constrained Estimates	48
2.4 CRB for Estimates of Invariants	49
2.5 CRB for Projection Errors	52
	<b>iii</b>



2.6	Numerical Examples	53
2.7	Concluding Remarks	58
	Appendix 2.A Proof of Proposition 2	59
	Bibliography	61
<b>3</b>	<b>SUBSPACE METHOD FOR BLIND IDENTIFICATION AND DECONVOLUTION</b>	<b>63</b>
3.1	Introduction	63
3.2	Subspace Identification of SIMO Channels	65
3.2.1	Practical Considerations	69
3.2.2	Simplifications in the Two-Channel Case	70
3.3	Subspace Identification of MIMO Channels	71
3.3.1	Rational Spaces and Polynomial Bases	72
3.3.2	The Structure of the Left Nullspace of a Sylvester Matrix	76
3.3.3	The Subspace Method	78
3.3.4	Advanced Results	82
3.4	Applications to the Blind Channel Estimation of CDMA Systems	84
3.4.1	Model Structure	84
3.4.2	The Structured Subspace Method: The Uplink Case	88
3.4.3	The Structured Subspace Method: The Downlink Case	89
3.5	Undermodeled Channel Identification	92
3.5.1	Example: Identifying a Significant Part of a Channel	99
3.5.2	Determining the Effective Impulse Response Length	100
	Appendix 3.A	102
3.A.1	Proof of Theorem 1	103
3.A.2	Proof of Proposition 3	104
3.A.3	Proof of Theorem 4	105
3.A.4	Proof of Proposition 5	106
	Bibliography	108
<b>4</b>	<b>BLIND IDENTIFICATION AND EQUALIZATION OF CHANNELS DRIVEN BY COLORED SIGNALS</b>	<b>113</b>
4.1	Introduction	114
4.2	FIR MIMO Channel	115
4.2.1	Original Model	115
4.2.2	Slide-Window Formulation	115
4.2.3	Noise Variance and Number of Input Signals	116
4.3	Identifiability Using SOS	117
4.3.1	Identifiability Conditions	117
4.3.2	Some Facts of Polynomial Matrices	118
4.3.3	Proof of the Conditions	120
4.3.4	When the Input is White	121
4.4	Blind Identification via Decorrelation	121
4.4.1	The Principle of the BID	121

4.4.2	Constructing the Decorrelators	126
4.4.3	Removing the GCD of Polynomials	128
4.4.4	Identification of the SIMO Channels	130
4.5	Final Remarks	135
	Bibliography	135
<b>5</b>	<b>OPTIMUM SUBSPACE METHODS</b>	<b>139</b>
5.1	Introduction	139
5.2	Data Model and Notations	140
5.2.1	Scalar Valued Communication Systems	140
5.2.2	Multi Channel Communication Systems	141
5.2.3	A Stacked System Model	143
5.2.4	Correlation Matrices	145
5.2.5	Statistical Assumptions	147
5.3	Subspace Ideas and Notations	148
5.3.1	Basic Notations	149
5.4	Parameterizations	151
5.4.1	A Noise Subspace Parameterization	151
5.4.2	Selection Matrices	153
5.5	Estimation Procedure	154
5.5.1	The Signal Subspace Parameterization	155
5.5.2	The Noise Subspace Parameterization	156
5.6	Statistical Analysis	156
5.6.1	The Residual Covariance Matrices	157
5.6.2	The Parameter Covariance Matrices	159
5.7	Relation to Direction Estimation	161
5.8	Further Results for the Noise Subspace Parameterization	162
5.8.1	The Results	163
5.8.2	The Approach	163
5.9	Simulation Examples	164
5.10	Conclusions	171
	Appendix 5.A	173
	Bibliography	174
<b>6</b>	<b>LINEAR PREDICTIVE ALGORITHMS FOR BLIND MULTICHANNEL IDENTIFICATION</b>	<b>179</b>
6.1	Introduction	179
6.2	Channel Identification Based on Second Order Statistics: Problem Formulation	181
6.3	Linear Prediction Algorithm for Channel Identification	183
6.4	Outer-Product Decomposition Algorithm	185
6.5	Multi-step Linear Prediction	188
6.6	Channel Estimation by Linear Smoothing (Not Predicting)	189
6.7	Channel Estimation by Constrained Output Energy Minimization	192

