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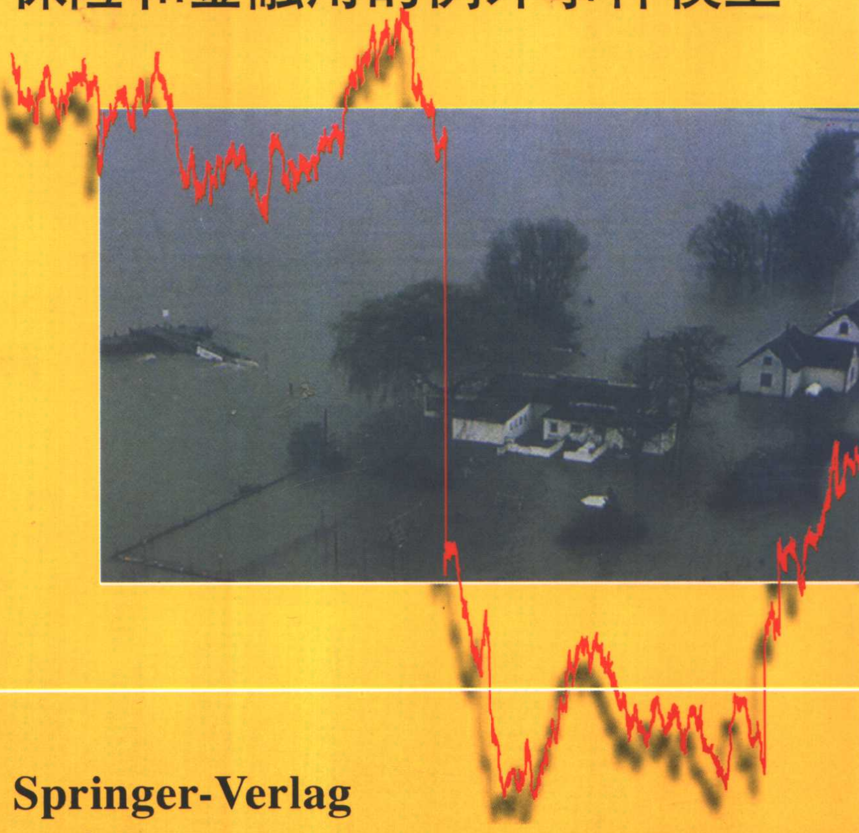
Modelling Extremal Events

for Insurance and Finance

保险和金融用的例外事件模型



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With 100 Figures

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Voor Gerda, Krispijn, Eline en Frederik.
Na al dit werk blijft één vraag onbeantwoord:
"Hoe kan ik jullie ooit danken voor de opoffering en steun?"

Paul

Meinen Eltern

Thomas

Preface

Qui scribunt libros caveant a judice multo
Cum multus judex talibus immineat

Abelard (1121)

Writers of books should beware of the verdict of the crowd
For the verdict of the crowd is prejudiced against them

Translation by Christopher Platt

In a recent issue, *The New Scientist* ran a cover story under the title: "Mission improbable. How to predict the unpredictable"; see Matthews [448]. In it, the author describes a group of mathematicians who claim that extreme value theory (EVT) is capable of doing just that: predicting the occurrence of rare events, outside the range of available data. All members of this group, the three of us included, would immediately react with: "Yes, but, ...", or, "Be aware ...". Rather than at this point trying to explain what EVT can and cannot do, we would like to quote two members of the group referred to in [448]. Richard Smith said, "There is always going to be an element of doubt, as one is extrapolating into areas one doesn't know about. But what EVT is doing is making the best use of whatever data you have about extreme phenomena." Quoting from Jonathan Tawn, "The key message is that EVT cannot do magic – but it can do a whole lot better than empirical curve–fitting and guesswork. My answer to the sceptics is that if people aren't given well–founded methods like EVT, they'll just use dubious ones instead."

These two quotes set the scene for the book you are holding. Over many years we have been in contact with potential users of EVT, such as actuaries, risk managers, engineers, ... Whatever theory can or cannot predict about extremal events, in practice the problems are there! As scientists, we cannot duck the question of the height of a sea dyke to be built in Holland, claiming that this is an inadmissible problem because, to solve it, we would have to extrapolate beyond the available data. Likewise, reinsurers have for a long time known a great deal about extremal events; in their case, premiums have to be set which both cover the insured in case of a claim, and also are calculated in such a way that in the event of a catastrophe, the company stays solvent. Finally, recent developments in the financial markets create products such as catastrophe–linked bonds where the repayment value is contingent on the occurrence of some well–defined catastrophe. These and many more examples benefit from a well–established body of theory which is now referred to as EVT. Our book gives you an introduction to the mathematical and statistical theory underlying EVT. It is written with a broad audience of potential users in mind. From the subtitle however, it is clear that the main target group is in the financial industry. A reason for this emphasis is that the latter have been less exposed to EVT methodology. This is in contrast to hydrologists and reliability engineers, for instance, where for a long time EVT has belonged to the standard toolkit.

