

高等学校试用

# 英语理工科教材选

第四分册 电工学

徐民寿(主编) 汤克驷 朱大芳 选编

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机械工业部部属院校选编

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# 英语理工科教材选

Book IV

Electrical Engineering

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本分册内容选自 Ralph J. Smith 所著 «Circuits  
Devices And Systems» 英文原版书 (第三版)  
中的第 2、8、22、25 等四章。

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## 英语理工科教材选

(第四分册 电工学)

徐民寿(主编) 汤克疆 朱大芳 选编

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## 编者的话

为了提高机械工业部部属院校学生的外语水平,培养学生阅读英语科技书刊的能力,我们选编了这套“英语理工科教材选”。整套“教材选”共分九个分册,内容包括数学、物理、理论力学、材料力学(与理论力学合为一个分册)、电工学、工业电子学、金属工艺学、机械原理、机械零件(与机械原理合为一个分册)、计算机算法语言、管理工程等十一门业务课程。各业务课都选了三章英语原版教材(个别也有选四章),供机械工业部部属院校试用。

在业务课中使用部分外语原版教材,这是我们的一次尝试,也是业务课教材改革、吸收国外先进科学技术的探索。在选材时,我们考虑了我国现行各课程的体系、内容以及学生的外语程度,尽可能选用适合我国实际的外国材料。

本“教材选”的选编工作,是在机械工业部教育局的直接领导下,由部属院校的有关教研室做了大量调查研究后选定的,并进行注释和词汇整理工作。由马泰来、卢思源、李国瑞、柯秉衡、谢卓杰、戴炜华、戴鸣钟等同志(以姓氏笔划为序)组成的审编小组,对选材的文字、注释、词汇作了审校。戴鸣钟教授担任整套“教材选”的总审。

由于时间仓促,选材、注释和编辑必有不尽完善之处,希广大读者提出宝贵意见,以利改进。

1983年4月

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## Vocabulary

# Chapter 2

## Circuit Principles

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Circuit Laws

Network Theorems

Nonlinear Networks

The emphasis so far has been on the behavior of individual components and the ideal circuit elements that serve as models. Now we are ready to consider① combinations of elements into circuits, using two well-known experimental laws. Our first objective is to learn how to formulate and solve circuit equations.

Circuits of considerable complexity or generality are called *networks* and circuit principles capable of general application are called *network theorems*. Some of these theorems are useful in reducing complicated networks to simple ones. Other theorems enable us to draw general conclusions about network behavior. This chapter focuses on networks consisting of resistances and steady (dc) sources; later the theorems will be extended to include networks containing other elements and other sources.

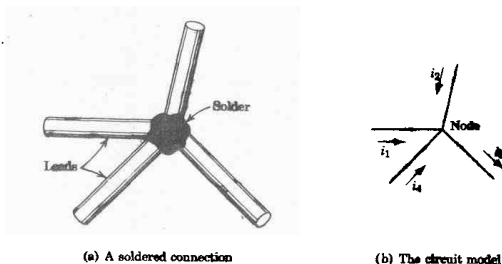
We start with *linear* networks, i.e., combinations of components that can be represented by the ideal circuit elements  $R$ ,  $L$ , and  $C$ , and ideal energy sources. However, most electronic devices and many practical components are nonlinear. Therefore, we consider how linear methods can be used in analyzing some nonlinear networks, and we learn some new techniques applicable to nonlinear devices.

### Circuit Laws

The foundations of network theory were laid about 150 years ago by Gustav Kirchhoff, a German university professor, whose careful experiments resulted in the laws that bear his name. In the following discussion, a *branch* is part of a circuit with two terminals to which connections can be made, a *node* is the point where two or more branches come together, and a *loop* is a closed path formed by connecting branches.

**Kirchhoff's Current Law**

To repeat Kirchhoff's experiments in the laboratory, we could arrange to measure the currents in a number of conductors or "leads" soldered together (Fig. 2.1a). In every case, we would find that the sum of the currents flowing



(a) A soldered connection  
 Figure 2.1 Kirchhoff's current experiment.

(b) The circuit model

into the common point (a *node*) at any instant is just equal to the sum of the currents flowing out.