6.8	Discussion	195
6.8.1	Channel Conditions	195
6.8.2	Data Conditions	196
6.8.3	Noise Effect	196
6.9	Simulation Results	197
6.10	Summary	198
	Bibliography	207
<b>7</b>	<b>SEMI-BLIND METHODS FOR FIR MULTICHANNEL ESTIMATION</b>	<b>211</b>
7.1	Introduction	212
7.1.1	Training Sequence Based Methods and Blind Methods	212
7.1.2	Semi-Blind Principle	213
7.2	Problem Formulation	214
7.3	Classification of Semi-Blind Methods	217
7.4	Identifiability Conditions for Semi-Blind Channel Estimation	218
7.4.1	Identifiability Definition	218
7.4.2	TS Based Channel Identifiability	219
7.4.3	Identifiability in the Deterministic Model	219
7.4.4	Identifiability in the Gaussian Model	222
7.5	Performance Measure: Cramér-Rao Bounds	224
7.6	Performance Optimization Issues	226
7.7	Optimal Semi-Blind Methods	227
7.8	Blind DML	229
7.8.1	Denoised IQML (DIQML)	230
7.8.2	Pseudo Quadratic ML (PQML)	231
7.9	Three Suboptimal DML Based Semi-Blind Criteria	232
7.9.1	Split of the Data	232
7.9.2	Least Squares-DML	232
7.9.3	Alternating Quadratic DML (AQ-DML)	233
7.9.4	Weighted-Least-Squares-PQML (WLS-PQML)	235
7.9.5	Simulations	236
7.10	Semi-Blind Criteria as a Combination of a Blind and a TS Based Criteria	236
7.10.1	Semi-Blind SRM Example	237
7.10.2	Subspace Fitting Example	239
7.11	Performance of Semi-Blind Quadratic Criteria	242
7.11.1	$M_U$ and $M_K$ infinite	243
7.11.2	$M_U$ infinite, $M_K$ finite	243
7.11.3	Optimally Weighted Quadratic Criteria	247
7.12	Gaussian Methods	247
7.13	Conclusion	249
	Bibliography	250

---

<b>8</b>	<b>A GEOMETRICAL APPROACH TO BLIND SIGNAL ESTIMATION</b>	<b>255</b>
8.1	Introduction	256
8.2	Design Criteria for Blind Estimators	258
8.2.1	The Constant Modulus Receiver	260
8.2.2	The Shalvi-Weinstein Receiver	261
8.3	The Signal Space Property and Equivalent Cost Functions	263
8.3.1	The Signal Space Property of CM Receivers	263
8.3.2	The Signal Space Property of SW Receivers	264
8.3.3	Equivalent Cost Functions	265
8.4	Geometrical Analysis of SW Receivers: Global Characterization	266
8.4.1	The Noiseless Case	268
8.4.2	The Noisy Case	270
8.4.3	Domains of Attraction of SW Receivers	275
8.5	Geometrical Analysis of SW Receivers: Local Characterizations	277
8.5.1	Local Characterization	277
8.5.2	MSE of CM Receivers	281
8.6	Conclusion and Bibliography Notes	282
8.6.1	Bibliography Notes	283
	Appendix 8.A Proof of Theorem 5	285
	Bibliography	288
<b>9</b>	<b>LINEAR PRECODING FOR ESTIMATION AND EQUALIZATION OF FREQUENCY-SELECTIVE CHANNELS</b>	<b>291</b>
9.1	System Model	293
9.2	Unifying Filterbank Precoders	296
9.3	FIR-ZF Equalizers	301
9.4	Jointly Optimal Precoder and Decoder Design	306
9.4.1	Zero-order Model	306
9.4.2	MMSE/ZF Coding	308
9.4.3	MMSE Solution with Constrained Average Power	309
9.4.4	Constrained Power Maximum Information Rate Design	311
9.4.5	Comparison Between Optimal Designs	313
9.4.6	Asymptotic Performance	317
9.4.7	Numerical Examples	318
9.5	Blind Symbol Recovery	320
9.5.1	Blind Channel Estimation	322
9.5.2	Comparison with Other Blind Techniques	324
9.5.3	Statistical Efficiency	330
9.6	Conclusion	332
	Bibliography	332

