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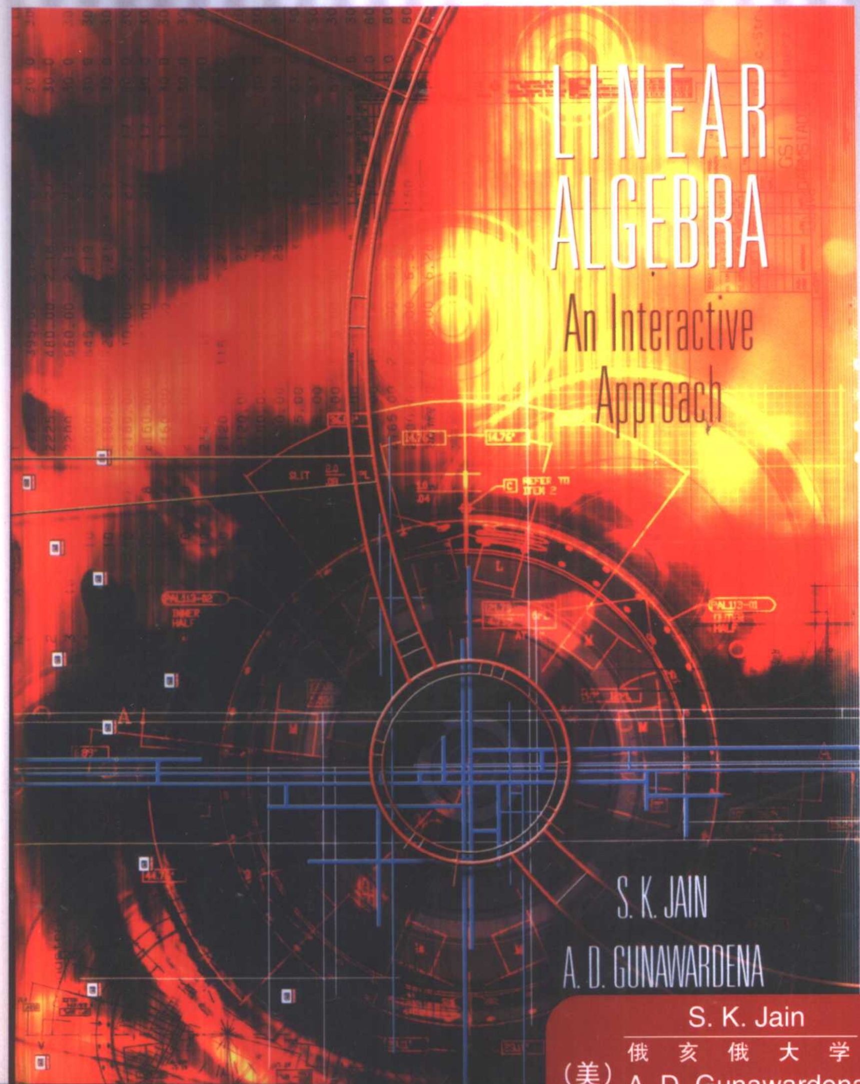

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机械工业出版社
China Machine Press

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Linear Algebra: An Interactive Approach

（美） S. K. Jain 俄亥俄大学
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Preface

Introduction

The purpose of this book is to provide an introduction to linear algebra, a branch of mathematics dealing with matrices and vector spaces. Matrices have been introduced here as a handy tool for solving systems of linear equations. But their utility goes far beyond this initial application. There is hardly any area of modern mathematics in which matrices do not have some application. They have also many applications in other disciplines, such as statistics, economics, engineering, physics, chemistry, biology, geology, and business.

The present text in linear algebra is designed for a general audience of sophomore-level students majoring in any area of art, science, or engineering. The only prerequisites are two or three years of high school mathematics with some knowledge of calculus. A special feature of this book is that it can be used in a course taught in a traditional manner as well as in a course using technology. Those using technology may refer to complete solutions of selected exercises (marked as drills) using Matlab at the end of the book, while others using Maple or Mathematica may refer to corresponding solutions on the web page. The readers would find the examples and solutions to drills using technology quite helpful and illustrative to solve similar problems. Concepts and practical methods for solving problems are illustrated through plenty of examples. The theorems and facts underlying these methods are clearly stated as they arise, but their proofs are provided in a separate section called "Proofs of Facts" near the end of the chapter.

