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复变函数及应用

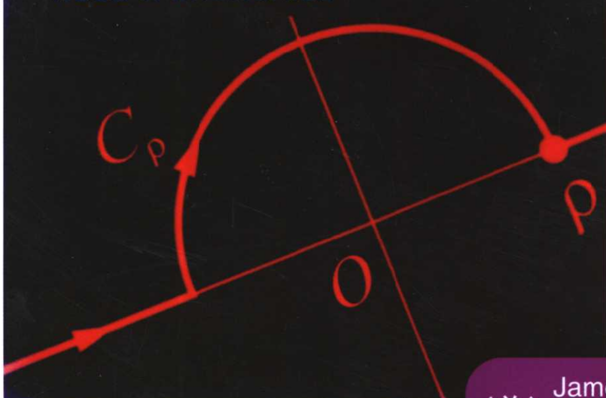
（英文版·第7版）

INTERNATIONAL EDITION

COMPLEX VARIABLES
and
APPLICATIONS

SEVENTH EDITION

JAMES WARD BROWN
RUEL V. CHURCHILL



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James Ward Brown is Professor of Mathematics at The University of Michigan–Dearborn. He earned his A.B. in Physics at Harvard University and his A.M. and Ph.D. in Mathematics at The University of Michigan in Ann Arbor, where he was an Institute of Science and Technology Predoctoral Fellow. He was coauthor with Dr. Churchill of the sixth edition of *Fourier Series and Boundary Value Problems.* A past director of a research grant from the National Science Foundation, he is the recipient of a Distinguished Teaching Award from his institution, as well as a Distinguished Faculty Award from the Michigan Association of Governing Boards of Colleges and Universities. He is listed in *Who's Who in America.*

Ruel V. Churchill is Late Professor of Mathematics at The University of Michigan, where he began teaching in 1922. He received his B.S. in Physics from the University of Chicago and his M.S. in Physics and Ph.D. in Mathematics from The University of Michigan. He was coauthor with Dr. Brown of the recent sixth edition of *Fourier Series and Boundary Value Problems,* a classic text that he first wrote over sixty years ago. He was also the author of *Operational Mathematics,* now in its third edition. Throughout his long and productive career, Dr. Churchill held various offices in the Mathematical Association of America and in other mathematical societies and councils.

To the memory of my father,
GEORGE H. BROWN,
and of my long-time friend and coauthor,
RUEL V. CHURCHILL.
These distinguished men of science for years influenced
the careers of many people, including myself.

J.W.B.

PREFACE

This book is a revision of the sixth edition, published in 1996. That edition has served, just as the earlier ones did, as a textbook for a one-term introductory course in the theory and application of functions of a complex variable. This edition preserves the basic content and style of the earlier editions, the first two of which were written by the late Ruel V. Churchill alone.

In this edition, the main changes appear in the first nine chapters, which make up the core of a one-term course. The remaining three chapters are devoted to physical applications, from which a selection can be made, and are intended mainly for self-study or reference.

Among major improvements, there are thirty new figures; and many of the old ones have been redrawn. Certain sections have been divided up in order to emphasize specific topics, and a number of new sections have been devoted exclusively to examples. Sections that can be skipped or postponed without disruption are more clearly identified in order to make more time for material that is absolutely essential in a first course, or for selected applications later on. Throughout the book, exercise sets occur more often than in earlier editions. As a result, the number of exercises in any given set is generally smaller, thus making it more convenient for an instructor in assigning homework.

As for other improvements in this edition, we mention that the introductory material on mappings in Chap. 2 has been simplified and now includes mapping properties of the exponential function. There has been some rearrangement of material in Chap. 3 on elementary functions, in order to make the flow of topics more natural. Specifically, the sections on logarithms now directly follow the one on the exponential function; and the sections on trigonometric and hyberbolic functions are now closer to the ones on their inverses. Encouraged by comments from users of the book in the past several years, we have brought some important material out of the exercises and into the text. Examples of this are the treatment of isolated zeros of analytic functions in Chap. 6 and the discussion of integration along indented paths in Chap. 7.

The *first objective* of the book is to develop those parts of the theory which are prominent in applications of the subject. The *second objective* is to furnish an introduction to applications of residues and conformal mapping. Special emphasis is given to the use of conformal mapping in solving boundary value problems that arise in studies of heat conduction, electrostatic potential, and fluid flow. Hence the book may be considered as a companion volume to the authors' "Fourier Series and Boundary Value Problems" and Ruel V. Churchill's "Operational Mathematics," where other classical methods for solving boundary value problems in partial differential equations are developed. The latter book also contains further applications of residues in connection with Laplace transforms.

