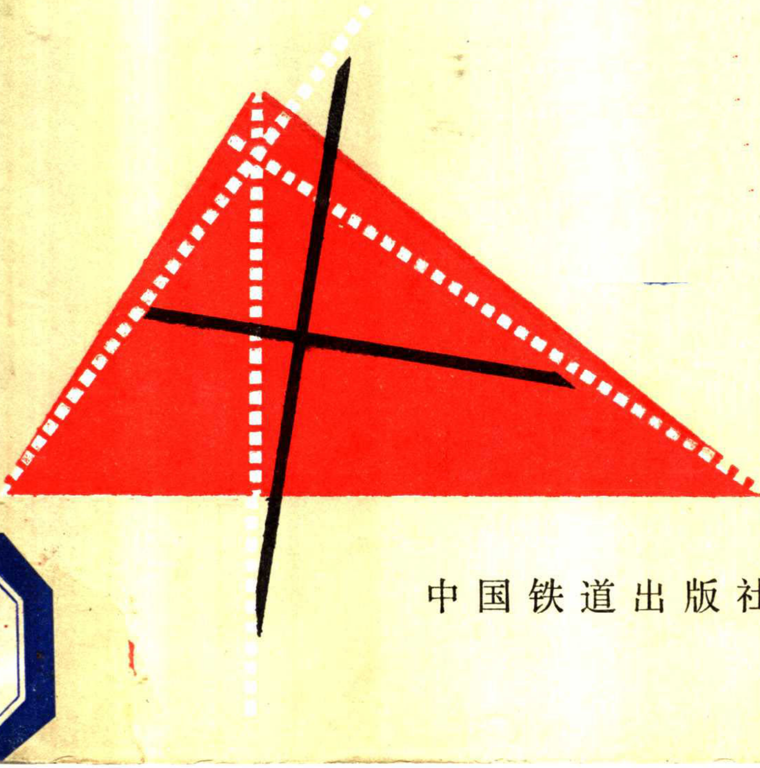
初等数学趣探

罗 河 著



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The Novel Problems of Elementary Mathematics

Prof. LO HO

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出版者的话

本书是罗河教授若干数学论文的汇编。

罗河教授二十年代初考入唐山交通大学土木系。在大学时即致力于初等数学的研究，当时曾在《科学》12卷11期上发表有关数学论文。1945年至1947年罗教授前往英国进行研究工作时，曾在国外发表了两篇论文。

全国解放后，罗教授在唐山交大任教并担任教务长职务。在百忙中，仍孜孜不倦地研究初等数学，先后在《数学学报》、《测量制图学报》发表文章，并于1953年正式出版他的专著《图算原理》。

本书中阐述的初等数学九个问题，有些是个人的发现，有些是世界著名数学家已作出解答，著者又从另一途径作了新的解法。

数学是分析事物关系的科学，与其他一切科学一样是探求真理的科学。真理的发展是没有止境的，就是已有数千年发展历史的初等数学，也并不是已为人们所完全认识和发现。本书不仅是作者对数学富有浓厚兴趣而进行探讨的结果，告诉人们几个饶有趣味的问题解答，同时也将启发读者以锲而不舍的精神去探求科学的真理，为实现四个现代化而拚搏。

序　言

这里所说的初等数学是指一般高中所讲授的数学，如代数、平面几何、平面三角和解析几何等课程的内容。这几种学科有的已有几千年历史，较年青的也存在几百年了。这个领域已经古今中外亿万人学习、推敲、探索过，尚待开发的处女地似乎不多了。

专业数学工作者志在登高，把注意力放在新开发的领域里。初等数学只是攀登高峰必经之路，内容平淡，没有什么值得留恋的。所以近两世纪以来，初等数学是定了型的学科，未曾出现实质性的新内容。

作者是学工程的，以工程教学为职业，对于数学虽有兴趣，但限于条件，无意高攀，只就初等数学里一些细小问题作为业余爱好进行思考，主要是自我消遣，与象棋、桥牌爱好者有类似旨趣。大半个世纪以来，我的业余时间多消磨在自我挑选的若干问题上，其间虚功不少，也有一些收获。这里收录的材料都是发前人之所未发的一点心得。

当前全国上下正在向四化进军，形势大好，令人振奋。吾老矣，力不从心，虽不能和大家一道，为振兴中华上阵拚搏，岂能袖手旁观。乃不顾肤浅，将散轶在中外刊物上的早期文稿，加以修整并作必要补充，汇成此册，以飨奋战在各条战线上为四化献身的亿万健儿。抛砖引玉，预祝更多更大的胜利不断到来。