While our readership is expected to be broad, we do require a certain mathematical level. Through the availability of standardised software, EVT can be at the fingertips of many. However, a clear understanding of its capabilities and limitations demands a fair amount of mathematical knowledge. Basic courses in linear algebra, calculus, probability and statistics are essential. We have tried hard to keep the technical level minimal, stressing the understanding of new concepts and results rather than their detailed discussions and proofs. Plentiful examples and figures should make the introduction of new methodology more digestible.

Those who have no time to read the book from cover to cover, and rather want a fairly streamlined introduction to EVT in practice, could immediately start with Chapter 6. Do however read the Guidelines first. From the applied techniques presented in Chapter 6, you will eventually discover relevant material from other chapters.

A long list of references, together with numerous sections of Notes and Comments should guide the reader to a wealth of available material. Though our list of references is long, as always it reflects our immediate interest. Many important papers which do not fit our presentation have been omitted. Even in more than 600 pages, one cannot achieve completeness; the biggest gap is doubtless multivariate extreme value theory. This is definitely a shortcoming! We feel that mathematical theory has to go hand in hand with statistical theory and computer software before it can safely be presented to the end-user, but for the multivariate case, despite important recent progress, we do not feel that the theory has reached a stage as well-established as the one-dimensional one.

As with any major project, we owe thanks to lots of people. First of all, there are those colleagues and friends who have helped us in ways which go far beyond what normally can be hoped for. Charles Goldie was a constant source of inspiration and help, both on mathematical issues, as well as on stylistic ones. He realised early on that three authors who are not native English speakers, when left alone, will produce a Flemish-Dutch-German-Swiss version of the English language which is bound to bemuse many. In his typical diplomatic manner, Charles constructed confidence bands around proper English which he hoped we would not overstep too often. The fine tuning and final decisions were of course always in our hand, hence also the full responsibility for the final outcome.

Gabriele Baltes, Jutta Gonska and Sigrid Hoffmann made an art out of producing numerous versions in LaTeX of half readable manuscripts at various stages. They went far beyond the support expected from a secretary. The many computer graphs in the book show only the tip of the iceberg. For each one produced, numerous were proposed, discussed, altered, ... We owe many thanks, also for various other support throughout the project, to Franco Bassi, Klemens Binswanger, Milan Borkovec, Hansjörg Furrer, Natascha Jung, Anne Kampovsky, Alexander McNeil, Patricia Müller, Annette Schärf and Alfred Schöttl. For the software used we thank Alexander McNeil, John Nolan and Richard Smith.

Many colleagues helped in proofreading parts of the book at various stages: Gerd Christoph, Daryl Daley, Rüdiger Frey, Jan Grandell, Maria Kafetzakis, Marcin Kotulski, Frank Oertel, Sid Resnick, Chris Rogers, Gennady Samorodnitsky, Hanspeter Schmidli and Josef Steinebach. Their critical remarks kept us on our toes! Obviously there has been an extensive exchange with the finance industry as potential end-user, in the form of informal discussions, seminars or lectures. Moreover, many were generous in sharing their data with us. We hope that the final outcome will also help them in their everyday handling of extremal events: Alois Gisler (Winterthur Versicherungen), René Held and Hans Fredo List (Swiss Reinsurance), Richard Olsen (Olsen and Associates), Mette Rytgaard (Copenhagen Reinsurance) and Wolfgang Schmidt (Deutsche Bank).

All three of us take pleasure in thanking our respective home institutions and colleagues for their much appreciated support. One colleague means something special to all three of us: Hans Bühlmann. His stimulating enthu-

siasm for the beauty and importance of actuarial mathematics provided the ideal environment for our project to grow. We have benefitted constantly from his scholarly advice and warm friendship.

The subtitle of the book "For Insurance and Finance" hints at the potential financial applications. The "real thing", be it either Swiss Francs, German Marks or Dutch Guilders, was provided to us through various forms of support. Both the Forschungsinstitut für Mathematik (ETH) and the Mathematisches Forschungsinstitut Oberwolfach provided opportunities for face-to-face meetings at critical stages.

