许协庆
水动力学论文集

中国水利水电科学研究院 编

中国水利水电出版社

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内 容 提 要

本书的主要内容包括用计算方法求解工程中的水力学问题，所涉及的方面包括涡线的形成和消散；计算明渠急流的特征线法；用三组特征曲线计算河床演变；边界积分方法计算沿进水口、闸墩头部和边墙突体的压力分布；可变区域有限元法计算自由面重力流；特征线有限元法计算双曲型方程和明渠急流问题；优化差分格式计算泊松方程；固液两相流中的空泡溃灭计算；明渠急流的特征有限元解；用于对流扩散方程的高精度优化差分格式；射流对河床冲击作用的计算，水机转轮和蜗壳内部的层流和湍流计算；冲坑内水流扩散的数值模拟等。书中的计算结果可供参考，所提出的方法也可用来计算其他问题。

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序　言

许协庆先生的论文集终于编辑出版了，我应邀作序，感到很高兴。我和许先生相识甚早，70年代末，作为著名的水力学家，他被聘请为清华大学水利系兼职教授，自此我们接触的机会更多。他在水力学，尤其是计算水力学方面的创造性工作，给我留下了深刻的印象。许先生为人直率、忠厚，学风严谨、朴实，我们之间学术上和个人间的联系都使我受益匪浅。

许协庆先生原籍江苏南京，1918年出生于北京。他的父亲是我国早期留学日本的学生，不幸49岁时英年早逝，当时许协庆先生方5岁，母亲在困难的条件下将他抚养成人，并鼓励他刻苦攻读，这些成了他终生奋斗的动力。1936年他以优异的成绩毕业于江苏省立南京中学，并在全省中学会考中名列第五。同年8月，考入上海交通大学土木工程学院就读。次年，日本侵略军进攻上海，11月他随亲戚去汉口。1938年转入当时已迁至湖南湘潭的交通大学唐山工程学院土木系，不久即随该校去贵州平越（今福泉）。在迁校过程中，他和其他无经济来源的沦陷区学生，常每日步行几十公里去上学，这也锻炼了他吃苦耐劳的能力。1940年大学毕业后任助教两年，后历任岷江电厂清水溪灌溉工程处工务员，宜宾电厂助理工程师等职。1945年他考取在重庆招考的公费留美研究生，1946年去美国艾奥瓦（Iowa）大学力学与水力学系就读。1947年获硕士学位，1950年获博士学位。受聘为艾奥瓦水力学研究所副研究员，从事漩涡与湍流方面的研究。1952年去纽约，先后任工程公司设计工程师，纽约市立大学助理教授等职。中华人民共和国成立后，他于1955年7月回到了阔别9年的祖国，满怀热情地投入到新中国的建设行列。30多年来，他一直在水利水电科学研究院任高级工程师（教授级），他把全部知识和精力献给了我国水利水电方面的研究工作。许协庆先生曾任中国力学学会第三届理事会理事，北京力学学会三届理事会常务理事，现任国际水力学会能量交换流体现象分部委员。

这本论文集共收集了63篇论文，按发表时间先后排序，大致可分为四个阶段。第一阶段包括两篇论文（1～2），是作者在美国时，在他的导师C. J. Posey和Hunter Rouse指导下完成的，研究了涡的发生和发展过程。第二阶段包括11篇论文（3～13），是他回国后到60年代对国内水利水电工程问题的研究成果。第3篇论文中所介绍的水锤压力图解法，成为当时一种常用的简明而有效的方法。其余论文内容主要是围绕各种水利设施的水流计算，给出了压力分布和初生空化数的计算方法，以及特征线解法的研究。第13篇是第12篇论文的译文。第三阶段包括“文化革命”后期至80年代以前的4篇论文（14～17），可以看成是前一阶段工作的继续和新的工作的开始。其中第15篇论文是国内首次把在弹性力学中发展起来的有限元理论应用于流体力学计算，也是首次用数值方法计算了自由水面问题。第18篇以后属于第四阶段的工作。从1982～1992这短短的11年中，他和他的助手和学生们总共发表了46篇（18～63）论文，这当然和这段时间有良好的科研环境有关。也是他对科

