高等学校试用

英语理工科教材选

第一分册 数学

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机械工业部部属院校选编

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Book I

Mathematics

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本分册内容选自 Robert Ellis 与 Denny Gulick 所著 Calculus with Analytic Geometry 英文原版书（1978年版）中的第11、12、13 三章

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（第一分册　数学）

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编者的话

为了提高机械工业部部属院校学生的外语水平，培养学生阅读英语科技书刊的能力，我们选编了这套"英语理工科教材选"。整套"教材选"共分九个分册，内容包括数学、物理、理论力学、材料力学（与理论力学合为一个分册）、电工学、工业电子学、金属工艺学、机械原理、机械另件（与机械原理合为一个分册）、计算机算法语言、管理工程等十一门业务课程。各业务课都选了三章英语原版教材（个别也有选四章），供机械工业部部属院校试用。

在业务课中使用部分外语原版教材，这是我们的一次尝试，也是业务课教材改革、汲收国外先进科学技术的探索。在选材时，我们考虑了我国现行各课程的体系、内容以及学生的外语程度，尽可能选用适合我国实际的外国材料。

本"教材选"的选编工作，是在机械工业部教育局的直接领导下，由部属院校的有关教研室做了大量调查研究后选定的，并进行注释和词汇整理工作。由马泰来、卢思源、李国瑞、柯秉衡、谢卓杰、戴炜华、戴鸣钟等同志（以姓氏笔划为序）组成的审编小组，对选材的文字、注释、词汇作了审校。戴鸣钟教授担任整套"教材选"的总审。

由于时间仓促，选材、注释和编辑必有不尽完善之处，希广大读者提出宝贵意见，以利改进。

1983年4月

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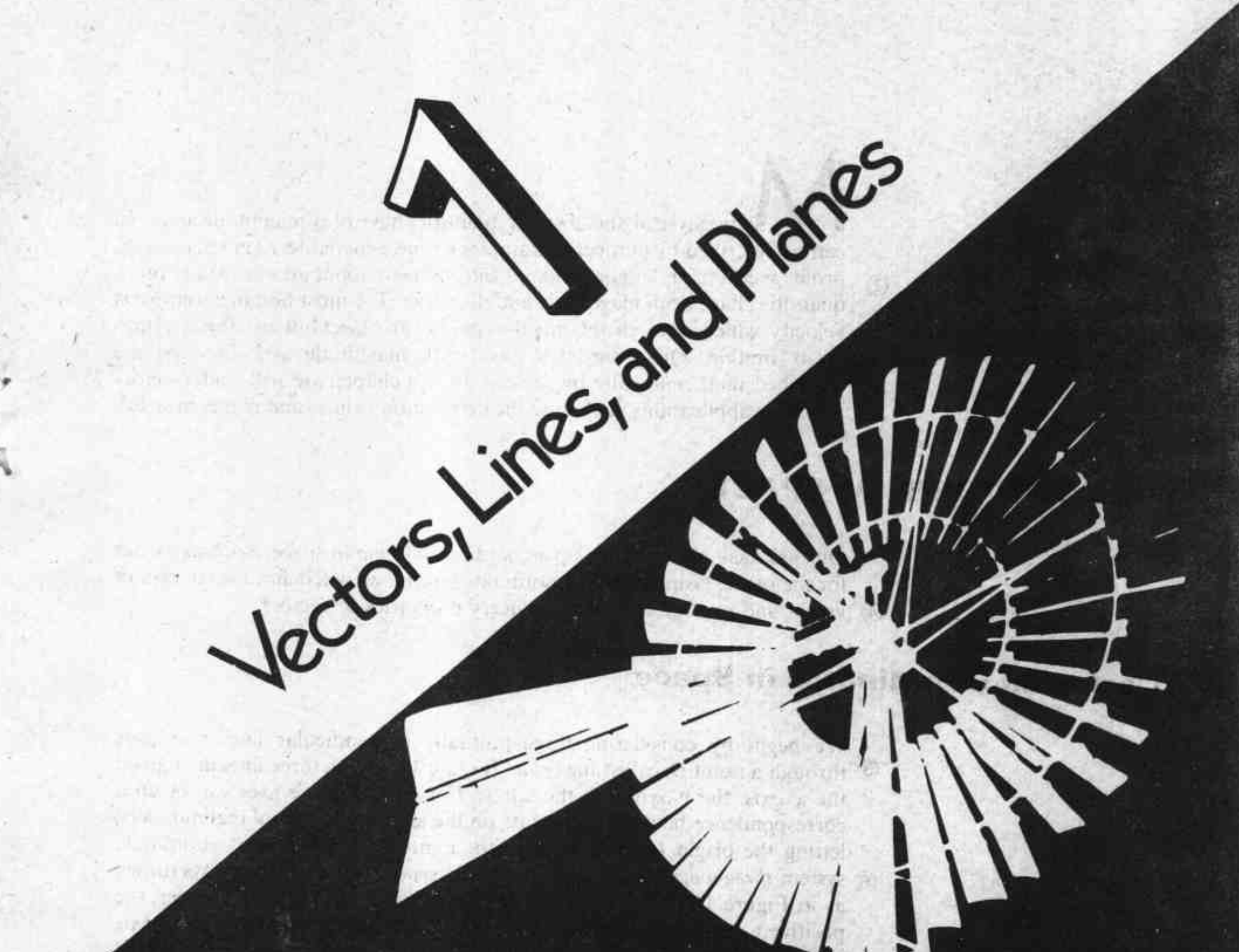