本书第八题里例题的计算工作是由西南交通大学刘其舒同志编写程序在电子计算机上完成的，特此致谢。又在书稿的编写过程里，吾妻王敏增负责家务并做些抄写工作，对本书也起了促成的作用。

罗　河　1983年9月30日

The Novel Problems of Elementary Mathematics

(An Abstract)

This compilation of prof. Lo-Ho's original works in elementary mathematics, published eleswhere since the year 1927, consists of nine topics which are either entirely new discoveries, or new solutions of well-known existing problems. The problems treated in this collection are as follows:

1. Four Pairs of Mutually Perpendicular Lines of a Plane Triangle.

There exist in every plane triangle four pairs of mutually perpendicular lines, each of which intersects the three altitudes of the triangle at points equidistant from the corresponding vertices of the triangle; the two lines of a pair intersect at the incentre or excentre at right angle; and the equal distances intercepted on the altitudes are equal to R±D, where R is the radius of the circumscribed circle of the triangle, and D is the distance from the incentre or excentre to the circumcentre.

This theorem was discovered in 1925 and published in 1931.

2. Analytic Expression of $\cos\frac{2\pi}{17}$ in Terms of the Number 17.

A new method of solution of the problem based on the elementary theory of plane trigonometry was found in 1927, in addition to the method based on the algebraic theory of equations discovered by K.F. Gauss in 1796, thus making the solution of this well-known problem intelligible to middle school students.

3. Analytic Expression of $\cos\frac{2\pi}{257}$ in Terms of the Number 257.

This problem may be solved in the same manner as that of $\cos\frac{2\pi}{17}$; but to avoid the elaborate successive expansion of the products of a large number of sines, an alternate method is used instead, which is made possible by a simple method of sub-dividing the 128 inherent cosines into groups smaller and smaller, and a definite procedure of determining the values of these groups of cosines. At last it is evident that $\cos\frac{2\pi}{257}$ can be found as an anglytic expression in terms of 257 which is to be repeated 2232 times under various forms of quadratic radicals.

4. Geometric Solution of Linear Equation System with the Unknowns Represented by Slopes of Straight Lines.

Although the analytic solution of simultaneous linear equations involves only the four arithmatic operations–addition, subtraction, multiplication and division of rational numbers, and these operations can easily be performed with the traditional geometric instruments, ruler and compasses, no successive attempt of geometric solution of the general linear equation system has been reported till the thirtieth of the present century, when this historical vacancy of plane geometry was filled by the publication of two methods of pure geometric solution of the problem by the author of this book.

In the first method, the linear equations are converted into a number of broken lines of which the corresponding parts are parallel to each other, and the slopes of the different parts are the values of the unknowns of the equations, to be determined by pure geometric method of construction.

5. Geometric Solution of Linear Equation System with the Unknowns Represented by Horizontal Distances.

In this method the equations are represented by a number of broken lines with their corresponding turning points on common vertical lines, and the location of the positions of these vertical lines is the object of geometrical construction.

6. A new Formula of Approximation to the Real Roots of Equation.

For this problem solved long ago by Newton and Horner, an entirely new method of approach has been found, which is simple in theory and yields a rapid approach to the roots required.

7. A new Method of Construction of Alignment Nomogram.

Alignment nomogram is the simplest device of solving certain type of equations. But the classical method of its construction is limited to equations of simple composition. The new method of construction enlarges its scope of application to a large number of unknown functional relations including those embodied in a mass of empirical data.

The method is realized by constructing a new surface circumscribing two pairs of vertical cross sectional curves of an unknown surface. The new surface supported on the four enclosing vertical curves is a good approximation to the enclosed portion of the original surface of the unknown function; and the vertical distance between them at any point may be computed with a general formula.

8. Alignment Nomogram with Minimum Mean Square Error.

The alignment nomogram constructed for a table of numerical values of an unknown function, from four sets of values along four lines, has errors at all points within the covered area with the only exception of those points on the four lines of coincidence. In this articie a method of adjustment is established to improve the data on the four lines used in the construction, so that the resulting errors at all points including those used in the construction should be such that their mean square error is minimum in value, which, as shown in several actual sample computations, only amounts to about one third of the m.s.e. before adjustment. The resulting nomogram based on the adjusted data is much improved in theoretical accuracy.

The performance of the alignment nomogram constructed by the new method is essentially the determination of a large number of data from the limited few used in the construction. This fact is the basic essence of the method of interpolation; and the values of the unknown function so determined may be further improved so that their mean square error is minimum in value. For bivariate data, there is so far no adequate method of adjustment and interpolation so efficient and so accurate.