- In Fig. 2.1b, a circuit model is used to represent an actual connection. The arrows define the *reference* direction for positive current where current is defined by the motion of positive charges. The quantity " $i_1$ " specifies the *magnitude* of the current and its *algebraic sign* with respect to the reference direction. If  $i_1$  has a value of "+5 A," the effect is as if positive charge is moving toward the node at the rate of 5 C/s. If  $i_1 = +5$  A flows in a metallic conductor in which charge is transported in the form of negative electrons, the electrons are actually moving away from the node but the effect is the same. If  $i_2$  has a value of "-3 A," positive charge is in effect moving away from the node. In a practical situation the direction of current flow is easy to determine; if an ammeter reads "upscale," current is flowing from the meter terminal marked + through the meter to the terminal marked -.

By Kirchhoff's current law, the *algebraic sum of the currents into a node at any instant is zero*. It is sometimes convenient to abbreviate this statement, and write " $\Sigma i = 0$ ," where the Greek letter sigma stands for "summation." As applied to Fig. 2.1b,

$$\Sigma i = 0 = i_1 + i_2 - i_3 + i_4 \quad (2-1)$$

Obviously some of the currents may be negative.

## Example 1

In Fig. 2.2, if  $i_1 = +5$  A,  $i_2 = -3$  A, and  $i_4 = +2$  A, what is the value of  $i_3$ ?

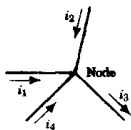


Figure 2.2 Current calculation.

By Kirchhoff's current law,

$$\Sigma i = 0 = i_1 + i_2 - i_3 + i_4$$

$$\Sigma i = 0 = +5 - 3 - i_3 + 2$$

or

$$i_3 = +5 - 3 + 2 = +4$$

Note that we could just as well say that the algebraic sum of the currents *leaving* a node is zero.

## Kirchhoff's Voltage Law

The current law was originally formulated on the basis of experimental data. The same result can be obtained from the principle of conservation of charge and the definition of current. Kirchhoff's voltage law also was based on experiment, but the same result can be obtained from the principle of conservation of energy and the definition of voltage.

To repeat Kirchhoff's observations about voltages, we could set up an electrical circuit and arrange to measure the voltages across a number of components that form a closed path. Only a portion of the circuit is shown in Fig. 2.3, but the combination of a battery, a resistor, an inductor, and the associated leads forms the desired closed path. In every case, we would find that the voltages around the loop, when properly combined, add up to zero. ⑥

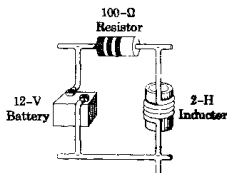


Figure 2.3 A voltage experiment.

The circuit model of Fig. 2.4 is more convenient to work with. There the voltages are labeled and the + signs define the *reference* direction for positive voltage or potential difference. The quantity  $v_R$  specifies the *magnitude* of the source voltage and its algebraic sign with respect to the reference. If  $v_R$  has a value of +12 V, the voltage of node *b* with respect to node *a* is positive. Since, by definition, voltage is energy per unit charge, a positive charge of 1 C moving from node *a* to *b* gains 12 J of energy from the voltage source. If  $v_R$  has a value of +5 V, a positive charge of 1 C moving from *b* to *c* loses 5 J of energy; this energy is removed from the circuit and dissipated in the resistance *R*. If  $v_L$  has ⑦



a value of  $-7\text{ V}$ , node  $d$  is at a higher voltage than  $c$  and a positive charge of  $1\text{ C}$  moving from  $c$  to  $d$  gains  $7\text{ J}$  of energy. A current flowing from  $c$  to  $d$  could receive energy previously stored in the magnetic field of inductance  $L$ .