学研究不倦追求的结果。这一时期，他的论文涉及到明渠流、水轮机内部流场等广泛的研究领域以及许多工程实际问题的计算，同时还提出了变区域有限元，特征线有限元及曲率缓变曲线等具有很高学术和应用价值的方法。

许协庆先生是我国计算水力学的先驱。60年代起，他用数值计算方法求解各种水力学问题，如沿任意曲线边界的水流压力分布问题，空化和空蚀问题，提出计算河床演变的三组特征线法等。80年代以后他先后提出了变区域有限元，特征线有限元等一系列理论，把他的方法推到了新的高度。许协庆先生还是一位教育家，作为清华大学、河海大学的兼职教授，他曾为学生授课，在任职和兼职单位共培养出硕士13名，博士8名。

许协庆先生的论文集即将出版，我向他表示衷心的祝贺。相信许先生这本论文集的出版，必将推动我国水力学研究特别是计算水力学的研究更迅速的发展。

董曾南

1994年元月

前　言

许协庆先生是我国著名的计算水动力学专家。他早年就学于交通大学上海工程学院和交通大学唐山工程学院。1946年去美国艾奥瓦大学就读，1947年获硕士学位，1950年获博士学位，导师H. Rouse。毕业后先后任该校水力学研究所副研究员，工程公司设计工程师以及纽约市立大学助理教授之职。1955年回国后，一直在水利水电科学研究院工作。多年来，他把现代计算技术应用来解决水利水电工程中的实际问题和理论问题，为推动计算水动力学的发展和应用作出了卓越的贡献。

许协庆先生也是一位教育家，他除了担任中国水利水电科学研究院教授级高级工程师从事科研工作以外，还作为清华大学和河海大学的兼职教授，至今已培养出博士和硕士各十余名。

许协庆先生曾担任北京力学学会常务理事，中国力学学会理事，国际水力学会能量交换分部委员。他曾参加编写了中国大百科全书出版社出版的《中国大百科全书·力学卷》，《力学词典》，高等教育出版社出版的《工程力学手册》等。他的名字被列入中国名人研究中心编的《中国当代名人录》（1991、1993年版，上海人名出版社），《中国电力人物志》（1992年，水利电力出版社），《中国现代水利人物志》（1994年，水利电力出版社）以及美国传记中心（American Biographical Institute）和美国国际传记中心（International Biographical Center）出版的几种名人辞典。

张世雄

1996年

目　录

How the Vortex Affects Orifice Discharge

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To what extent does the vortex affect discharge through an outlet? Will it, once started, grow to serious proportions? These uncertainties often cause designers of vertical shaft spillways and other outlets to call for expensive structures to eliminate the vortex and insure radial flow.

Studies recently completed at the State University of Iowa shed new light on those fundamental questions which have kept designers from designing for—instead of against—the vortex.

A 6-ft dia. cylindrical tank with a 4-in. sharp-edged, circular orifice at the center of its bottom was set up with provision for making the inflow radial, tangential or any combination of the two.

It was found that when inflow has a tangential component, a stable vortex forms which has a definite effect on discharge. If the tangential component of inflow is shut off after the vortex has become stable, it will soon die out. With purely radial inflow the vortex is small and transitory; its effect on discharge is negligible.

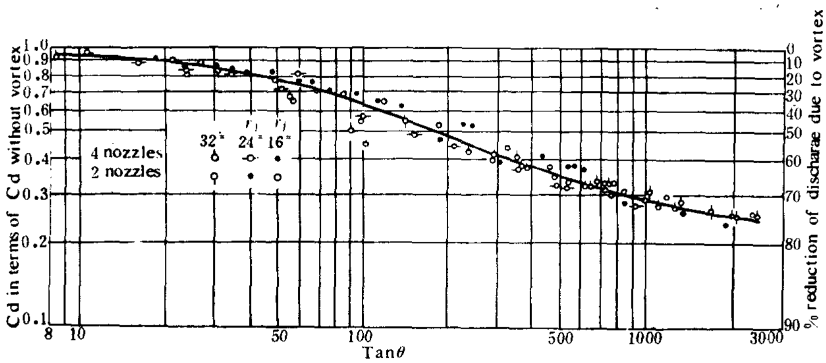
The tank used in all tests had a false bottom, below which radial flow Q_r was guided in through a baffle and radial vanes. Tangential flow Q_t was directed into the tank immediately above the false bottom through four 1-in. pipes. These branched from a 3-in.-dia. pipe above the tank, extending out, then down to the outer edge of the tank's false bottom to follow its periphery for 30 in. They could be swung about their vertical legs to vary the distance of tangential inflow jets from the orifice.