Many physical and abstract quantities have only magnitude and thus can be described by numbers. Examples we have encountered are mass, cost,
① profit, speed, area, length, volume, and moment about an axis. Many other quantities have both magnitude and direction. The most notable example is velocity, which involves not only the speed of an object but also the direction
② of its motion. Quantities that have both magnitude and direction are
③ described mathematically by vectors. In this chapter we will study vectors and their applications, including the description of lines and planes in space.

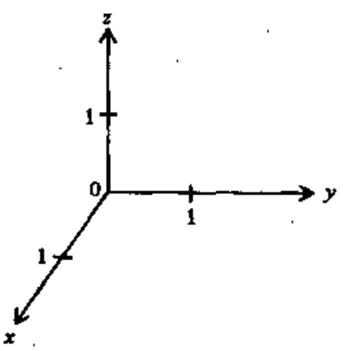
11.1 VECTORS IN SPACE

Our first task will be to set up a coordinate system in space, much as we did for the plane. Using the new coordinate system, we will define the concept of
④ vector and present various elementary properties of vectors.

Cartesian Coordinates in Space

We begin by considering three mutually perpendicular lines that pass
⑤ through a point O, called the *origin* (Figure 11.1). The three lines are named the *x axis*, the *y axis*, and the *z axis*. For each of these axes we set up a correspondence between the points on the axis and the set of real numbers, letting the origin O correspond to the number 0. We call this coordinate
⑥ system *three-dimensional space*, or simply *space*. The axes are always drawn as in Figure 11.1, with the positive x axis pointing toward the viewer, the positive y axis pointing toward the right on the page, and the positive z axis pointing upward on the page.