By Kirchhoff's voltage law, the algebraic sum of the voltages around a loop at any instant is zero. As an abbreviation, we may write " $\Sigma v = 0$ ." As applied to Fig. 2.4,

$$\Sigma v = 0 = v_{ba} + v_{cb} + v_{dc} + v_{ad} \quad (2-2)$$

where  $v_{ba}$  means "the voltage of  $b$  with respect to  $a$ "; if  $v_{ba}$  is positive, terminal  $b$  is at a higher potential than terminal  $a$ . In applying the law, the loop should be traversed in one continuous direction, starting at one point and returning to the same point. Starting from node  $a$  in this case,

$$\Sigma v = 0 = v_s - v_R - v_L + 0 \quad (2-3)$$

A minus sign is affixed to the  $v_R$  term because the plus sign on the diagram indicates that node  $b$  is nominally at a higher potential than  $c$  or  $v_{cb} = -v_R$ .

### Example 2

In Fig. 2.4, if  $v_s = +12\text{ V}$  and  $v_R = +5\text{ V}$ , what is the value of  $v_L$ ?

By Kirchhoff's voltage law,

$$\Sigma v = 0 = v_s - v_R - v_L + 0$$

$$\Sigma v = 0 = +12 - 5 - v_L + 0$$

or

$$v_L = +12 - 5 = +7\text{ V}$$

The loop can be traversed in either direction, starting at any point. Going counterclockwise, starting at node  $c$ ,

$$\Sigma v = 0 = +v_R - v_s + 0 + v_L$$

which agrees with Eq. 2-3.

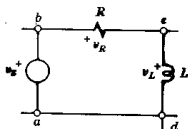


Figure 2.4 Voltage calculation.

### Application of Kirchhoff's Laws

To illustrate the application of Kirchhoff's laws in solving electric circuits, consider the circuit shown in Fig. 2.5. In this case, the source voltages and the resistances are given and the element currents and voltages are to be determined.

The first step is to label the unknown currents arbitrarily; as drawn, the arrow to the right indicates the reference direction of  $i_1$ . If the value of  $i_1$  is calculated and found to be positive, current  $i_1$  actually flows to the right and an ammeter inserted between node  $a$  and the  $2\text{-}\Omega$  resistor with the  $+$  terminal at  $a$  would

read *upscale* or positive. If the value of  $i_1$  is found to be negative, current  $i_1$  actually flows to the left (the ammeter inserted as previously would read *downscale* or negative).

Applying Kirchhoff's current law to node  $a$ ,

$$\Sigma i_a = 0 = +i_4 - i_1$$

or

$$i_4 = i_1$$

From this we conclude that the current is the same at every point in a series circuit. Since  $i_4 \equiv i_1$ , it is not another unknown and will be considered no further. Note that  $i_2$  is the current in the 20-V source as well as in the 4- $\Omega$  resistance.

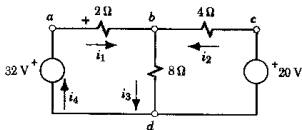


Figure 2.5 Application of Kirchhoff's laws.

The next step is to define the unknown element voltages in terms of the arbitrarily assumed currents. In flowing from  $a$  to  $b$ , the positive charges constituting current  $i_1$  lose energy in the 2- $\Omega$  resistance; this loss in energy indicates that the potential of  $a$  is higher than that of  $b$ , or  $v_{ab} = Ri_{ab} = +2i_1$ . The left-hand terminal of the 2- $\Omega$  resistance is then marked with a neat + to indicate the polarity of the element voltage in terms of the assumed current direction. Following the same analysis, the upper end of the 8- $\Omega$  resistance and the right-hand end of the 4- $\Omega$  resistance could be marked + in accordance with the following element equations:

$$\begin{aligned} v_{ab} &= +2i_1 & \text{or} & & v_{ba} &= -2i_1 \\ v_{bd} &= +8i_3 & \text{or} & & v_{db} &= -8i_3 \\ v_{cb} &= +4i_2 & \text{or} & & v_{bc} &= -4i_2 \end{aligned} \quad (2-4)$$

Additional relations are obtained in the form of *connection equations*. Applying Kirchhoff's current law to node  $b$ ,

$$\Sigma i_b = 0 = +i_1 + i_2 - i_3 \quad (2-5)$$

Since we are summing the currents *into* node  $b$ ,  $i_3$  is shown with a minus sign. Applying the current law to node  $d$ ,

$$\Sigma i_d = 0 = -i_1 - i_2 + i_3$$

Note that this equation is *not independent*; it contributes no new information.