This book has been used for many years in a three-hour course given each term at The University of Michigan. The classes have consisted mainly of seniors and graduate students majoring in mathematics, engineering, or one of the physical sciences. Before taking the course, the students have completed at least a three-term calculus sequence, a first course in ordinary differential equations, and sometimes a term of advanced calculus. In order to accommodate as wide a range of readers as possible, there are footnotes referring to texts that give proofs and discussions of the more delicate results from calculus that are occasionally needed. Some of the material in the book need not be covered in lectures and can be left for students to read on their own. If mapping by elementary functions and applications of conformal mapping are desired earlier in the course, one can skip to Chapters 8, 9, and 10 immediately after Chapter 3 on elementary functions.

Most of the basic results are stated as theorems or corollaries, followed by examples and exercises illustrating those results. A bibliography of other books, many of which are more advanced, is provided in Appendix 1. A table of conformal transformations useful in applications appears in Appendix 2.

In the preparation of this edition, continual interest and support has been provided by a number of people, many of whom are family, colleagues, and students. They include Jacqueline R. Brown, Ronald P. Morash, Margret H. Höft, Sandra M. Weber, Joyce A. Moss, as well as Robert E. Ross and Michelle D. Munn of the editorial staff at McGraw-Hill Higher Education.

James Ward Brown

CONTENTS

CHAPTER

1

COMPLEX NUMBERS

In this chapter, we survey the algebraic and geometric structure of the complex number system. We assume various corresponding properties of real numbers to be known.

1. SUMS AND PRODUCTS

Complex numbers can be defined as ordered pairs (x, y) of real numbers that are to be interpreted as points in the *complex plane,* with rectangular coordinates x and y, just as real numbers x are thought of as points on the real line. When real numbers x are displayed as points $(x, 0)$ on the *real axis,* it is clear that the set of complex numbers includes the real numbers as a subset. Complex numbers of the form $(0, y)$ correspond to points on the y axis and are called *pure imaginary numbers.* The y axis is, then, referred to as the *imaginary axis.*

It is customary to denote a complex number (x, y) by z, so that

(1) $$z = (x, y).$$

The real numbers x and y are, moreover, known as the *real and imaginary parts* of z, respectively; and we write

(2) $$\operatorname{Re} z = x, \quad \operatorname{Im} z = y.$$

Two complex numbers $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ are equal whenever they have the same real parts and the same imaginary parts. Thus the statement $z_1 = z_2$ means that z_1 and z_2 correspond to the same point in the complex, or z, plane.

The *sum* $z_1 + z_2$ and the *product* $z_1 z_2$ of two complex numbers $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ are defined as follows:

(3) $$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2),$$

(4) $$(x_1, y_1)(x_2, y_2) = (x_1 x_2 - y_1 y_2, y_1 x_2 + x_1 y_2).$$

Note that the operations defined by equations (3) and (4) become the usual operations of addition and multiplication when restricted to the real numbers:

$$(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0),$$
$$(x_1, 0)(x_2, 0) = (x_1 x_2, 0).$$

The complex number system is, therefore, a natural extension of the real number system.

Any complex number $z = (x, y)$ can be written $z = (x, 0) + (0, y)$, and it is easy to see that $(0, 1)(y, 0) = (0, y)$. Hence

$$z = (x, 0) + (0, 1)(y, 0);$$

and, if we think of a real number as either x or $(x, 0)$ and *let i denote the imaginary number* $(0, 1)$ (see Fig. 1), it is clear that*

(5) $$z = x + iy.$$

Also, with the convention $z^2 = zz$, $z^3 = zz^2$, etc., we find that

$$i^2 = (0, 1)(0, 1) = (-1, 0),$$

or

(6) $$i^2 = -1.$$

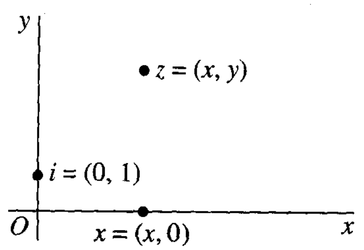

FIGURE 1

In view of expression (5), definitions (3) and (4) become

(7) $$(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2),$$

(8) $$(x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(y_1 x_2 + x_1 y_2).$$

---

* In electrical engineering, the letter j is used instead of i.