In three-dimensional space there are three *coordinate planes*: the *xy*



Three-dimensional space

FIGURE 11.1

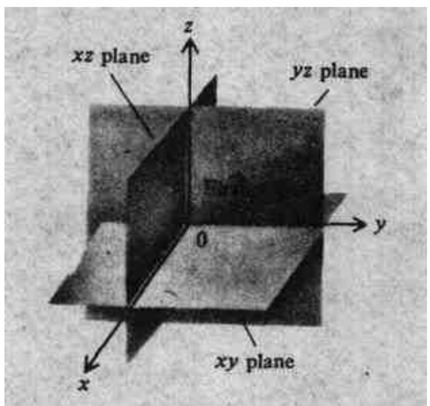


FIGURE 11.2

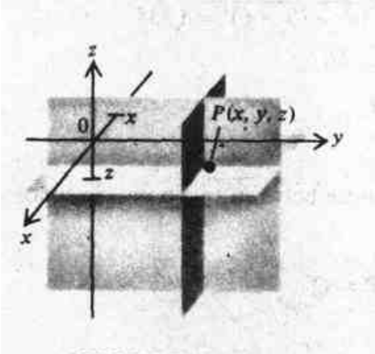


FIGURE 11.3

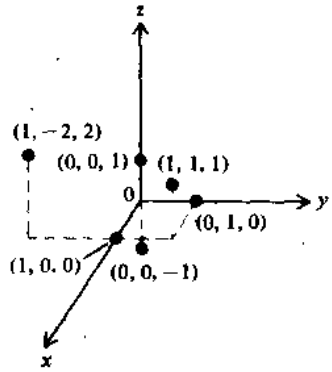


FIGURE 11.4

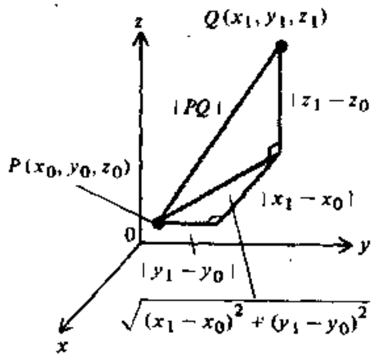


FIGURE 11.5

plane, which contains the x and y axes; the xz plane, which contains the x and z axes; and the yz plane, which contains the y and z axes. Since each plane divides space into two parts, the three coordinate planes together divide space into eight regions, called *octants* (Figure 11.2). The octant containing the positive x, y, and z axes is called the *first octant.*

If P is any point in space, then the three planes through P perpendicular to the three axes intersect the x axis, the y axis, and the z axis at points corresponding to numbers x, y, and z, respectively (Figure 11.3). Therefore, ⑦
we associate P with the ordered triple of numbers (x, y, z) and call (x, y, z) the *rectangular coordinates* (or *Cartesian coordinates*) of P. Figure 11.4 exhibits a few points in space, along with their Cartesian coordinates.

Under the association we have just described, each point in space is identified with an ordered triple of numbers. Conversely, each ordered triple (x, y, z) is identified with a single point whose coordinates are (x, y, z). This correspondence will enable us to describe geometric objects in space by means of equations. ⑧

Just as with points in the plane, the notion of distance between two points in space is fundamental. The formula for the *distance* $|PQ|$ between ⑨
two points $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$ is

$$|PQ| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} \qquad (1)$$

as you can prove by two successive applications of the Pythagorean Theorem (Figure 11.5). If $Q = O$, then (1) reduces to

$$|PO| = |OP| = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

Example 1 Let $P = (-1, 3, 6)$ and $Q = (4, 0, 5)$. Find $|PQ|$, $|PO|$, and $|OQ|$

Solution We calculate that

$$|PQ| = \sqrt{(4-(-1))^2 + (0-3)^2 + (5-6)^2} = \sqrt{35}$$
$$|PO| = \sqrt{(-1)^2 + 3^2 + 6^2} = \sqrt{46}$$
$$|OQ| = \sqrt{4^2 + 0^2 + 5^2} = \sqrt{41} \quad \square$$

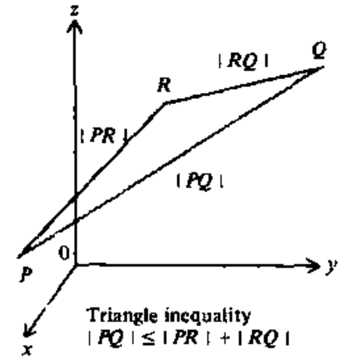


FIGURE 11.6

The three basic laws governing the distance between two points in space are

$$|PQ| = 0 \quad \text{if and only if} \quad P = Q$$
$$|PQ| = |QP|$$
$$|PQ| \leq |PR| + |RQ| \quad \text{for any third point } R$$

The first two laws follow directly from (1). The third law, which is known as the "triangle inequality," implies that the length of any side of a triangle does not exceed the sum of the lengths of the other two sides (Figure 11.6). (See Exercise 25 in Section 11.2 for a proof of the triangle inequality.)