Applying the voltage law to the left-hand loop  $abda$ , and considering each element in turn,

$$\Sigma v = 0 = v_{ba} + v_{ab} + v_{ad} \quad (2-6)$$

For the right-hand loop  $bcdb$ ,

$$\Sigma v = 0 = v_{cb} + v_{dc} + v_{bd} \quad (2-7)$$

Kirchhoff's voltage law applies to any closed path; for the outside loop  $abda$ ,

$$\Sigma v = 0 = v_{ba} + v_{cb} + v_{dc} + v_{ad}$$

But this equation is just the sum of Eqs. 2-6 and 2-7 and no new information is obtained. Although this equation is not independent, it can be valuable in checking.

- (12) It is always possible to write as many independent equations as there are unknowns. For a circuit with six unknowns (three voltages and three currents), we have written six equations (three element and three connection equations). Other equations could be written but they would contribute no new information. If the currents are of primary interest, the unknown voltages may be eliminated by substituting Eqs. 2-4 into Eqs. 2-6 and 2-7, which become

$$\Sigma v_{abda} = 0 = -2i_1 - 8i_3 + 32$$

$$\Sigma v_{bcdb} = 0 = +4i_2 - 20 + 8i_3$$

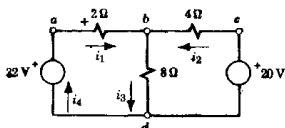


Figure 2.6 Writing circuit equations.

When these are rewritten along with Eq. 2-5, we have for Fig. 2.6

$$i_1 + i_2 - i_3 = 0 \quad (2-8)$$

$$+2i_1 + 8i_3 = 32 \quad (2-9)$$

$$+4i_2 + 8i_3 = 20 \quad (2-10)$$

To evaluate the unknowns, these three equations are to be solved simultaneously. Some commonly employed methods are illustrated in the solution of this problem.

### Solution by Determinants

The method of determinants is valuable because it is systematic and general; it can be used to solve complicated problems or to prove general theorems. The first step is to write the equations in the *standard form* of Eqs. 2-8, 2-9, and 2-10 with the constant terms on the right and the corresponding current terms aligned on the left.

A determinant is an array of numbers or symbols; the array of the coefficients of the current terms is a *third-order determinant*

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 8 \\ 0 & 4 & 8 \end{vmatrix} \quad (2-11)$$

The value of a determinant of second order is

$$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \quad (2-12)$$

The value of a determinant of third order is

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = +a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1 \quad (2-13)$$

A simple rule for second- and third-order determinants is to take the products "from upper left to lower right" and subtract the products "from upper right to lower left." Determinants of higher order can be evaluated by the process called "expansion by minors."<sup>†</sup>

Repeating the first two columns for clarity and using the rule of Eq. 2-13,

$$D = \begin{vmatrix} 1 & 1 & -1 & 1 & 1 \\ 2 & 0 & 8 & 2 & 0 \\ 0 & 4 & 8 & 0 & 4 \end{vmatrix} = + (1)(0)(8) + (1)(8)(0) + (-1)(2)(4) \\ - (1)(8)(4) - (1)(2)(8) - (-1)(0)(0) \\ = 0 + 0 - 8 - 32 - 16 - 0 = -56$$

Replacing the coefficients of  $i_3$  with the constant terms on the right-hand side of the equations in the standard form yields a new determinant:

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 32 \\ 0 & 4 & 20 \end{vmatrix} = 0 + 0 + 0 - 128 - 40 - 0 = -168 \quad (2-14)$$

By Cramer's rule for the solution of equations by determinants,

$$i_3 = \frac{D_3}{D} \quad \text{or} \quad i_3 = \frac{D_3}{D} = \frac{-168}{-56} = 3 \text{ A}$$

Currents  $i_1$  and  $i_2$  could be obtained in a similar manner.