Observe that the right-hand sides of these equations can be obtained by formally manipulating the terms on the left as if they involved only real numbers and by replacing i^2 by -1 when it occurs.

2. BASIC ALGEBRAIC PROPERTIES

Various properties of addition and multiplication of complex numbers are the same as for real numbers. We list here the more basic of these algebraic properties and verify some of them. Most of the others are verified in the exercises.

The commutative laws

$$z_1 + z_2 = z_2 + z_1, \quad z_1 z_2 = z_2 z_1 \tag{1}$$

and the associative laws

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3), \quad (z_1 z_2) z_3 = z_1 (z_2 z_3) \tag{2}$$

follow easily from the definitions in Sec. 1 of addition and multiplication of complex numbers and the fact that real numbers obey these laws. For example, if $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$, then

$$z_1 + z_2 = (x_1 + x_2, y_1 + y_2) = (x_2 + x_1, y_2 + y_1) = z_2 + z_1.$$

Verification of the rest of the above laws, as well as the distributive law

$$z(z_1 + z_2) = z z_1 + z z_2, \tag{3}$$

is similar.

According to the commutative law for multiplication, $iy = yi$. Hence one can write $z = x + yi$ instead of $z = x + iy$. Also, because of the associative laws, a sum $z_1 + z_2 + z_3$ or a product $z_1 z_2 z_3$ is well defined without parentheses, as is the case with real numbers.

The additive identity $0 = (0, 0)$ and the multiplicative identity $1 = (1, 0)$ for real numbers carry over to the entire complex number system. That is,

$$z + 0 = z \quad \text{and} \quad z \cdot 1 = z \tag{4}$$

for every complex number z. Furthermore, 0 and 1 are the only complex numbers with such properties (see Exercise 9).

There is associated with each complex number $z = (x, y)$ an additive inverse

$$-z = (-x, -y), \tag{5}$$

satisfying the equation $z + (-z) = 0$. Moreover, there is only one additive inverse for any given z, since the equation $(x, y) + (u, v) = (0, 0)$ implies that $u = -x$ and $v = -y$. Expression (5) can also be written $-z = -x - iy$ without ambiguity since

(Exercise 8) $-(iy) = (-i)y = i(-y)$. Additive inverses are used to define subtraction:

(6) $$z_1 - z_2 = z_1 + (-z_2).$$

So if $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$, then

(7) $$z_1 - z_2 = (x_1 - x_2, y_1 - y_2) = (x_1 - x_2) + i(y_1 - y_2).$$

For any *nonzero* complex number $z = (x, y)$, there is a number z^{-1} such that $zz^{-1} = 1$. This multiplicative inverse is less obvious than the additive one. To find it, we seek real numbers u and v, expressed in terms of x and y, such that

$$(x, y)(u, v) = (1, 0).$$

According to equation (4), Sec. 1, which defines the product of two complex numbers, u and v must satisfy the pair

$$xu - yv = 1, \quad yu + xv = 0$$

of linear simultaneous equations; and simple computation yields the unique solution

$$u = \frac{x}{x^2 + y^2}, \quad v = \frac{-y}{x^2 + y^2}.$$

So *the* multiplicative inverse of $z = (x, y)$ is

(8) $$z^{-1} = \left(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2}\right) \qquad (z \neq 0).$$

The inverse z^{-1} is not defined when $z = 0$. In fact, $z = 0$ means that $x^2 + y^2 = 0$; and this is not permitted in expression (8).

EXERCISES

1. Verify that
   (*a*) $(\sqrt{2} - i) - i(1 - \sqrt{2}i) = -2i$; (*b*) $(2, -3)(-2, 1) = (-1, 8)$;
   (*c*) $(3, 1)(3, -1)\left(\frac{1}{5}, \frac{1}{10}\right) = (2, 1)$.
2. Show that
   (*a*) $\operatorname{Re}(iz) = -\operatorname{Im} z$; (*b*) $\operatorname{Im}(iz) = \operatorname{Re} z$.
3. Show that $(1 + z)^2 = 1 + 2z + z^2$.
4. Verify that each of the two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.
5. Prove that multiplication is commutative, as stated in the second of equations (1), Sec. 2.
6. Verify
   (*a*) the associative law for addition, stated in the first of equations (2), Sec. 2;
   (*b*) the distributive law (3), Sec. 2.