The *sphere* with center $P_0 = (x_0, y_0, z_0)$ and *radius a* is defined to be the set of all points P such that

$$|P_0P| = a$$

(Figure 11.7). Thus a point $P = (x, y, z)$ lies on that sphere if and only if

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = a$$

or equivalently, if and only if

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$$

This is an equation of a sphere in space.

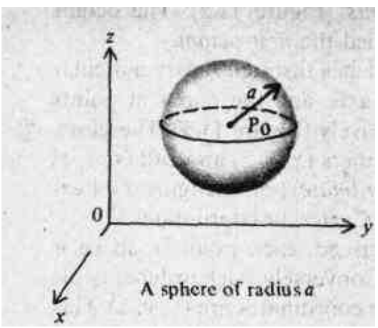


FIGURE 11.7

Example 2 Show that

$$x^2 + y^2 + z^2 = 2x + 4y - 6z$$

is an equation of a sphere. Find the center and the radius of the sphere.

Solution We transpose terms from the right side to the left side of the equation; complete the squares, and obtain

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) + (z^2 + 6z + 9) = 1 + 4 + 9$$

or

$$(x-1)^2 + (y-2)^2 + (z+3)^2 = 14$$

This is an equation of the sphere with center $(1, 2, -3)$ and radius $\sqrt{14}$. $\square$

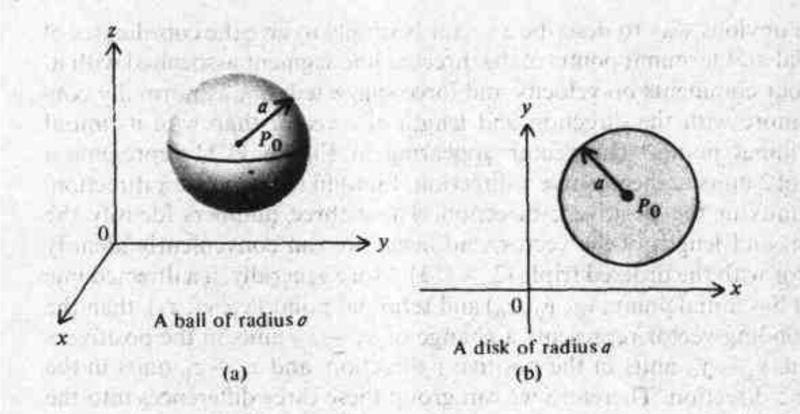


FIGURE 11.8

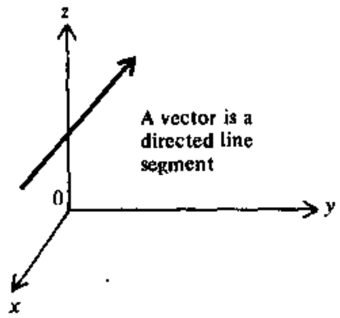


FIGURE 11.9

The *ball* with center $P_0 = (x_0, y_0, z_0)$ and radius a is the collection of points $P = (x, y, z)$ such that $|P_0 P| \leq a$, or such that

$$\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \leq a$$

(Figure 11.8(a)). This is equivalent to

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \leq a^2$$

Thus if $P_0 = (1, 2, -3)$, then $P = (x, y, z)$ is in the ball with center P_0 and radius 3 provided that $|P_0 P| \leq 3$, or that

$$\sqrt{(x - 1)^2 + (y - 2)^2 + (z + 3)^2} \leq 3$$

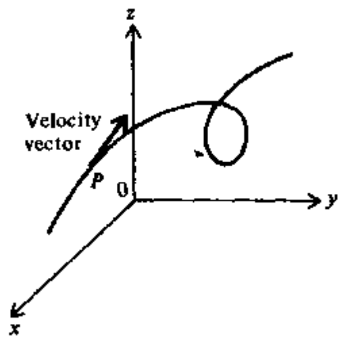


FIGURE 11.10

A ball is the solid region in space enclosed by a sphere. The correspond- ⑩
ing region in the plane is enclosed by a circle, and we call such a region a *disk*. The disk with center $P_0 = (x_0, y_0)$ and radius a is the collection of points $P = (x, y)$ such that $|P_0 P| \leq a$, or such that