Three piezometer connections, made 13 in. from the orifice center, were connected by tube to a stilling well where water surface elevation was measured by a hook gage. Experiments were done under constant head—1.63 ft. Variables were the tangential and radial discharge, the number of nozzles, and radius of the nozzle jet circle, r_j, from center of orifice.

Six sets of runs were done for different combinations of nozzles, 2 or 4, and the distances r_j, 16 in., 24 in. or 32 in. In each set the tangential and the radial flow were varied. From total inflow and head the coefficients of discharge C_d were computed.

It was found that (regardless of the number of nozzles or their distance from the center) the size of the stable vortex that forms depends upon the ratio of the average tangential component of velocity to the average radial component of velocity. This ratio equals the tangent of θ, the angle that the resultant velocity makes with the radial. And tan θ is constant for any radial distance from the center of orifice, if no appreciable tangential force or momentum is introduced.

The relationship between tan θ and the reduction in the coefficient of discharge is shown by the curve below. This relationship can be used to estimate the economy of using radial guide vanes to eliminate the vortex. Also, where the vortex is used to decrease orifice discharge, its effect can be estimated from the results plotted above.



Orifice discharge drops, as strength of vortex above the orifice—measured in terms of tan θ-increases

The tests described were made at the Iowa Institute of Hydraulic Research as a part of the graduate training program of the department of mechanics and hydraulics. Prof. Hunter Rouse is institute director and Prof. J. W. Howe is department head.

On the Growth and Decay of a Vortex Filament*

Hunter Rouse Hsieh-ching Hsu

STATEMENT OF THE PROBLEM

KNOWLEDGE of the variation in velocity and pressure throughout the field of a vortex filament is of basic importance in various phases of fluid mechanics. On the one hand, the formation of vortices in the wake of a cylindrical body is directly related to the variable force exerted upon the body, and the subsequent diffusion and dissipation of the vortex energy are essential factors in the study of wakes in general. On the other hand, although present-day analysis of fully developed turbulence no longer emphasizes the vortex nature of the turbulence structure, evaluation of instantaneous pressure fluctuations from a temporal velocity record may well be facilitated by consideration of the elementary vortex pattern. This possibility becomes the more worthy of attention with recognition of the fact that success in predicting incipient cavitation in zones of turbulence generation depends primarily upon the accuracy with which minimum local pressures can be foretold.

Since the time of Rankine[1] the velocity and pressure fields in the vicinity of a vortex filament have been expressed in terms of his so-called "combined vortex." As indicated in Fig. 1, this idealization consists of a central rotational zone (or "forced" vortex) of constant vorticity, in which the velocity varies directly with radial distance from the axis, and a surrounding irrotational zone (or "free" vortex) of constant circulation, in which the velocity varies inversely with radial distance. As described in most textbooks on the subject[2], the resulting distribution of pressure—or elevation of a free liquid surface to which the axis is normal—will have the form shown in the figure, the maximum pressure intensity or surface elevation being that of the undisturbed fluid at an infinite radius and the minimum occurring at the centerline and depending upon the diameter and vorticity of the central filament.

Evident from examination of the structure of Rankine's combined vortex is the partial independence of the two zones of flow. In other words, while the circulation of the irrotational zone and the diameter and vorticity of the rotational zone are uniquely interrelated, the same irrotational field may accompany filaments having an infinite variety of diameters and vorticities. A question hence arises as to just what governs the size of an actual vortex—and, more-

* 本文刊载于 *Proceedings of the first national congress of applied mechanics*, 1951.

over, to what extent such a vortex differs from Rankine's idealization. Although it is indicated in a few references[3] that the existence of viscous shear necessarily modifies the velocity pattern somewhat in accordance with the heavy broken lines in Fig. 1, systematic measurements of the phenomenon appear to be wholly lacking. In fact, the only example of vortex motion which is conveniently subject to measurement is that which occurs over an open drain in a shallow tank; this, however, depends for its stability upon the existence of a radial as well as a tangential component of flow, which makes it inapplicable except in a general way to the problem at hand.

