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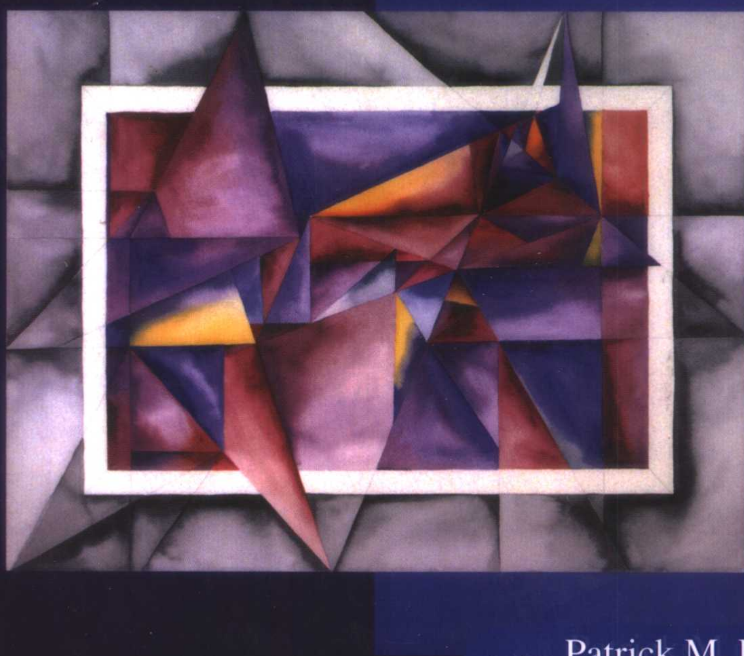
（英文版）

ADVANCED CALCULUS

A COURSE IN MATHEMATICAL ANALYSIS

Patrick M. Fitzpatrick



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THIS BOOK IS DEDICATED TO THE MEMORY OF MY FATHER,

MICHAEL JOSEPH FITZPATRICK (1912–1989)

In order to put his system into mathematical form at all, Newton had to devise the concept of differential quotients and propound the laws of motion in the form of differential equations—perhaps the greatest advance in thought that a single individual was ever privileged to make.

Albert Einstein

from an essay

On the one hundredth anniversary of Maxwell's birth

James Clerk Maxwell: A Commemorative Volume

Preface

The goal of this book is to rigorously present the fundamental concepts of mathematical analysis in the clearest, simplest way, within the context of illuminating examples and stimulating exercises. I hope that the student will assimilate a precise understanding of the subject, together with an appreciation of its coherence and significance. The full book is suitable for a year-long course; the first nine chapters are suitable for a one-semester course on functions of a single variable.

Mathematical analysis has been seminal in the development of many branches of science. Indeed, the importance of the applications of the computational algorithms that are a part of the subject often leads to courses in which familiarity with implementing these algorithms is emphasized at the expense of the ideas that underlie the subject. While these techniques are very important, without a genuine understanding of the concepts that are at the heart of these algorithms, it is possible to make only limited use of these computational possibilities.

I have tried to emphasize the unity of the subject. Mathematical analysis is not a collection of isolated facts and techniques, but is, instead, a coherent body of knowledge. Beyond the intrinsic importance of the actual subject, the study of mathematical analysis instills habits of thought that are essential for a proper understanding of many areas of pure and applied mathematics.