PE recalls fondly the most stimulating visit he had, as part of his sabbatical in the autumn of 1996, at the School of ORIE at Cornell University. The splendid social and academic environment facilitated the successful conclusion of the book. CK worked on this project partly at ETH Zürich and partly at the Johannes Gutenberg University of Mainz. During most of the time she spent on the book in Zürich she was generously supported by the Schweizerische Lebensversicherungs- und Rentenanstalt, the Schweizerische Rückversicherungs-Gesellschaft (Swiss Re), Winterthur-Versicherungen, and the Union Rückversicherungs-Gesellschaft. Her sincere thanks go to these companies. TM remembers with nostalgia his time in New Zealand where he wrote his first parts of the book. The moral support of his colleagues at ISOR of the Victoria University of Wellington allowed him to concentrate fully on writing. He gratefully acknowledges the financial support of a New Zealand FRST Grant.

Last but not least, we thank our students! One of the great joys of being an academic is being able to transfer scientific knowledge to young people. Their questions, projects and interest made us feel we were on the right track. We hope that their eagerness to learn and enthusiasm to communicate is felt throughout the pages of this book.

March, 1997

PE, Zürich
CK, Mainz
TM, Groningen

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Reader Guidelines

The basic question each author should pose him/herself, preferably in the future tense before starting, is

Why have we written this book?

In our case the motivation came from many discussions we had with mathematicians, economists, engineers and physicists, mainly working in insurance companies, banks or other financial institutions. Often, these people had as students learnt the more classical theory of stochastics (probability theory, stochastic processes and statistics) and were interested in its applications to insurance and finance. In these discussions notions like *extremes, Pareto, divergent moments, leptokurtosis, tail events, Hill estimator* and many, many more would appear. Invariably, a question would follow, "Where can I read more on this?" An answer would usually involve a relatively long list of books and papers with instructions like "For this, look here, for that, perhaps you may find those papers useful, concerning the other, why not read ...". You see the point! After years of frustration concerning the non–existence of a relevant text we decided to write one ourselves. You now hold the fruit of our efforts: a book on the modelling of extremal events with special emphasis on applications to insurance and finance. The latter fields of application were mainly motivated by our joint research and teaching at the ETH where various chapters have been used for many years as *Capita Selecta* in the ETH programme on insurance mathematics. Parts of the book have also formed the basis for a Summer School of the Swiss Society of Actuaries (1994) and the Master's Programme in Insurance and Finance at ESSEC, Paris (1995). These trials have invariably led to an increase in the size of the book, due to questions like "Couldn't you include this or that?". Therefore, dear reader, you are holding a rather hefty volume. However, as in insurance and finance where everything is about "operational time" rather than real time, we hope that you will judge the "operational volume" of this book, i.e. measure its value not in physical weight but in "information" weight.

For whom have we written this book?

As already explained in the previous paragraph, in the first place for all those working in the broader financial industry faced with questions concerning extremal or rare events. We typically think of the actuarial student, the professional actuary or finance expert having this book on a corner of the desk ready for a quick freshen–up concerning a definition, technique, estimator or example when studying a particular problem involving extremal events. At the same time, most of the chapters may be used in teaching a special–topics course in insurance or mathematical finance. As such both undergraduate as well as graduate students interested in insurance and/or finance related subjects will find this text useful: the former because of its development of specific techniques in analysing extremal events, the latter because of its comprehensive review of recent research in the larger area of extreme value theory. The extensive list of references will serve both. The emphasis on economic applications does not imply that the intended readership is restricted to those working on such problems. Indeed, most of the material presented is of a much more general nature so that anyone with a keen interest in extreme value theory, say, or more generally interested in how classical probabilistic results change if the underlying assumptions allow for larger shocks in the system, will find useful material in it. However, the reader should have a good background in mathematics, including stochastics, to benefit fully. This brings us to the key question

What is this book about?

Clearly about extremal events! But what do we mean by this?

In the introduction to their book on *Outliers in Statistics*, Barnett and Lewis [51], the authors write: "When all is said and done, the major problem in outlier study remains the one that faced the very earliest research workers in the subject – what is an outlier?" One could safely repeat this sentence for our project, replacing *outlier* by *extremal event*. In their case, they provide methodology which allows for a possible description of outliers (influential observations) in statistical data. The same will be true for our book: we will mainly present those models and techniques that allow a precise mathematical description of certain notions of extremal events. The key question to what extent these theoretical notions correspond to specific events in practice is of a much more general (and indeed fundamental) nature, not just restricted to the methodology we present here. Having said that, we will not shy away from looking at data and presenting applied techniques designed for the user. It is all too easy for the academic to hide constantly behind the screen of theoretical research: the actuary or finance expert facing the real problems has to take important decisions based on the data at hand. We shall provide him or her with the necessary language, methods, techniques and examples which will allow for a more consistent handling of questions in the area of extremal events.