Text Organization

The subject matter is laid out in a leisurely manner with plenty of examples to illustrate concepts and applications. Most of the sections contain a fairly large number of exercises, some of which relate to real-life problems. Chapters 1 and 2 deal with linear systems, leading naturally to matrices and to algebraic operations on matrices. Chapter 3 introduces the vector space F^n of n-tuples, linear dependence and independence of vectors. The concept of rank is introduced in Chapter 4 and is followed by more applications of elementary row operations in Chapter 5. Specifically, we show in Chapter 5 how to find the inverse of an invertible matrix, the LU-decomposition and full rank factorization of a matrix. Chapter 6 provides a working knowledge of determinants, which are later considered in a rigorous fashion in Chapter 10. Eigenvalue problems and inner product spaces are given in Chapters 7 and 8, respectively. An interesting feature of Chapter 7 is a method for finding eigenvalues without using determinants. Two methods for finding the least-squares solution of an inconsistent linear system are given in Chapter 8. The first method is geometric and uses the notion of shortest distance: the second is algebraic and uses the concept of generalized inverses. Vector spaces are revisited in Chapter 9. where a formal definition of a vector space is given and important examples of vector spaces of functions are considered.

In addition to the answers to all exercises, we have provided hints or solutions to selected ones. Complete Matlab solutions have been provided for the exercises that are marked as drills. The student's solution manual that contains solutions to the odd-numbered exercises is available separately as is the complete solution manual for the instructor.

Electronic Text on a CD

This text–CD package includes a CD that contains the entire contents of the book neatly organized into an electronic text. In addition, the CD also contains concept demonstrations, Matlab drills, solutions, projects, and chapter review questions. The electronic material is supported by a well-designed graphical user interface that allows the user to navigate to any part of the text by clicking the mouse. The ability to read the text electronically, find any topic you need, do an on-line test, or perform a drill activity using Matlab makes this electronic text a useful instrument for learning linear algebra. We believe that the use of technology in mathematics enriches the learning experience and encourages exploration of computationally hard problems that might not be easy to solve by hand. The experience of using technology to solve mathematical problems is an essential skill for today's graduates entering a high-tech dominated world. Although we have chosen Matlab as our technology tool, the printed text can also be used in a traditional setting. For the instructor who is interested in the use of technology in teaching, this CD contains a wealth of material for teaching linear algebra in a computer lab setting. It provides an interactive environment that encourages a hands-on approach.

Suggestions for Implementation

The book is suitable for a one-semester course in linear algebra. It may also be used for a one-quarter course by skipping certain sections in Chapters 5–10. We suggest the following guidelines for teaching the course in a computer lab setting. First, begin with demos in the electronic textbook to illustrate a new concept. Then advise the students to read related material in the text to reinforce the concept. Next, explain the Matlab operations needed in a given chapter. The list of basic Matlab operations is provided not only on the CD but also in the printed text. Pick a drill in a chapter and work it out fully in the class, and assign Matlab exercises to work in the lab or at home. Exercises marked as drills with a picture of a CD have Matlab solutions in the CD as well as at the end of the printed text. Solutions of drills using Mathematica/Maple are given on the Web page. Finally, encourage students to do the projects using Matlab.

Acknowledgments

We would like to thank, among others, Jonathan Golan, Dinh V. Huynh, Pramod Kanwar, Sergio R. Lopez-Permouth, S. R. Nagpaul, M. S. K. Sastry, and Charles Withee for using the manuscript of this book in their traditional or on-line classes and for their helpful comments. We would like to express special thanks to Pramod Kanwar for his assistance various ways, including the preparation of the CD.