<b>10 BLIND CHANNEL IDENTIFIABILITY WITH AN ARBITRARY LINEAR PRECODER</b>	<b>339</b>
10.1 Introduction	339
10.2 Basic Theory of Polynomial Equations	344
10.2.1 Definition of Generic	344
10.2.2 General Properties of Polynomial Maps	344
10.2.3 Generic and Non-Generic Points	346
10.2.4 Invertibility Criteria	347
10.3 Inherent Scale Ambiguity	348
10.4 Weak Identifiability and the CRB	348
10.5 Arbitrary Linear Precoders	349
10.6 Zero Prefix Precoders	351
10.7 Geometric Interpretation of Precoding	354
10.7.1 Linear Precoders	354
10.7.2 Zero Prefix Precoders	355
10.8 Filter Banks	355
10.8.1 Algebraic Analysis of Filter Banks	357
10.8.2 Spectral Analysis of Filter Banks	358
10.9 Ambiguity Resistant Precoders	360
10.10 Symbolic Methods	361
10.11 Conclusion	362
Bibliography	363
<b>11 CURRENT APPROACHES TO BLIND DECISION FEEDBACK EQUALIZATION</b>	<b>367</b>
11.1 Introduction	367
11.2 Notation	370
11.3 Data Model	373
11.4 Wiener Filtering	374
11.4.1 Unconstrained Length MMSE Receivers	375
11.4.2 Constrained Length MMSE Receivers	377
11.4.3 Example: Constrained Versus Unconstrained Length Wiener Receivers	379
11.5 Blind Tracking Algorithms	380
11.5.1 DD-DFE	381
11.5.2 CMA-DFE	388
11.5.3 Algorithmic and Structural Modifications	389
11.5.4 Summary of Blind Tracking Algorithms	391
11.6 DFE Initialization Strategies	391
11.6.1 Generic Strategy	391
11.6.2 Multistage Equalization	395
11.6.3 CMA-IIR Initialization	397
11.6.4 Local Stability of Adaptive IIR Equalizers	398
11.6.5 Summary of Blind Initialization Strategies	399

---

11.7 Conclusion	400
Appendix 11.A Spectral Factorization	402
Appendix 11.B CL-MMSE-DFE	403
Appendix 11.C DD-DFE Local Convergence	405
Appendix 11.D Adaptive IIR Algorithm Updates	406
Appendix 11.E CMA-AR Local Stability	409
Bibliography	411

# CHANNEL ESTIMATION AND EQUALIZATION USING HIGHER-ORDER STATISTICS

**Jitendra K. Tugnait**

*Department of Electrical & Computer Engineering  
Auburn University  
Auburn, Alabama 36849, USA*

## **1.1 Introduction**

Two major sources of impairments of digital communications signals as they propagate through channels are multipaths and limited bandwidth. This leads to intersymbol interference (ISI) at the receiver which, in turn, may lead to high error rates in symbol detection. Equalizers are designed to compensate for these channel distortions. One may directly design an equalizer given the received signal, or one may first estimate the channel impulse response and then design an equalizer based on the estimated channel. The received signals are sampled at the baud (symbol) or higher (fractional) rate before processing them for channel estimation and/or equalization. Depending upon the sampling rate, one has either a single-input/single-output (SISO) (baud rate sampling), or a single-input/multiple-output (SIMO) (fractional sampling), complex discrete-time equivalent baseband channel in the single user case, and multiple-input/multiple-output (MIMO), complex discrete-time equivalent baseband channel in the multi-user case.

A training sequence (known to the receiver) may be transmitted during start-up (acquisition mode). In the operational stage, the receivers typically switch to a decision-directed mode where the previously equalized and detected symbols are used as a (pseudo-)training sequence together with the received data to update the channel or the equalizer coefficients. The various issues involved and the trade-offs among various competing approaches (linear, decision-feedback, maximum-likelihood sequence estimation, least mean-square vs. recursive least-squares, baud rate vs. fractional rate, etc.) are fairly well-understood and documented; see the well-known text [12] and references therein. More recently, there has been much interest in blind (self-recovering) channel estimation and blind equalization where no training sequences are available or used. In multipoint networks, whenever a channel from the master to one of the tributary stations goes down, it is clearly not feasible (or desirable) for the master to start sending a training sequence to reboot a particular line. In digital communications over fading/multipath channels, a restart is required following a temporary path loss due to a severe fade. In in-service transmission impairment monitoring, the training sequences are obviously not supplied by the transmitter.