In addition to the absolutely essential topics, other important topics have been arranged in such a way that selections can be made without disturbing the coherence of the course. As three examples of such optional topics, I mention the approximation methods for estimating integrals, the Weierstrass Approximation Theorem, and metric spaces. Precise estimates for the errors incurred in the approximation of integrals were always present in the classical courses in mathematical analysis. Nowadays, they do not appear so frequently. In view of the recent growth in computational capability and the attendant need to estimate errors in approximation methods, this topic seems to me worthy of consideration for inclusion in a course. This material is presented in the last section of Chapter 7; subsequent material is independent of this section. An approximation theorem of quite a different flavor is the Weierstrass Approximation Theorem. This stands as one of the singular jewels of classical analysis. It can be presented as a companion to the discussion of approximation of functions by Taylor polynomials; this theorem is proven in the last section of Chapter 8, and again, the subsequent material is independent of this section. The third topic, metric spaces, is more abstract than other topics in the book, but it is an abstraction that is wholly justified by its synthetic power and the applicability of the general theory to important specific problems. On the other hand, however, two of the most important examples of metric spaces, Euclidean spaces and function spaces, are of sufficient independent interest that it can be argued that they deserve a separate, self-contained discussion. The choice I have made is to begin the study of functions of several variables in Chapter 10 with the study of Euclidean space $\mathbb{R}^n$, and then, in Chapter 11, to study functions and mappings between Euclidean spaces. Then I have included a separate chapter, Chapter 12, on metric spaces. The student will have already seen important specific realizations of the general theory, namely the concept of uniform convergence for sequences of functions and the study of subsets of Euclidean space, and with these examples in mind can better appreciate the general theory. The Contraction Mapping Principle is proved and used to establish the fundamental existence result on the solvability of nonlinear differential equations. This serves as a powerful example of the use of brief, general theory to furnish concrete information about specific problems. Once more, none of the subsequent material depends on Chapter 12.

At the beginning of this course it is necessary to establish a base on which the subsequent proofs will be built. It has been my experience that in order to cover, within the allotted time, a substantial amount of analysis, it is not possible to provide a detailed construction of the real numbers starting with a serious treatment of set theory. I have chosen to codify the properties of the real numbers as three groups of axioms. In the Preliminaries, the arithmetic and order properties are codified in the Field and Positivity Axioms; a detailed discussion of the consequences of these axioms, which certainly are familiar to the student, is provided in Appendix A. The least familiar of these axioms, the Completeness Axiom, is presented in the first section of the first chapter, Section 1.1.

In Chapter 2, convergent sequences are studied. Monotonicity and linearity properties of convergent sequences are proven, and the Completeness Axiom is recast as the Bolzano-Weierstrass Theorem and the Nested Interval Theorem for convergent sequences. The material from this chapter is used repeatedly throughout the book. For instance, in Chapter 3, continuity of functions and limits of functions are defined in terms of sequential convergence. The linearity properties of convergent sequences from Chapter 2 immediately imply corresponding linearity properties for continuous functions, limits, derivatives, and later, in Chapter 7, for integrals.

Chapter 4 is devoted to differentiation. In Section 4.5, Darboux's Theorem is proven: It asserts that in order for a real-valued function that is defined on an open interval to be the derivative of another function, it is necessary that the given function possess the intermediate value property. This is the first result regarding the solvability of differential equations.

The students will be familiar with the properties of the logarithmic and trigonometric functions and their inverses, although, most probably, they will not have seen a rigorous analysis of these functions. In Chapter 5, the natural logarithm, the sine, and the cosine functions are introduced as the (unique) solutions of particular differential equations; on the provisional assumption that these equations have solutions, an analytic derivation of the properties of these functions and their inverses is provided. Later, after the differentiability properties of functions defined by integrals and by power series have been established, it is proven that these differential equations do indeed have solutions, and so the provisional assumptions of Chapter 5 are removed.

Integration is studied in Chapters 6 and 7. The integral is defined in terms of Darboux sums, and later its property of being the limit of appropriate sequences of Riemann sums is established. The relationship between integration and differentiation is described in two theorems, which I call the First and the Second Fundamental Theorems of Calculus. This is done to emphasize the distinction between the formula for evaluating the integral of a function that is known to be the derivative of another function and the related, but different, matter of understanding the conditions under which a given function is the derivative of some other function and providing integral representations of solutions of differential equations. The study of the approximation of functions by Taylor polynomials is the subject of Chapter 8. In Chapter 9, we consider a sequence of functions that converges to a limit function and study the way in which the limit function inherits properties possessed by the functions that are the terms of the sequence; the distinction between pointwise and uniform convergence is emphasized. This concludes the study of functions of a single variable.