$$\sqrt{(x - x_0)^2 + (y - y_0)} \leq a$$

(Figure 11.8(b)). This is equivalent to

$$(x - x_0)^2 + (y - y_0) \leq a^2$$

Vectors

Intuitively, a vector is a directed line segment in space, often described by an arrow (Figure 11.9). Vectors appear constantly in the study of motion in space. For instance, consider a particle moving along the path shown in Figure 11.10. If at a given time t the particle is at point P, we can assign to P a vector, called the *velocity vector*, which points in the direction of the particle's motion and has length equal to the speed of the particle. Vectors are also used to describe force; the vector describing a force points in the direction in which the force acts and has length equal to the magnitude of the force. ⑪

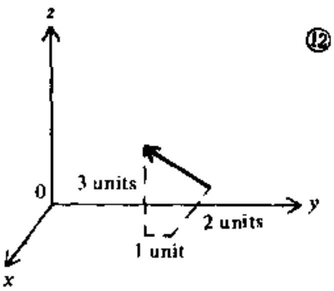


FIGURE 11.11

One obvious way to describe a vector is simply to give the coordinates of the initial and terminal points of the directed line segment associated with it. But as our comments on velocity and force suggested, one is normally concerned more with the direction and length of a vector than with its initial
(12) and terminal points. The vector appearing in Figure 11.11 represents a change of 2 units in the positive x direction, 1 unit in the negative y direction, and 3 units in the positive z direction. These three numbers identify the direction and length of the vector, and hence we can conveniently identify the vector with the ordered triple $(2, -1, 3)$. More generally, if a directed line segment has initial point (x_0, y_0, z_0) and terminal point (x_1, y_1, z_1), then the corresponding vector represents a change of $x_1 - x_0$ units in the positive x direction, $y_1 - y_0$ units in the positive y direction, and $z_1 - z_0$ units in the positive z direction. Therefore we can group these three differences into the ordered triple $(x_1 - x_0, y_1 - y_0, z_1 - z_0)$ and identify the vector with this ordered triple.

DEFINITION 11.1

A *vector* is an ordered triple (a_1, a_2, a_3) of numbers. The numbers a_1, a_2, and a_3 are called the *components* of the vector. The vector associated with the directed line segment with initial point $P = (x_0, y_0, z_0)$ and terminal point $Q = (x_1, y_1, z_1)$ is $(x_1 - x_0, y_1 - y_0, z_1 - z_0)$ and is
(13) denoted $\overrightarrow{PQ}$.

Two vectors (a_1, a_2, a_3) and (b_1, b_2, b_3) are equal if and only if their components are equal, that is,

$$a_1 = b_1, \quad a_2 = b_2, \quad \text{and} \quad a_3 = b_3$$

In the arrow notation, $\overrightarrow{PQ} = \overrightarrow{RS}$ if $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are represented by the same ordered triple.

Example 3 Let $P = (1, 3, 7)$, $Q = (-1, 0, 6)$, $R = (0, -1, -2)$, and $S = (-2, -4, -3)$. Show that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are the same vector.

Solution Applying Definition 11.1 to the components of P, Q, R, and S, we find that

$$\overrightarrow{PQ} = (-2, -3, -1) = \overrightarrow{RS} \quad \square$$

In print, vectors are almost always represented by boldface letters, although the choice of the particular letters used for vectors varies widely
(14) (depending on the context). To distinguish vectors from numbers, which are

also called *scalars*. we will normally denote vectors by lower case boldface letters near the beginning of the alphabet, such as **a**, **b**, and **c**. Other letters denote special vectors; for example, the zero vector (0, 0, 0) is denoted **0**. Since it is difficult to write a boldface letter by hand, vectors are usually written by placing an arrow over a symbol or expression. Thus we would write **a** as $\vec{a}$.