Whatever definition one takes, most will agree that Table 1, taken from *Sigma* [582] contains extremal events. When looked upon as single events, each of them exhibits some common features.

- *Their (financial) impact on the (re)insurance industry is considerable.* As stated in *Sigma* [582], at $US 150 billion, the total estimated losses in 1995 amounted to ten times the cost of insured losses – an exceptionally high amount, more than half of which was accounted for by the Kobe earthquake. Natural catastrophes alone caused insured losses of $US 12.4 billion, more than half of which were accounted for by four single disasters costing some billion dollars each; the Kobe earthquake, hurricane "Opal", a hailstorm in Texas and winter storms combined with floods in Northern Europe. Natural catastrophes also claimed 20 000 of the 28 000 fatalities in the year of the report.
- *They are difficult to predict a long time ahead.* It should be noted that 28 of the insurance losses reported in Table 1 are due to natural events and only 2 are caused by man-made disasters.
- If looked at within the larger context of all insurance claims, *they are rare events.*

Extremal events in insurance and finance have (from a mathematical point of view) the advantage that they are mostly quantifiable in units of money. However most such events have a non-quantifiable component which more and more economists are trying to take into account. Going back to the data presented in Table 1, extremal events may clearly correspond to individual (or indeed grouped) claims which by far exceed the capacity of a single insurance company; the insurance world's reaction to this problem is the creation of a reinsurance market. One does not however have to go to this grand scale. Even looking at standard claim data within a given company one is typically confronted with statements like "In this portfolio, 20% of the claims are

Losses	Date	Event	Country
16 000	08/24/92	Hurricane "Andrew"	USA
11 838	01/17/94	Northridge earthquake in California	USA
5 724	09/27/91	Tornado "Mireille"	Japan
4 931	01/25/90	Winterstorm "Daria"	Europe
4 749	09/15/89	Hurricane "Hugo"	P. Rico
4 528	10/17/89	Loma Prieta earthquake	USA
3 427	02/26/90	Winter storm "Vivian"	Europe
2 373	07/06/88	Explosion on "Piper Alpha" offshore oil rig	UK
2 282	01/17/95	Hanshin earthquake in Kobe	Japan
1 938	10/04/95	Hurricane "Opal"	USA
1 700	03/10/93	Blizzard over eastern coast	USA
1 600	09/11/92	Hurricane "Iniki"	USA
1 500	10/23/89	Explosion at Philips Petroleum	USA
1 453	09/03/79	Tornado "Frederic"	USA
1 422	09/18/74	Tornado "Fifi"	Honduras
1 320	09/12/88	Hurricane "Gilbert"	Jamaica
1 238	12/17/83	Snowstorms, frost	USA
1 236	10/20/91	Forest fire which spread to urban area	USA
1 224	04/02/74	Tornados in 14 states	USA
1 172	08/04/70	Tornado "Celia"	USA
1 168	04/25/73	Flooding caused by Mississippi in Midwest	USA
1 048	05/05/95	Wind, hail and floods	USA
1 005	01/02/76	Storms over northwestern Europe	Europe
950	08/17/83	Hurricane "Alicia"	USA
923	01/21/95	Storms and flooding in northern Europe	Europe
923	10/26/93	Forest fire which spread to urban area	USA
894	02/03/90	Tornado "Herta"	Europe
870	09/03/93	Typhoon "Yancy"	Japan
865	08/18/91	Hurricane "Bob"	USA
851	02/16/80	Floods in California and Arizona	USA

Table 1 *The* 30 *most costly insurance losses 1970–1995. Losses are in million $US at 1992 prices. For a precise definition of the notion of catastrophic claim in this context see* Sigma [582].

responsible for more than 80% of the total portfolio claim amount". This is an extremal event statement as we shall discuss more in detail in Section 8.2.

By stating above that the quantifiability of insurance claims in monetary units makes the mathematical modelling more tractable, we do not want to trivialise the enormous human suffering underlying such events. It is indeed striking that, when looking at the 30 worst catastrophes, in terms of fatalities over the same period in Table 2 only one event (the Kobe earthquake) figures