9. A New Theory of Interpolation.

In this article, a structural formula $\sum_{i=1}^{r} H_n(t, u, v, \cdots z)_i$ is established as a general form of approximation to all single-valued continuous function $f(t, u, v, \cdots, z)$ of n variables, so that $\sum_{i=1}^{r} H_n(t,u,v,\cdots,z)_i$ and $f(t,u,v,\cdots,z)$ are identical on $n(2r)^{n-1}$ occasions, when any $(n-1)$ variables are assigned one of their 2r specific values used in the construction of the general formula $\sum_{i=1}^{r} H_n(t,u,v,\cdots,z)_i$ which may be used to determine the unknown intermediate values of the unknown function from its known values on the $n(2r)^{n-1}$ loci of known values.

For the function of two variables, the surfaces of $f(t,u)$ and $\sum_{i=1}^{r} H_2(t,u)_i$ have 4r loci of intersection in two directions, which is a mathematical achievement hitherto unthinkable.

目　录

第 一 题

三角形里的四对垂直线*

提　　要

三角形是几何图形的基本形式，几何学中很大一部分是三角形内部关系的分析。几何关系的探索，有些类似于在棋盘上走棋，沒有一定法则可循。有些关系可根据某种设想在纸上划图试探。也有一些关系，在沒有发现之前是很难设想到的。三角形的四对垂直线就是这类关系；它是通过某种分析计算过程偶然发现的。是不是还有其他尙在隐藏中的类似关系呢？这个问题只能由未来的漫长岁月逐步明确。

1—1　导　　言

三角形是几何图形的基本形式，是普通几何学的主要组成部分。经过几千年来亿万人的探索，三角形的各种内部关系已被揭露出来很多，其中以九点圆与内切圆和旁切圆相切的关系最为突出。是不是还有其他类似关系尚未发现呢？三角形的四对垂直线是本世纪里的新发现，是对上述问题的一个正面回答。可以肯定，问题尚未结束，仍在等待新的补充。

1—2　定　　理

在同一平面三角形里均存在四对相互垂直的直线，分别

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* 参见罗河：三角形的几何学之一问题。《科学》第 15 卷，第 12 期，1931年。

在内心和旁心上相互直交；每一条线在三个垂高上的交点到其相应顶点的距离是相等的；这个相等的距离分别等于$R\pm D$，其中R为三角形外接圆的半径，D为外心到内心或旁心的距离。

1—3 证明第一部分

在图1—1中虚线AP、BP和CP是三角形ABC的三个垂高、O为其外心(外接圆心)，I为其内心(内切圆心)。在垂高AP和BP上分别取O_a，O_b两点，使AO_a和BO_b均等于外接圆的半径R；在AP和BP上另各取两点A_1，A_2，B_1和B_2如图1—1所示，使

$$O_aA_1=O_aA_2=O_bB_1=O_bB_2=OI=D。$$

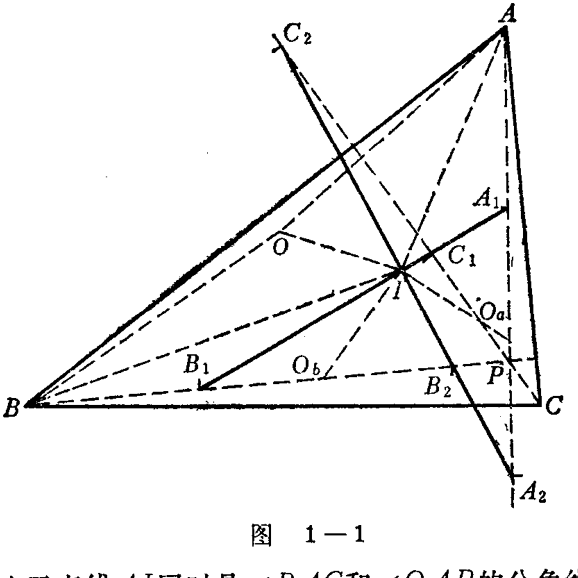

图 1—1

由于直线AI同时是$\angle BAC$和$\angle OAP$的分角线；

并且 $$AO=AO_a$$

故 $$IO_a=IO=A_1O_a=D$$

$$\angle IA_1O_a = 90° - \frac{1}{2}\angle A_1O_aI = 90° - \frac{1}{2}\angle IOA$$

同样 $$\angle O_bB_1I = 90° - \frac{1}{2}\angle BOI$$

故 $$\angle IA_1O_a + \angle O_bB_1I = 180° - \frac{1}{2}\angle BOA$$
$$= 180° - \angle APB$$