A number of people have played an important role in the production of the book. Among them are John Tynan of Marietta College, Ohio; Yong UK Cho of Silla University (Korea); graduate students Adel Al-Ahmadi and Hussain Al-Hazmi who have been of great assistance in the tedious task of proofreading; and Larry E. Snyder and Roland Swardson for their help in editing some portions of the manuscript. We also mention with great pleasure the cooperation and assistance offered by individuals at Brooks/Cole, especially our publisher Bob Pirtle, editor John-Paul Ramin, and assistant editor Molly Nance.

S. K. Jain would like to thank his wife and his family; A. D. Gunawardena would like to thank his family, Sriya, Neomi, and Naoka, for their understanding and patience.

We wish to acknowledge the inspiration given by the late Professor P. B. Bhattacharya in initiating this project. We dedicate this book to his memory.

Finally, we are very grateful to the following who have reviewed this text and offered valuable comments: Roman Hilcher, Michigan State University; Michael Tsatsomeros, Washington State University; Jane Day, San Jose State Unversity; David Miller, William Paterson University; and Alan Schuchat, Wellesley College.

S. K. Jain

A. D. Gunawardena

Notes to Students

Chapter 1

An important activity that occurs throughout the book is reducing a system of linear equations to a system that is readily solvable. This is equivalent to reducing a certain matrix into a specific form, known as row echelon form, discussed in this chapter. Sections 1.1 and 1.2 guide the reader to the central theme of the book: reducing a matrix into row echelon form. Most instructors would come to Section 1.3 quickly after just giving a glimpse of the material discussed earlier. This is fine because it allows time to cover more topics. You will be learning how to solve a linear system of equations by Gauss elimination method. We suggest that you familiarize yourself with (1) the three elementary row operations, (2) the steps for reducing a matrix to row echelon form, and (3) the method for solving a linear system by using backward substitution, as discussed in Section 1.4. Reducing the augmented matrix into reduced echelon form is not necessary. However, if you do so, you will get the solution(s) without using the backward substitution method. In practice, doing backward substitution amounts to less work than reducing a row echelon matrix further to reduced row echelon form.

Chapter 2

Matrix addition, matrix subtraction, and scalar multiplication are defined naturally, and these operations are easy to apply. But matrix multiplication, although motivated by linear systems of equations for finding their solutions, does not appear natural, and you might find it difficult to appreciate the formula for multiplication of matrices. We suggest that you practice matrix multiplication problems and work on the many interesting applications to real-life problems given in worked-out examples and in exercises.

Chapter 3

You will find this chapter challenging. Some of you might not like the abstract definition of subspace and proving or disproving whether a given subset is a subspace or not. If you get an intuitive picture of a subspace, you may proceed and come back as needed. Linear independence and linear dependence are central concepts that are discussed in this chapter and will appear throughout the book. The definitions of these concepts are easy, but still you may have difficulty in reproducing the definitions correctly and applying them to problems. There is no magic prescription! You simply go through many worked-out examples in the book and solve exercises. Other concepts discussed in this chapter are basis and dimension. Finding a basis of a subspace may require long computations. We suggest that you work on the examples in the book and practice some problems. A much shorter method of finding a basis of a subspace is given in Chapter 4 after more machinery has been developed. The proofs of facts contained in Section 3.4 are nontrivial.

Chapter 4

Computing the rank of a matrix is easy once you are given its definition and the procedure to compute it. Computing the nullity of a matrix is straightforward but involves somewhat long computations. Nullity of a matrix turns out to be the number of arbitrary unknowns in the solution set of a homogeneous linear system whose coefficient matrix is the given matrix. The rank-nullity theorem connects the rank of a matrix with its nullity by stating "rank + nullity = number of columns." This fact has many applications in linear algebra. It can be used, for example, to compute the nullity once you know its rank unless you are required to compute it by using definition only. You will need to know the effect of performing elementary row or column operations on a matrix as given in the last section. This will be needed to understand the theoretical reason behind computing the inverse of a matrix and other factorizations discussed in Chapter 5. If you skip this section or forget the results contained therein, you might still be able to compute the inverse and perform factorizations correctly by following the steps given in the text but you wouldn't know the reasons behind your work.