### Substitution Method

When the number of unknowns is not large, the method of *substitution* is sometimes more convenient than using determinants. In this method, one of the equations is solved for one of the unknowns and the result is substituted in the other equations, thus eliminating one unknown. Solving Eq. 2-8 for  $i_3$  yields

$$i_3 = i_1 + i_2$$

Substituting in Eqs. 2-9 and 2-10 yields

$$2i_1 + 8(i_1 + i_2) = 32 \\ 4i_2 + 8(i_1 + i_2) = 20$$

<sup>†</sup> See Britton and Snively: *College Algebra*, Holt, Rinehart & Winston, New York, 1953, or any other college algebra text. An abbreviated discussion of determinants is given in the Appendix of H. H. Skilling: *Electrical Engineering Circuits*, 2nd ed., John Wiley & Sons, New York, 1965.

which can be rewritten as

$$(2 + 8)i_1 + 8i_2 = 32 \quad (2-15)$$

$$8i_1 + (8 + 4)i_2 = 20 \quad (2-16)$$

Solving Eq. 2-15 for  $i_1$  yields

$$i_1 = \frac{32 - 8i_2}{10}$$

Substituting in Eq. 2-16 yields

$$8 \frac{32 - 8i_2}{10} + 12i_2 = 20 \quad \text{or} \quad i_2 = \frac{-56}{56} = -1 \text{ A}$$

and the voltage across the  $4\text{-}\Omega$  resistance is

$$v_{cb} = 4i_2 = 4(-1) = -4 \text{ V}$$

The interpretation of the minus sign is that in reality node  $b$  is at a higher potential than  $c$ , and positive current flows to the right in the  $4\text{-}\Omega$  resistance.

### Loop-Current Method

The reduction in the number of unknowns and in the number of simultaneous equations achieved by substitution can be obtained automatically by an ingenious approach to circuit analysis. A *loop current*  $I_1$  is assumed to circulate around

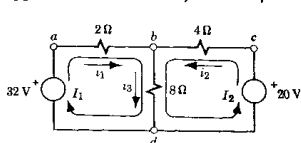


Figure 2.7 Loop currents.

loop  $abda$  in Fig. 2.7, and another loop current  $I_2$  is assumed to circulate around loop  $cbdc$ . By inspection, the branch current  $i_1$  is equal to loop current  $I_1$ , and  $i_2$  is equal to  $I_2$ ; but branch current  $i_3$  is the sum of loop currents  $I_1$  and  $I_2$ .

By applying Kirchhoff's voltage law to the two loops we obtain

$$\Sigma v_{abda} = 0 = -2I_1 - 8(I_1 + I_2) + 32$$

$$\Sigma v_{cbdc} = 0 = -4I_2 - 8(I_1 + I_2) + 20$$

which can be rewritten as

$$(2 + 8)I_1 + 8I_2 = 32 \quad (2-17)$$

$$8I_1 + (8 + 4)I_2 = 20 \quad (2-18)$$

These are similar to Eqs. 2-15 and 2-16, but the physical interpretation is different. Equation 2-17 says: "The source voltage in loop 1 is equal to the sum of two voltage drops. The first,  $(2 + 8)I_1$ , is the product of loop current  $I_1$  and the sum of all the resistances in loop 1. The second,  $8I_2$ , is the product of loop

current  $I_2$  and the sum of all resistances that are common to loops 1 and 2." The *self-resistance* of loop 1 is  $2 + 8 = 10\ \Omega$ ; the *mutual resistance* between the two loops is  $8\ \Omega$ . A similar statement could be made by using Eq. 2-18. To obtain  $I_1$ , multiply Eq. 2-17 by 3 and Eq. 2-18 by  $(-2)$  and add the resulting equations. Then

$$\begin{array}{rcl} 30I_1 + 24I_2 & = & 96 \\ -16I_1 - 24I_2 & = & -40 \\ \hline 14I_1 & = & 56 \quad \text{or} \quad I_1 = 4\ \text{A} = i_1 \end{array}$$

### Checking

In solving complicated circuits there are many opportunities for mistakes and a reliable check is essential. This is particularly true when the solution is obtained in a mechanical way, using determinants. A new equation such as that obtained by writing Kirchhoff's voltage law around the outside loop of Fig. 2.7 provides (17) a good check. With a little experience you will be able to write (by inspection, without first writing the element equations)

$$\Sigma v = 0 = -2i_1 + 4i_2 - 20 + 32$$

Substituting  $i_1 = 4\ \text{A}$  and  $i_2 = -1\ \text{A}$ ,

$$\Sigma v = 0 = -2(4) + 4(-1) - 20 + 32 = 0$$

and the values of  $i_1$  and  $i_2$  are checked.