Each vector $\mathbf{a} = (a_1, a_2, a_3)$ can be associated with a directed line segment having an arbitrary initial point P (Figure 11.12), and different initial points in space give us different representations of the same vector. If P is the origin, then the vector $\mathbf{a} = (a_1, a_2, a_3)$ is associated with the point (a_1, a_2, a_3) in space and with the directed line segment from the origin to (a_1, a_2, a_3) (Figure 11.12).

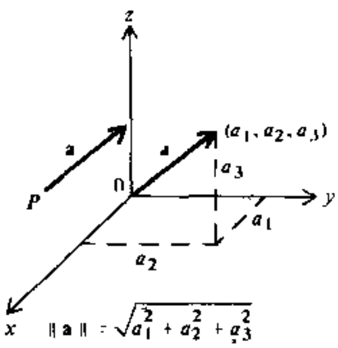


FIGURE 11.12

A natural way to assign a length to a vector $\mathbf{a} = (a_1, a_2, a_3)$ is assign it the length of the directed line segment from the origin to the point (a_1, a_2, a_3).

DEFINITION 11.2

The *length* (or *norm*) of a vector $\mathbf{a} = (a_1, a_2, a_3)$ is denoted $\|\mathbf{a}\|$ and is defined by 16

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

A *unit vector* is a vector having length 1.

For example, if $\mathbf{a} = (-1, 3, 6)$, then

$$\|\mathbf{a}\| = \sqrt{(-1)^2 + 3^2 + 6^2} = \sqrt{46}$$

This is the same number we found in Example 1 for the distance between the point $(-1, 3, 6)$ and the origin. In general, the length $\|\overrightarrow{PQ}\|$ of the vector $\overrightarrow{PQ}$ is the same as the distance $|PQ|$ between P and Q.

There are three special unit vectors that will simplify describing and operating on vectors:

$$\mathbf{i} = (1, 0, 0), \qquad \mathbf{j} = (0, 1, 0), \qquad \mathbf{k} = (0, 0, 1)$$

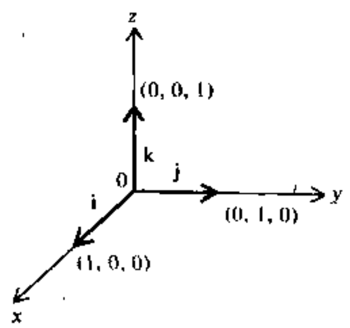


FIGURE 11.13

(Figure 11.13). It is obvious from the definition of length that

$$\|\mathbf{i}\| = \|\mathbf{j}\| = \|\mathbf{k}\| = 1$$

so that **i**, **j**, and **k** are indeed unit vectors.

Combinations of Vectors

Numbers can be added, subtracted, and multiplied; vectors can be combined in similar ways.

DEFINITION 11.3

Let $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ be vectors, and let c be a number. Then we define the *sum* $\mathbf{a} + \mathbf{b}$, the *difference* $\mathbf{a} - \mathbf{b}$, and the *scalar multiple* $c\mathbf{a}$ by

$$\mathbf{a} + \mathbf{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
$$\mathbf{a} - \mathbf{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$
$$c\mathbf{a} = (ca_1, ca_2, ca_3)$$

Sometimes we write $\mathbf{a}/c$ for $(1/c)\mathbf{a}$. Thus we might express $\frac{1}{5}\mathbf{a}$ as $\mathbf{a}/5$.

Two types of products of vectors will be defined in Sections 11.2 and 11.3. However, we will not define any quotients of vectors.

There are many laws governing the operations in Definition 11.3. For example

$$\mathbf{0} + \mathbf{a} = \mathbf{a} + \mathbf{0} = \mathbf{a} \qquad \mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$
$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-1)\mathbf{b} \qquad c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$$
$$0\mathbf{a} = \mathbf{0} \qquad 1\mathbf{a} = \mathbf{a}$$
$$\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$$

Example 4 Let $\mathbf{a} = (1, -3, 2)$ and $\mathbf{b} = (-4, -1, 0)$. Find $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, and $-\frac{1}{2}\mathbf{a}$.