Chapter 5

This chapter is straightforward. Read the steps to compute the inverse and to obtain various factorizations and practice some problems. Read the properties of the inverse, especially about the inverse of the product of invertible matrices.

Chapter 6

You will find this chapter as easy as Chapter 5. You will often be using the theorem on expanding the determinant with respect to any row or column (Theorem 6.3.3) and properties of determinants. To find the determinant of numerical matrices, you will mostly be using the method of reducing the matrix into row echelon form. You are urged to read and remember the properties of determinants, as they are heavily used in expanding determinants to reduce the amount of calculations.

Chapter 7

Computations of eigenvalues and eigenvectors are straightforward once you know their definitions. Two methods are presented in the book. One is traditional as given in Section 7.2; the other method given in Section 7.3 is not commonly given in texts, but we recommend that you read both methods and then choose which one you prefer. Indeed, you might find it helpful to know both techniques, because one technique may be superior to the other for a given problem. Read and remember the properties of eigenvalues and eigenvectors, in particular that eigenvectors corresponding to distinct eigenvalues are linearly independent. As a consequence, a matrix with distinct eigenvalues always is diagonalizable. This is a highly important fact that is used often. That every matrix is triangularizable is also proved and is illustrated with examples. This fact has many applications in matrix theory. But this topic might not be taught because of time. The Cayley-Hamilton Theorem also has many applications, but it might be skipped if time is short. We give a very short proof that we believe is new in the sense that it is not in the literature. It is extremely short and easy to follow. The traditional proof using adjoint of matrices is also given. If this theorem is taught and its applications are shown, you will find them interesting and easy to understand.

Chapter 8

This chapter has the closest connection with geometry. Memorize the Gram-Schmidt method of finding an orthonormal set. The important topics are diagonalization of symmetric matrias, singular value decomposition, applications of the principal-axis theorem, and best approximate solutions of inconsistent linear systems. Not all topics from this chapter are covered in a beginning-level course in linear algebra. However, these are very interesting topics and have applications to real-life problems. Most of these problems involve long computations, and it is helpful to use technology. You will find systemic steps given in the chapter for tackling a given problem on diagonalizing a symmetric matrix and finding a best approximate solution of an inconsistent linear system.

Chapter 9

This chapter is an abstract version of the concepts discussed in Chapter 3. It deals with abstract vector spaces and functions between them. You will appreciate it if you have a mathematical bent of mind for viewing concepts in a general setting. Not all schools might find this appropriate at your level or might not have time to teach abstract vector spaces. However, functions between concrete vector spaces known as linear mappings (also called linear transformations) are likely to be introduced at some stage. You should read the sections on linear mappings without being deterred by the abstract definition of vector space. An important fact in the theory of linear mappings is that there is one-to-one correspondence between linear mappings and matrices of suitable sizes (depending on the dimensions of vector spaces between which the linear mapping is defined). Follow the steps to work on the problems to compute a matrix of a linear mapping.

Chapter 10

This contains the abstract definition of the determinant of a square matrix introduced in Chapter 6. The proofs of various properties are given in this chapter. Not all schools might teach determinants in such a rigorous manner. Some instructors may combine Chapters 6 and 10. Read as advised. Those who are simply interested in applications of determinants can skip this chapter.

Suggested Syllabus

One-Quarter First Course in Linear Algebra

Chapter 1, Chapter 2, Chapter 3, Chapter 4, Chapter 5 (omit Sections 5.3–5.4), Chapter 6 (omit Section 6.4), Chapter 7 (Section 7.4 optional), Chapter 8 (omit Sections 8.4–8.6). Presenting all proofs may require omission of some material toward the end.

One-Semester First Course in Linear Algebra

One should be able to cover most of the material, possibly omitting some section or proofs to find more time for later chapters, and for the projects.