### Example 3

Given the circuit of Fig. 2.8, calculate the current in the  $10\text{-}\Omega$  resistance using loop currents

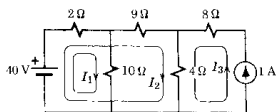


Figure 2.8 Loop-current analysis.

1. Assume loop currents so that the desired current is  $I_1$  by definition and  $I_3 = 1\ \text{A}$  as given.
2. Applying Kirchhoff's voltage law to two loops,

$$\begin{aligned} \Sigma v &= 0 = 40 - 2(I_1 + I_2) - 10I_1 \\ \Sigma v &= 0 = 40 - 2(I_1 + I_2) - 9I_2 - 4(I_2 + 1) \end{aligned}$$

In terms of self and mutual resistance,

$$(10 + 2)I_1 + 2I_2 = 40 \quad (2-19)$$

$$2I_1 + (2 + 9 + 4)I_2 + 4 \times 1 = 40 \quad (2-20)$$

From Eq. 2-20,  $I_2 = (36 - 2I_1)/15$ .

Substituting into Eq. 2-19,

$$12I_1 + (72 - 4I_1)/15 = 40$$

or

$$I_1 = \frac{600 - 72}{180 - 4} = \frac{528}{176} = 3\ \text{A}$$

## Node-Voltage Method

- ⑮ The wise selection of loop currents can significantly reduce the number of simultaneous equations to be solved in a given problem. In the preceding example, the number of equations was reduced from three to two by using loop currents. Would it be possible to get all the important information in a single equation with a single unknown? In this particular case, the answer is "yes" if we choose as the unknown the voltage of node *b* with respect to a properly chosen reference. In some practical devices many components are connected

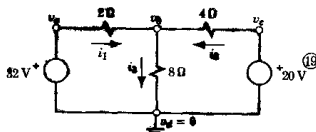


Figure 2.9 Node voltages.

to a metal "chassis," which in turn is "grounded" to the earth. Such a ground, often shown as a common lead at the bottom of a circuit diagram, is a convenient reference. In this problem the greatest simplification will result if we choose node *d*; now the voltage of any node is understood to be with respect to node *d*, i.e., the voltage of node *d* is zero. (See Fig. 2.9.)

Next, apply Kirchhoff's current law to each independent node. Here there is only one independent node (since the voltages of nodes *a* and *c* are fixed by the voltage sources), so the sum of the currents into node *b* is

$$\sum i_b = 0 = i_1 + i_2 - i_3 \quad (2-21)$$

The current  $i_1$  to node *b* from node *a* is equal to the difference in potential between nodes *a* and *b* divided by the resistance between *a* and *b*, or

$$i_1 = \frac{v_a - v_b}{R_{ab}} \quad (2-22)$$

Similarly,

$$i_2 = \frac{v_b - v_d}{R_{bd}} \quad \text{and} \quad i_3 = \frac{v_b - v_c}{R_{bc}} \quad (2-23)$$

where all voltages are in reference to node *d*. In this particular problem,  $v_a$  is given as +32 V, and  $v_c$  is given as +20 V.

- ⑯ Finally, bearing in mind Eq. 2-21 and the concepts represented by Eqs. 2-22 and 2-23, we write one equation with a single unknown,

$$\sum i_b = 0 = \frac{32 - v_b}{2} + \frac{20 - v_b}{4} - \frac{v_b - 0}{8} \quad (2-24)$$

Multiplying through by 8,

$$128 - 4v_b + 40 - 2v_b - v_b = 0$$

Therefore, the unknown voltage at independent node  $b$  is

$$v_b = \frac{168}{7} = 24 \text{ V}$$

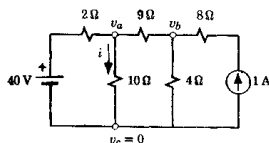
By inspection of Fig. 2.9 we can write

$$i_3 = \frac{24}{8} = 3 \text{ A}, \quad i_1 = \frac{32 - 24}{2} = 4 \text{ A}, \quad \text{and} \quad i_2 = \frac{20 - 24}{4} = -1 \text{ A}$$

which agree with the results previously obtained. As illustrated in Example 4, the node-voltage approach can be applied to more complicated circuits.