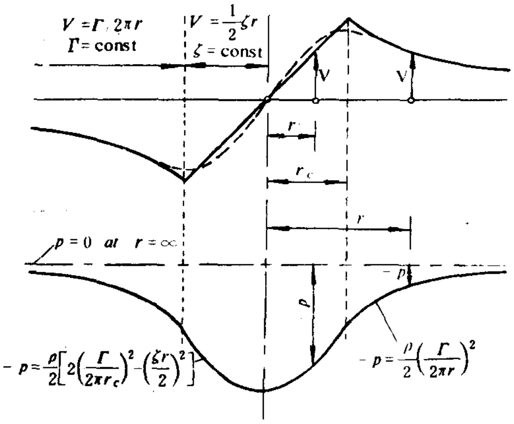


Fig. 1. Velocity and pressure fields expressed in terms of rankine's "combined vortex"

If, as suggested by the broken line in Fig. 1, one considers the development of a vortex to result from viscous shear, two existing analyses for rather idealized boundary conditions warrant at least preliminary examination. The first, by Lamb[4], assumes an irrotational field of constant circulation to exist initially, and the subsequent alteration of the velocity field is expressed as a function of time through use of the analogy between conduction of heat and diffusion of vorticity. The second, by Goldstein[5], assumes a fluid body initially at rest but containing an isolated filament of constant diameter and vorticity, the effect of which upon the surrounding velocity field is likewise expressed as a function of time, although in the form of a differential equation of which the solution is not expressible in terms of elementary functions.

Consideration of the two analyses as means of describing vortex behavior at once reveals serious departures from the actual state of motion. In the first, a question immediately arises as to the original source of the irrotational field; this becomes the more problematic when it is realized that the initial kinetic energy of such a field is of infinite magnitude, and that the ultimate dissipation must hence also be infinitely great. The method has nevertheless been used by Hooker[6] as a basis for studying the diffusion of the Kármán vortex trail. In the second analysis, the postulation of a finite core of constant vorticity (in effect, a solid cylinder of unchanging rotational speed) is likewise highly artificial, except perhaps as merely the initial phase of development. In the latter event, no solution has been found in the literature which might characterize the further behavior of such a filament once its vorticity was permitted to change.

In view of these circumstances, an attempt has been made by the writers to formulate in accordance with the Navier-Stokes equations a reasonable expression for the growth and decay of an isolated vortex filament. Three specific requirements were given due consideration in this analysis: First, continuity of the function with respect to both time and space; second, agreement with actual vortex characteristics beyond some reference stage; and, third, expressibility in terms of significant parameters. As described in the following sections, these ends were attained through three successive steps. The velocity field produced in an initially still fluid by the action of a single line vortex was first expressed as a function of vortex strength, fluid viscosity, and passage of time. The modification of this field occurring after abrupt elimination of the generating vortex was then evaluated in terms of the original strength and length of generation, the viscosity, and the continued passage of time. Finally, the resulting function was rewritten in terms of parameters depending upon the centerline vorticity, core diameter, and circulation of an actual vortex at some instant of its existence, the viscosity, and the passage of time beyond this instant.

DEVELOPMENT OF THE PRIMARY FUNCTION

For motion in concentric circles about the origin of polar coordinates, the Navier-Stokes equations for the tangential and the radial directions reduce to the following linear forms:

$$\frac{\partial v}{\partial t} = \nu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right) \tag{1}$$

$$\frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} \tag{2}$$

In the case of an infinitely long generating cylinder of radius r_g which is suddenly brought into rotation with the peripheral velocity v_g, the instantaneous fluid velocity at any radial distance from the axis will be, according to Goldstein[5],

$$v = \frac{v_g r_g}{r} + \frac{2v_g}{\pi}\int_0^\infty \exp(-\nu t x^2)\,\frac{J_1(xr)Y_1(xr_g) - Y_1(xr)J_1(xr_g)}{J_1^2(xr_g) + Y_1^2(xr_g)}\,\frac{\mathrm{d}x}{x} \tag{3}$$

in which J_1 and Y_1 are first-order Bessel functions of the first and second kind, respectively, and x is any variable.