As in the trained case, various approaches to blind channel estimation and equalization have been developed. When sampled at the baud rate, the received signal is discrete-time stationary and typically nonminimum-phase. When sampled at higher than baud rate (typically an integer multiple of baud rate), the signal is discrete-time scalar cyclostationary and equivalently, it may be represented as a discrete-time vector stationary sequence with an underlying SIMO model. With baud rate sampling, one has to exploit the higher-order statistics of the received signal either implicitly (as in [5] and [18] where direct design of equalizers is considered) or explicitly (as in [3], [6], [10], [11], [21] and [25]-[29] where the focus is on first estimating the channel impulse response using higher-order cumulants of the received signal). Higher-order statistics provide an incomplete characterization of the underlying non-Gaussian process. Joint channel and data estimation using maximum-likelihood and related approaches may be found in [9], [17] and references therein where a complete (non-Gaussian) probabilistic characterization of the signal is exploited. Computational complexity of these algorithms (explicit higher-order statistics and joint channel-data estimation) is large when the ISI spans many symbols (as in telephone channels) but they are relatively simple when ISI span is short (as in mobile radio channels). However, they may suffer from local convergence problems.

When there is excess channel bandwidth, baud rate sampling is below the Nyquist rate leading to aliasing and depending upon the symbol timing phase, in certain cases, causing deep spectral notches in sampled, aliased channel transfer function [4]. This renders the equalizer performance quite sensitive to symbol timing errors. Initially, in the trained case, fractional sampling was investigated to robustify the equalizer performance against timing error. More recently, in the blind context, it was discovered (see [24] and references therein) that oversampling provides some new information regarding the channel, which can be exploited for



blind channel estimation and equalization as long as some technical conditions are satisfied (the notorious “no common subchannel zeros” condition, also called channel disparity, for the underlying equivalent SIMO model). A similar SIMO model results if multiple sensors are used with or without fractional sampling. The work of [24] has spawned intense research activity in the use of second-order statistics for blind identification and equalization. It should be noted that the requisite technical conditions for applicability of these approaches are not always satisfied in practice; some examples are in [26].

In this chapter, we discuss higher-order statistics based approaches to blind channel estimation and equalization, with focus on estimation of the channel impulse response, for both single-user as well as multi-user systems, and both baud-rate as well as fractional-rate sampling. We consider a baseband equivalent discrete-time model with finite impulse response (although precise knowledge of the channel length is not crucial). Given the mathematical model, there are three broad classes of approaches to channel estimation, the distinguishing feature among them being the choice of the optimization criterion. All of the approaches involve (more or less) a least-squares error measure. The error definition differs, however, as follows:

- *Inverse Filter Error*: Filter (equalize) the data by an inverse filter (inverse to the channel in the noise-free, single-user case) and then minimize (or maximize) some function of the inverse filter output. The approaches of [5], [18] and [19] fall in this category. Typically the cost function is optimized with respect to the inverse filter coefficients (equalizer taps) and the optimized inverse filter output is the equalized output. This class of solutions results in a nonlinear optimization problem. Convergence to a local, non-desirable extremum is a possibility if (practical) finite length equalizers are used although schemes utilizing variable length equalizers exist to alleviate this problem [19]. On the other hand, precise knowledge of the channel length is not needed to apply these approaches. Inverse filter criteria based approaches utilize higher-order statistics implicitly only. They are treated in more detail elsewhere in this book and, therefore, will not be considered in any detail in this chapter except for the multi-user case discussed in Sec. 1.4.
- *Fitting Error*: Match the model-based statistics to the estimated (data-based) statistics in a least-squares sense to estimate the channel impulse response, as in [27], for example. This approach allows consideration of noisy observations. In general, it results in a nonlinear optimization problem. It requires availability of a good initial guess to prevent convergence to a local minimum. It yields estimates of the channel impulse response.
- *Equation Error*: It is based on minimizing an “equation error” in some equation which is satisfied ideally. The approaches of [3], [6], [21], [22] and [23] fall in this category. In general, this class of approaches results in a closed-form solution for the channel impulse response so that a global extremum is always guaranteed provided that the channel length (order) is known. These