### Example 4

Given the circuit of Fig. 2.10, calculate the current in the  $10\text{-}\Omega$  resistance using node voltages.



**Figure 2.10** Node-voltage analysis.

1. Choose node  $c$  (one end of the branch of interest) as the reference node.
2. Apply Kirchhoff's current law to independent nodes  $a$  and  $b$ .

$$\Sigma i_a = 0 = \frac{40 - v_a}{2} + \frac{v_b - v_a}{9} - \frac{v_a - 0}{10}$$

$$\Sigma i_b = 0 = \frac{v_a - v_b}{9} + 1 - \frac{v_b - 0}{4}$$

Multiplying through by 90 and 36, respectively,

$$1800 - 45v_a + 10v_b - 10v_a - 9v_a = 0$$

$$4v_a - 4v_b + 36 - 9v_b = 0$$

Collecting terms,

$$64v_a - 10v_b = 1800$$

$$4v_a - 13v_b = -36$$

3. Solving by determinants,

$$v_a = \frac{D_a}{D} = \frac{\begin{vmatrix} 1800 & -10 \\ -36 & -13 \end{vmatrix}}{\begin{vmatrix} 64 & -10 \\ 4 & -13 \end{vmatrix}} = \frac{-23,400 - 360}{-832 + 40}$$

$$\therefore v_a = 30 \text{ V} \quad \text{and} \quad i_{10} = v_a/10 = 3 \text{ A}$$

### Formulation of Equations

Many books are devoted to electrical network analysis at intermediate and advanced levels; these books give a thorough treatment of a variety of methods for solving networks of great complexity and generality. Although many significant problems can be solved by using the three approaches outlined in this



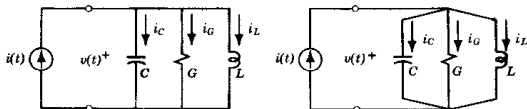


Figure 2.11 A parallel CGL circuit drawn in two ways.

chapter, the discussion here is intended to be introductory and illustrative rather than comprehensive.

- At this point you should be familiar with electrical quantities (their definitions, units, and symbols), you should understand the distinction between real circuit components and idealized circuit elements, you should know the voltage-current characteristics of active and passive circuit elements, and you should be able to apply Kirchhoff's laws to obtain circuit equations. In engineering, the solution of equations is usually less demanding than the formulation, and it is proper to place primary emphasis on the formulation of equations.

If voltages and currents are unchanging with time, as in the preceding example, the behavior of a circuit is determined by resistance alone. A more general problem is illustrated in Fig. 2.11, which resembles the tuning circuit found in every transistor radio. Following the procedure used previously, we write element equations and then connection equations. Assuming a voltage  $v(t)$  with the reference polarity indicated by the  $+$  sign, the corresponding current directions are as shown. Assuming no initial current in the inductance ( $i_L = 0$  at  $t = 0$ ), the element equations are

$$i_C = C \frac{dv}{dt}, \quad i_G = Gv, \quad i_L = \frac{1}{L} \int_0^t v \, dt + 0$$

Applying Kirchhoff's current law to the upper node to obtain the connection equation.

$$\Sigma i = 0 = i(t) - i_C - i_G - i_L$$

Substituting and rearranging,

$$C \frac{dv}{dt} + Gv + \frac{1}{L} \int_0^t v \, dt = i(t) \quad (2-25)$$

If the element values and  $i(t)$  are known, the solution of this *integro-differential* equation consists in finding  $v(t)$ . In the general case, this can be quite difficult (see Chapter 6).

As another illustration, consider the dynamics problem of Fig. 2.12a. A mass restrained by a spring slides on a friction surface under the action of an applied