Let it first be assumed that r_g approaches zero in such a manner that the product $v_g r_g$ maintains the constant value $\Gamma_g/2\pi$, the limiting magnitude of the integral then being evaluated. In determining the limit of the integrand, the following relationships are useful:

$$J_1(0) = J_1(\infty) = Y_1(\infty) = 0 \qquad Y_1(0) = \infty \tag{4}$$

$$Y_1(z) = \frac{1}{\pi}\left[2\left(\gamma + \ln\frac{z}{2}\right)J_1(z) - \frac{2}{z} - \sum_{m=0}^{\infty}\frac{(-1)^m(z/2)^{1+2m}}{m!(m+1)!}\right.$$
$$\left.\times\left(\frac{1}{1} + \frac{1}{2} + \cdots\frac{1}{m} + \frac{1}{1} + \frac{1}{2} + \cdots\frac{1}{1+m}\right)\right] \tag{5}$$

in which γ is the Euler constant. The limit of $r_g\ Y_1(xr_g)$ as r_g approaches zero may be expressed as follows:

$$\lim_{r_g\to 0} r_g Y_1(xr_g) = \frac{1}{\pi}\lim_{r_g\to 0}\left[2r_g\ln\left(\frac{xr_g}{2}\right)J_1(xr_g) - \frac{2r_g}{xr_g}\right] = -\frac{2}{\pi x} \tag{6}$$

Hence, in view of Equation (4),

$$\lim_{r_g\to 0}\frac{v_g J_1(xr)Y_1(xr_g) - v_g Y_1(xr)J_1(xr_g)}{J_1^2(xr_g) + Y_1^2(xr_g)} = \lim_{r_g\to 0}\frac{r_g v_g J_1(xr)}{r_g Y_1(xr_g)} = -\frac{\Gamma_g x J_1(xr)}{4}$$

Upon substitution of this limiting value into Equation (3), it is found that

$$v = \frac{\Gamma_g}{2\pi r} - \frac{\Gamma_g}{2\pi}\int_0^\infty \exp(-\nu t x^2)J_1(xr)\mathrm{d}x$$

which through use of the formulas

$$J_1(z) = -\frac{\mathrm{d}}{\mathrm{d}z}J_0(z) \tag{7}$$

and

$$\int_0^\infty J_0(ax)\exp(-p^2x^2)x\mathrm{d}x = \frac{1}{2p^2}\exp\frac{-a^2}{4p^2} \tag{8}$$

may be integrated to yield

$$v = \frac{\Gamma_g}{2\pi r}\exp\frac{-r^2}{4\nu t} \tag{9}$$

If the factor t be considered to approach the limit of infinity, Equation (9) will, paradoxically, become identical to that for the line vortex in an irrotational field of constant circulation. Evidently, this corresponds to an infinite input (as well as dissipation) of energy through everlasting rotation of the generating filament. The resulting velocity distribution, of course, is that indicated for the irrotational zone in Fig. 1, which approaches the limit of infinity as the radius approaches zero. If, on the other hand, the factor t is held to a finite value, the field surrounding the filament becomes restricted in its practical extent—even though the energy input is still infinite because of the infinite velocity at the axis.

Unlike the limiting case of the field of constant circulation, the flow characteristics of this field of decreasing circulation involve two parametric values: the circulation Γ_g of the generating filament and the time t_g during which it is assumed to operate. If, at the end of the generation period, the hypothetical filament suddenly ceases to exist, the parameters Γ_g and t_g will still characterize the subsequent dissipation. The analysis of the dissipation process over a particular time t_d is then reduced to finding a solution of Equation (1) which satisfies the condition

$$v = \frac{\Gamma_g}{2\pi r}\exp\frac{-r^2}{4\nu t_g} \tag{10}$$

for $t_d=0$, and the condition $v=0, r=0$ for $t_d>0$.