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1 Linear Systems and Matrices

Introduction

Mathematical modeling of real-world problems often involves a large number of variables and a large number of constraints, giving rise to linear systems of equations. Solving these linear systems requires restructuring of equations and reduction into a form that is easily solvable. In this chapter, we discuss a very useful and practical method called Gauss elimination for solving such systems. The method involves reducing a linear system or, equivalently, the *augmented matrix* of a system into a form known as *row echelon form* that readily yields a solution or solutions if such solutions exist.

1.1 Linear Systems of Equations

In plane geometry, an equation of the form

$$ax + by = c,$$

where a, b are not both 0, denotes a line, and so it is called a linear equation in variables x and y. A linear equation in three variables corresponds to a plane in three-dimensional space. When we have to write a linear equation in a large number, say n, of variables, it is convenient to list them by subscripts $1, 2, \ldots, n$ attached to the letter x. Then a linear equation in $x_1, x_2, \ldots, x_n$, is of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

When we have to write more than one linear equation in $x_1, x_2, \ldots, x_n$, it is convenient to use double subscripts for the coefficients:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1, \\ &\vdots \\ a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{ij}x_j + \cdots + a_{in}x_n &= b_i, \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m. \end{aligned} \tag{1}$$

In the above set of equations, a_{ij} is the coefficient in the ith equation of the jth variable and b_i is the constant term in that equation.

We do not read $a_{11}, a_{12}, \ldots$ as "a eleven," "a twelve," and so on; we read them as "a one-one," "a one-two," and so on, to mean that a_{11} is the coefficient in the first equation of the first variable x_1, a_{12} is the coefficient in the first equation of the second variable x_2, and so on. This notation a_{ij} is fundamental in linear algebra, and you are advised to become familiar with it.

The above set of m equations in n variables (also called unknowns), when considered as a single piece or entity, will be called an $m \times n$ linear system, or simply an $m \times n$ LS.

Solving an $m \times n$ linear system such as the one above amounts to finding a set of numbers $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that when $\alpha_1, \alpha_2, \ldots, \alpha_n$ are substituted for $x_1, \ldots, x_n$, respectively, in Equations (1), we get the m equalities

$$\begin{aligned} a_{11}\alpha_1 + \cdots + a_{1n}\alpha_n &= b_1, \\ &\vdots \\ a_{m1}\alpha_1 + \cdots + a_{mn}\alpha_n &= b_m. \end{aligned}$$

Such a list $\alpha_1, \alpha_2, \ldots, \alpha_n$ of numbers is called a solution of the $m \times n$ LS (1). The answers to questions such as whether an LS has a solution and how to solve it in the best possible way form the foundation of the subject called linear algebra.

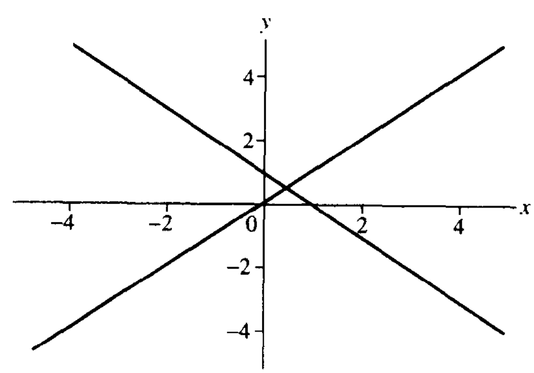
1.1.1 DEFINITION

Consistent and Inconsistent LS

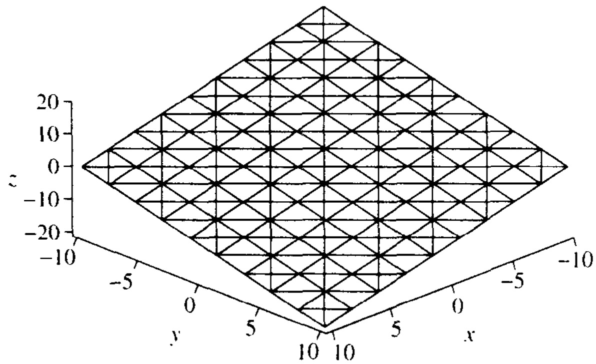
If a linear system has a solution, it is called *consistent*; if it has no solution, it is called *inconsistent*.