The study of functions of several variables begins with Chapters 10 and 11, which start with the study of the structure of Euclidean space $\mathbb{R}^n$ and then turn to the manner in which the results about sequences of numbers and functions of a single variable extend to sequences of points in $\mathbb{R}^n$, to functions defined on subsets of Euclidean space and to mappings between such spaces. There is no class of subsets of $\mathbb{R}^n$ that play the same distinguished role with regard to functions of several variables as do intervals with regard to functions of a single variable. For this reason, the general concepts of open, closed, compact, and connected are introduced for subsets of $\mathbb{R}^n$. The notions of compactness and connectedness for a subset of $\mathbb{R}^n$ are motivated by the necessity to extend the Intermediate Value Theorem and the Extreme Value Theorem to functions of several variables. As already mentioned, Chapter 12 is an independent chapter on metric spaces. The material related to differentiation of functions of several variables is covered in Chapters 13 and 14. Emphasis is placed on precise assertions of the way a function may be approximated, in a neighborhood of a point in its domain, by a simpler function.

The study of mappings between Euclidean spaces is the topic of Chapters 15, 16, and 17. Here, and at other points in the book, it is necessary to understand some linear algebra. In Section 15.1, the correspondence between linear mappings from $\mathbb{R}^n$ to $\mathbb{R}^m$ and $m \times n$ matrices is established. As for the other topics that involve linear algebra, in Appendix B the requisite topics in linear algebra are described, and using the cross product of two vectors in $\mathbb{R}^3$, full proofs are provided for the case of vectors and linear mappings in $\mathbb{R}^3$. In Chapter 15, differentiation is studied for mappings between Euclidean spaces: at each point in the domain of a continuously differentiable mapping there is defined the derivative matrix, together with the corresponding linear mapping called the differential. Approximation by linear mappings is studied and the chapter concludes with the Chain Rule for mappings. The Inverse Function Theorem and the

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《组织开发咨询工具精选》致力于帮助人们提高工作效率以及在其他方面实现有益的改变；共搜集了 50 种解决方案，这 50 种方案分属于五大方面：领导层优化、雇员层优化、团队优化、组织优化和战略规划与管理优化。本书为咨询师们提供了由资深咨询师设计的现成的组织优化工具，让他们在为客户提供咨询的时候能够有成形的解决方案可供应用。

《现代经理管理工具箱》（暂定）（The Active Manager's Toolkit）

梅尔·希尔伯曼博士 编著　开本：16 开　定价（估）：36.00 元 出版时间：2003 年 12 月

与传统经理（traditional manager）相比，现代经理（active manager）的业绩更为突出吗？当然是这样。让本书来告诉你这是为什么。

传统经理是消极应对的。他们坐等问题的出现，然后再想方设法予以解决。

现代经理是积极主动的，他们并非消极坐等问题的出现，而是事先进行预测；现代经理所做的许多工作都是传统经理不大重视的；现代经理不仅培养员工的个人能力，而且还组织和协调团队的整体合作；现代经理致力于收集信息和获取数据，其目的不仅仅是为了撰写报告，而是为了详尽地掌握工作的进展状况，获悉工作中出现的争议和问题，并且利用所掌握的信息来鉴别问题、解决问题；甚至在对员工进行培训的过程中，现代经理也充当着活跃者的角色，而不会置之不理，他们要么与人力资源和培训方面的专家协同工作，要么亲自进行员工培训……

作为一名现代经理，如何才能一应俱全地做到所有这些？

本书将成为你的良师益友，它从 5 个方面阐述了现代经理如何分配自己的精力。这 5 个方面是：

- 评定和提高自己的领导才能
- 培训和管理员工的个人业绩
- 创建协同合作的组织团队
- 制定团队计划和解决团队问题
- 倡导变革

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I am especially grateful to a teacher of mine and to a student of mine. As an undergraduate at Rutgers University, I was very fortunate to have Professor John Bender as my teacher. He introduced me to mathematical analysis. Moreover, his personal encouragement was what led me to pursue mathematics as a lifetime study. It is not possible to adequately express my debt to him. I also wish to single out for special thanks one of the many students who have contributed to this book. Alan Preis was a great help to me in the final preparation of the manuscript. His assistance and our stimulating discussions in this final phase made what could have been a very tiresome task into a pleasant one.

Patrick M. Fitzpatrick

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