Solution From the definitions we find that

$$\mathbf{a} + \mathbf{b} = (1 + (-4), -3 + (-1), 2 + 0) = (-3, -4, 2)$$
$$\mathbf{a} - \mathbf{b} = (1 - (-4), -3 - (-1), 2 - 0) = (5, -2, 2)$$
$$-\frac{1}{2}\mathbf{a} = \left(-\frac{1}{2}, \frac{3}{2}, -1\right) \quad \square$$

Using addition and scalar multiplication of vectors, we can express any vector $\mathbf{a} = (a_1, a_2, a_3)$ as a combination of the special unit vectors **i**, **j**, and **k**. Indeed

$$\begin{aligned}\mathbf{a} = (a_1, a_2, a_3) &= (a_1, 0, 0) + (0, a_2, 0) + (0, 0, a_3)\\ &= a_1(1, 0, 0) + a_2(0, 1, 0) + a_3(0, 0, 1)\\ &= a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\end{aligned}$$

In other words

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} \tag{2}$$

For example,

$$(1, -3, 2) = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

If $P = (x_0, y_0, z_0)$ and $Q = (x_1, y_1, z_1)$, then we can express $\overrightarrow{PQ}$ in the form of (2):

$$\overrightarrow{PQ} = (x_1 - x_0)\mathbf{i} + (y_1 - y_0)\mathbf{j} + (z_1 - z_0)\mathbf{k}$$

In our later study of calculus we will often find it useful to write a vector in the form of (2). Since the components of a vector **a** are the coefficients of the unit vectors **i**, **j**, and **k** in (2), we sometimes call a_1 the **i** component, a_2 the **j** component, and a_3 the **k** component of **a**.

Using the notation in (2), we restate the formulas for the length, sum, difference, and scalar multiple of vectors:

$$\|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\| = \sqrt{a_1^2 + a_2^2 + a_3^2} \tag{3}$$

$$\begin{aligned}(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) + (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) &= (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j}\\ &\quad + (a_3 + b_3)\mathbf{k}\\ (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) - (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) &= (a_1 - b_1)\mathbf{i} + (a_2 - b_2)\mathbf{j}\\ &\quad + (a_3 - b_3)\mathbf{k}\\ c(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) &= ca_1\mathbf{i} + ca_2\mathbf{j} + ca_3\mathbf{k}\end{aligned}$$

It follows from (3) that

$$|a_1| \le \|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\|, \qquad |a_2| \le \|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\|$$
$$|a_3| \le \|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}\|$$

In summary, we list the various ways of describing a single vector:

(a_1, a_2, a_3), an ordered triple of numbers
(a_1, a_2, a_3), a point in space
a directed line segment with initial point (x_0, y_0, z_0) and terminal point $(x_0 + a_1, y_0 + a_2, z_0 + a_3)$
$a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

Geometric Interpretations of Vector Operations

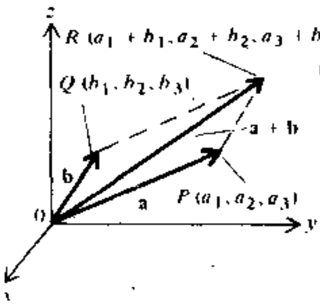


FIGURE 11.14

(18) The many geometric and physical meanings that can be attached to combin-
ations of vectors make vectors very powerful tools for scientists. To interpret
(19) the sum of two vectors geometrically, we begin by letting $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$. Then we can think of **a**, **b**, and $\mathbf{a} + \mathbf{b}$ as directed line segments from the origin to the points P, Q, and R having coordinates (a_1, a_2, a_3), (b_1, b_2, b_3), and $(a_1 + b_1, a_2 + b_2, a_3 + b_3)$, respec-
(20) tively (Figure 11.14). Now observe that if the directed line segment representing **b** is placed so that its initial point is P, then its terminal point will be R. Thus the vector $\mathbf{a} + \mathbf{b}$ can be obtained by placing the initial point of a representative of **b** on the terminal point of **a**. Notice that the two representations of **a** and **b** (solid and dashed) shown in Figure 11.14 determine a parallelogram whose diagonal is $\mathbf{a} + \mathbf{b}$.