即 $$\angle IA_1O_a + \angle O_bB_1I + \angle APB = 180°$$

在四边形PA_1IB_1里，已有三个内角之和等于180°，则第四角$\angle A_1IB_1$亦必为180°，即 A_1，I，B_1 三点在同一直线上。

设在垂高CP上同样取一点C_1，则用同样方法可证明C_1也和I与A_1共线，也就是这条经过内心 I 的直线在三个垂高上的交点A_1，B_1和C_1具有下列关系：

$$AA_1 = BB_1 = CC_1 = R - D$$

同样可证明，另一条在I点上和IA_1直交的直线在三个垂高上的交点A_2，B_2和C_2具有下列关系：

$$AA_2 = BB_2 = CC_2 = R + D$$

1—4 证明第二部分

在图 1—2 中，设I_b为三角形ABC中与顶角 B 相对的旁心，O为$\triangle ABC$的外心。在垂高BP上向对边方向取一点O_b使BO_b等于外接圆半径R；而在垂高AP上则在与对边相反的方向上取一点O_a，使AO_a等于R，在 AP、BP 上各取两点A_1，A_2，B_1，B_2如图 1—2 所示，使

$$O_aA_1 = O_aA_2 = O_bB_1 = O_bB_2 = OI_b = D_b$$

其中D_b代表由外心O到旁心I_b的距离。

由于直线AI_b是A点上外角的分角线，又是 $\angle OAP$ 外角的分角线，并且

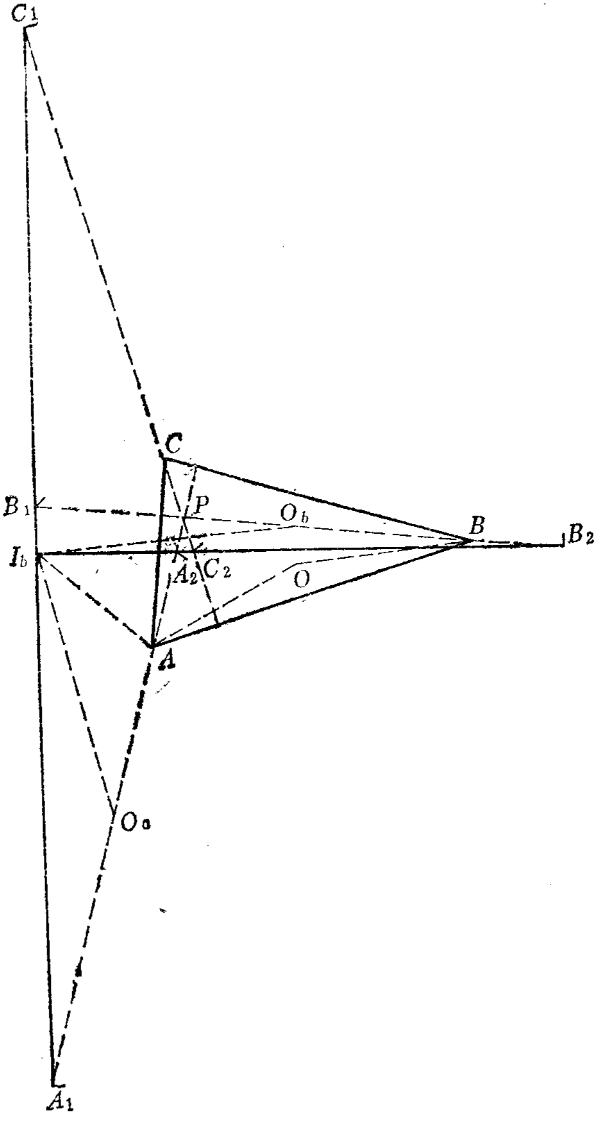

图 1—2

$$AO = AO_a$$

故 $$I_bO = I_bO_a = D_b$$

$$\angle O_aA_1I_b = \frac{1}{2}\angle AO_aI_b = \frac{1}{2}\angle I_bOA$$

同样 $$\angle I_bB_1O_b = \frac{1}{2}\angle I_bO_bB = \frac{1}{2}\angle BOI_b$$

故 $$\angle O_aA_1I_b + \angle I_bB_1O_b = \frac{1}{2}(\angle I_bOA + \angle BOI_b)$$

$$= \frac{1}{2}(360^\circ - \angle AOB) = 180^\circ - \angle ACB$$

$$= 180^\circ - \angle B_1PA$$

即 $$\angle O_aA_1I_b + \angle I_bB_1O_b + \angle B_1PA = 180^\circ$$

因此在四边形$PB_1I_bA_1$里已有三个顶角之和为180°，则第四顶点的角$\angle A_1I_bB_1$亦必为180°。

故A_1、B_1和I_b三点共线。

同样可证明C_1点也与I_b和A_1两点共线，也就是A_1、B_1、C_1和I_b四点共线。

同样可证明A_2、B_2、C_2和I_b等四点共线，并且此线和前一线在I_b点上直交。