One may show, through separation into two ordinary differential equations, that Equation (1) has a solution of the type

$$v = \exp(-\lambda^2 \nu t) J_1(\lambda r)$$

Since Equation (1) is linear, any linear combination of similar terms will also represent a solution. It is known, moreover, that any function subject to suitable restrictions may be expressed as a definite integral of the form

$$F(r) = \int_0^\infty \lambda \mathrm{d}\lambda \int_0^\infty F(x) J_1(\lambda x) J_1(\lambda r) x \mathrm{d}x$$

Thus, if $F(x)$ is replaced by $(\Gamma_g/2\pi x)\exp(-x^2/4\nu t_g)$, there results

$$\frac{\Gamma_g}{2\pi r}\exp\frac{-r^2}{4\nu t_g} = \frac{\Gamma_g}{2\pi}\int_0^\infty \lambda \mathrm{d}\lambda \int_0^\infty \exp\frac{-x^2}{4\nu t_g} J_1(\lambda x) J_1(\lambda r)\mathrm{d}x$$

The solution which satisfies the required conditions for $t_d=0$ and $t_d>0$ is

$$v = \frac{\Gamma_g}{2\pi}\int_0^\infty \lambda \exp(-\lambda^2 \nu t) J_1(\lambda r)\mathrm{d}\lambda \int_0^\infty \exp\frac{-x^2}{4\nu t_g} J_1(\lambda x)\mathrm{d}x$$

which, upon integration in accordance with Equations (7) and (8), assumes the desired form

$$v = \frac{\Gamma_g}{2\pi r}\left(\exp\frac{-r^2}{4\nu(t_g+t_d)} - \exp\frac{-r^2}{4\nu t_d}\right) \tag{11}$$

a generalized plot of which is shown in Fig. 2.

This equation indicates the radial distribution of velocity throughout the zone of motion at any time t_d for given values of the original parameters Γ_g, t_g, and ν. From it may be derived the corresponding functional relationships for the distributions of circulation, vorticity, and pressure, as well as the nominal size of the vortex core and the total kinetic energy of the motion. As such, Equation (11) represents the primary dissipation function, which serves as the basis of the subsequent interpretative reasoning.

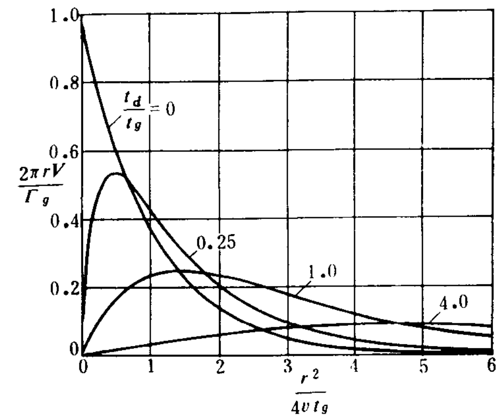


Fig. 2. Generalized flot indicating the radial distribution of velocity throughout the zone of motion at any time t_d for given vaiues of Γ_g, t_g, and ν

METHOD OF APPLICATION TO SPECIFIC CONDITIONS

Although the generation of an actual vortex is surely triggered by viscous action, and although dissipation just as surely begins with the onset of the generation process, in its early stages the latter may still be susceptible to approximate analysis as a case of unsteady potential flow—at least for fluids of small viscosity. For example, the characteristics of a vortex shortly after its generation would appear to depend upon three independent factors: the scale of the generating mechanism (such as a paddle or a wing tip), the relative velocity of the motion, and either a time interval or a displacement. (Whether this would indicate three, or only two, degrees of freedom in the vortex characteristics must remain for the present a moot question.)

Under such circumstances, Equation (11) seems at first glance to represent little more than a hypothetical case of flow, for the parameters Γ_g, t_g, and t_d are too artificial to have counterparts in the actual generation process. However, other types of flow have been described successfully in terms of parameters which have no physical significance beyond their usefulness as reference parameters. In other words, although the actual generation process may well differ radically from the sequence of conditions embodied in the primary function, the flexibility of this function may equally well permit it to represent with satisfactory approximation the state of motion at some later time. All subsequent phases of the motion would then be predictable through an extension of this function.

The proposal is therefore made that—until a more exact solution becomes available—Equation (11) be considered to represent the velocity distribution of a vortex at some instant after its generation, and that the three parameters Γ_g, t_g, and t_d be expressed in terms of three conveniently measurable and significant characteristics. Those proposed are the centerline vorticity ζ_0, the maximum circulation Γ_0 and the nominal core radius r_0.