Next we let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ be a vector and c be any number. It follows from the definition of length that

$$\|c\mathbf{a}\| = |c|\,\|\mathbf{a}\|$$

When **a** is multiplied by a positive number c, its length is multiplied by c and
(21) its direction does not change (Figure 11.15). The vector $-\mathbf{a}$ has the same length as **a** but has the opposite direction (Figure 11.15). Thus if **a** is multiplied by a negative number c, its length is multiplied by $|c|$ and its direction is reversed (Figure 11.15). Two nonzero vectors whose initial points are the origin are considered parallel only if they lie on the same line through the origin, and in that case they are multiples of one another. Hence we make the following definition.

DEFINITION 11.4

Two nonzero vectors a and b are *parallel* if and only if there is a
(22) **number c such that $\mathbf{b} = c\mathbf{a}$.**

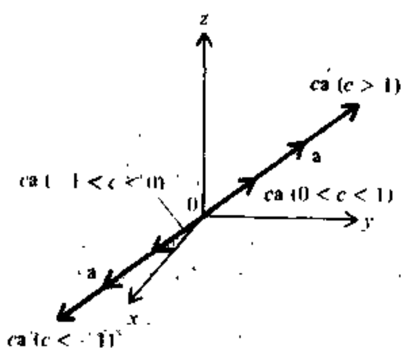


FIGURE 11.15

Example 5 Let $\mathbf{a} = 6\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. Determine whether $\mathbf{a}$ and $\mathbf{b}$ are parallel.

Solution You can check that $\mathbf{b} = -\frac{1}{2}\mathbf{a}$; consequently $\mathbf{a}$ and $\mathbf{b}$ are parallel. □

The *unit vector in the direction of a nonzero vector* $\mathbf{a}$ is $\mathbf{a}/\|\mathbf{a}\|$. Such vectors will be particularly important in Chapter 12.

Example 6 Find the unit vector in the direction of $4\mathbf{i} - \mathbf{j} - 3\mathbf{k}$.

Solution Since

$$\|4\mathbf{i} - \mathbf{j} - 3\mathbf{k}\| = \sqrt{4^2 + (-1)^2 + (-3)^2} = \sqrt{26}$$

it follows that the required vector is

$$\frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} - 3\mathbf{k}) \quad \square$$

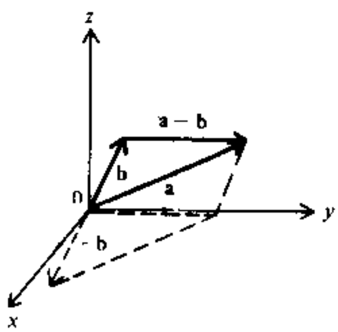


FIGURE 11.16

The difference $\mathbf{a} - \mathbf{b}$ is the sum $\mathbf{a} + (-\mathbf{b})$. Therefore our interpretations of sum and constant multiples of vectors tell us that if the directed line segments representing $\mathbf{a}$ and $\mathbf{b}$ both have the same initial point P (such as the origin), then the directed line segment from the terminal point of $\mathbf{b}$ to the terminal point of $\mathbf{a}$ represents the vector $\mathbf{a} - \mathbf{b}$ (Figure 11.16). We will occasionally employ this geometric interpretation of the difference of two vectors.

Applications of Vector Addition

Vector addition has been defined as in Definition 11.3 because so many physical quantities combine according to vector addition. For example, if several forces act at the same point P on an object, then the object reacts as though a single force equal to the vector sum of the several forces acts on the object

Example 7 Two children pull a sled by ropes 4 feet long attached to the front center of the sled. The smaller child holds the rope 2 feet above the sled and 2 feet to the side of the point of attachment, and the larger child holds the rope 3 feet above the sled and 2 feet to the opposite side of the point of attachment (Figure 11.17(a)). The smaller child exerts a force $\mathbf{F}_1$ of magnitude 5 pounds, and the taller child a force $\mathbf{F}_2$ of magnitude 7 pounds. Find the resultant force $\mathbf{F}_1 + \mathbf{F}_2$ on the sled.