7. Use the associative law for addition and the distributive law to show that

$$z(z_1 + z_2 + z_3) = zz_1 + zz_2 + zz_3.$$

8. By writing $i = (0, 1)$ and $y = (y, 0)$, show that $-(iy) = (-i)y = i(-y)$.

9. (*a*) Write $(x, y) + (u, v) = (x, y)$ and point out how it follows that the complex number $0 = (0, 0)$ is unique as an additive identity.

(*b*) Likewise, write $(x, y)(u, v) = (x, y)$ and show that the number $1 = (1, 0)$ is a unique multiplicative identity.

10. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing

$$(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$$

and then solving a pair of simultaneous equations in x and y.

Suggestion: Use the fact that no real number x satisfies the given equation to show that $y \neq 0$.

Ans. $z = \left(-\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right)$.

3. FURTHER PROPERTIES

In this section, we mention a number of other algebraic properties of addition and multiplication of complex numbers that follow from the ones already described in Sec. 2. Inasmuch as such properties continue to be anticipated because they also apply to real numbers, the reader can easily pass to Sec. 4 without serious disruption.

We begin with the observation that the existence of multiplicative inverses enables us to show that *if a product z_1z_2 is zero, then so is at least one of the factors z_1 and z_2*. For suppose that $z_1z_2 = 0$ and $z_1 \neq 0$. The inverse z_1^{-1} exists; and, according to the definition of multiplication, any complex number times zero is zero. Hence

$$z_2 = 1 \cdot z_2 = (z_1^{-1}z_1)z_2 = z_1^{-1}(z_1z_2) = z_1^{-1} \cdot 0 = 0.$$

That is, if $z_1z_2 = 0$, either $z_1 = 0$ or $z_2 = 0$; or possibly both z_1 and z_2 equal zero. Another way to state this result is that *if two complex numbers z_1 and z_2 are nonzero, then so is their product z_1z_2.*

Division by a nonzero complex number is defined as follows:

$$\frac{z_1}{z_2} = z_1z_2^{-1} \qquad (z_2 \neq 0). \tag{1}$$

If $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$, equation (1) here and expression (8) in Sec. 2 tell us that

$$\frac{z_1}{z_2} = (x_1, y_1)\left(\frac{x_2}{x_2^2 + y_2^2}, \frac{-y_2}{x_2^2 + y_2^2}\right) = \left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}, \frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2}\right).$$

That is,

$$\frac{z_1}{z_2} = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{y_1x_2 - x_1y_2}{x_2^2 + y_2^2} \qquad (z_2 \neq 0). \tag{2}$$

Although expression (2) is not easy to remember, it can be obtained by writing (see Exercise 7)

$$\frac{z_1}{z_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)}, \tag{3}$$

multiplying out the products in the numerator and denominator on the right, and then using the property

$$\frac{z_1 + z_2}{z_3} = (z_1 + z_2)z_3^{-1} = z_1z_3^{-1} + z_2z_3^{-1} = \frac{z_1}{z_3} + \frac{z_2}{z_3} \qquad (z_3 \neq 0). \tag{4}$$

The motivation for starting with equation (3) appears in Sec. 5.

There are some expected identities, involving quotients, that follow from the relation

$$\frac{1}{z_2} = z_2^{-1} \qquad (z_2 \neq 0), \tag{5}$$

which is equation (1) when $z_1 = 1$. Relation (5) enables us, for example, to write equation (1) in the form

$$\frac{z_1}{z_2} = z_1\left(\frac{1}{z_2}\right) \qquad (z_2 \neq 0). \tag{6}$$

Also, by observing that (see Exercise 3)

$$(z_1z_2)(z_1^{-1}z_2^{-1}) = (z_1z_1^{-1})(z_2z_2^{-1}) = 1 \qquad (z_1 \neq 0,\ z_2 \neq 0),$$

and hence that $(z_1z_2)^{-1} = z_1^{-1}z_2^{-1}$, one can use relation (5) to show that

$$\frac{1}{z_1z_2} = (z_1z_2)^{-1} = z_1^{-1}z_2^{-1} = \left(\frac{1}{z_1}\right)\left(\frac{1}{z_2}\right) \qquad (z_1 \neq 0,\ z_2 \neq 0). \tag{7}$$

Another useful identity, to be derived in the exercises, is

$$\frac{z_1z_2}{z_3z_4} = \left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right) \qquad (z_3 \neq 0,\ z_4 \neq 0). \tag{8}$$