For example, consider the following linear systems and their graphs.

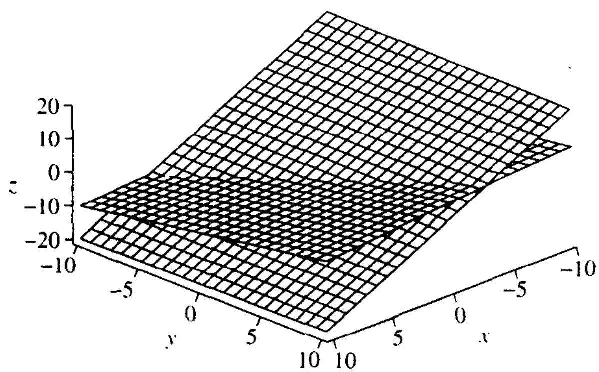
1. $x_1 + x_2 = 1,$
   $x_1 - x_2 = 0;$



2. $x_1 + x_2 + x_3 = 0;$



3. $x_1 - 2x_2 + 3x_3 = 1,$
$2x_1 + x_3 = 0;$

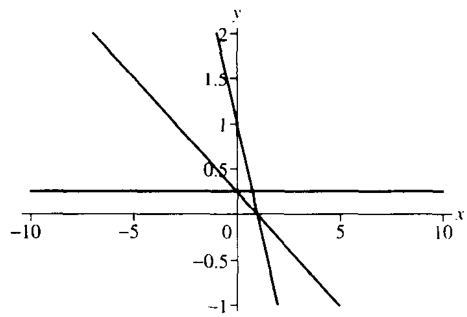


All three linear systems given above are consistent. Geometrically, the first LS represents two lines. The solution of this system is the point of intersection of the two lines in a plane. The second LS represents a plane, and every solution of the system is a point on the plane $x_1 + x_2 + x_3 = 0$. The solution of the third LS represents points on the line of intersection of the two planes. Both the second and third linear systems have infinitely many solutions.

Consider next the linear system

$$\begin{aligned} x_1 + x_2 &= 1, \\ 4x_2 &= 1, \\ x_1 + 4x_2 &= 1, \end{aligned}$$

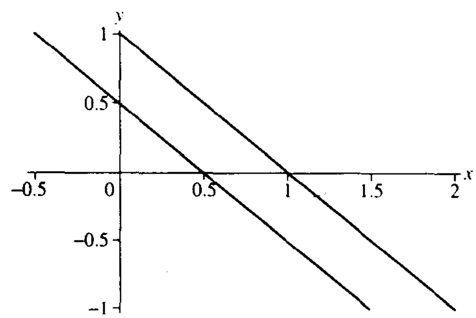
and its graph



This linear system represents three lines, and there is no point common to all of them. Thus this LS is inconsistent.

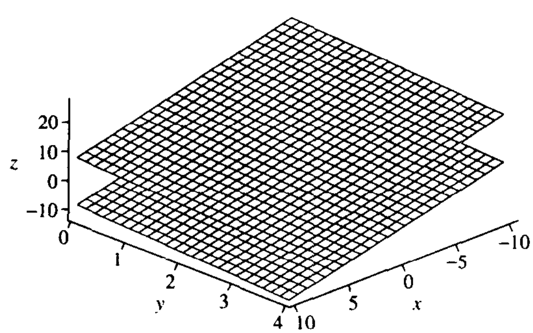
Furthermore, since any two parallel lines or planes do not intersect, the linear systems of the types

$$x_1 + x_2 = 1, \qquad 2x_1 + 2x_2 = 1;$$



and

$$x_1 + x_2 + x_3 = 1, \qquad 2x_1 + 2x_2 + 2x_3 = 35;$$



are inconsistent.