TEXT BOOKS FOR COLLEGES AND UNIVERSITIES

高等学校教学用书

ENGINEERING MATRIX METHODS

工程矩阵方法

Seung-Ping Li Professor
Fan Changxin Professor

李尚平（美籍）
樊昌信
编著

PUBLISHING HOUSE OF ELECTRONIC INDUSTRY

电子工业出版社

工 程 矩 阵 方 法

美国加州州立综合技术大学
电气和计算机工程系　教授　李尚平

中 国 西 安 电 子 科 技 大 学
信 息 工 程 系 教 授　樊昌信

電子工業出版社

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ENGINEERING MATRIX METHODS

SEUNG-PING LI, Professor

Department of Electrical and Computer Engineering
California State Polytechnic University,
Pomona U. S. A.

and

FAN CHANGXIN, Professor

Department of Information Engineering
XIDIAN University,
Xi'an China

Publishing House of Electronic Industry

内 容 提 要

本书用英汉对照的方法，简要地讲述了矩阵代数和矩阵微积分。主要内容包括矩阵的基本定义及运算，矩阵乘法和逆，特征矢量方程，矩阵函数，矢量空间和线性变换，线性规划等12章。每章均有详实的例题和习题，书后附录中给出了多个 FORTRAN 和 BASIC 计算机程序及各章的题解。

本书供理工科大学高年级学生和研究生使用，也可供科研和工程技术人员阅读。

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李尚平
樊昌信 著

责任编辑 吴金生

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电子工业出版社出版(北京万寿路)

北京科技印刷厂印刷

新华书店北京发行所发行 各地新华书店经售

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开本：787×1092 1/32 印张：14.75 字数：331千字

1988年3月第1版 1988年4月第1次印刷

印数：2500册 定价：2.50元

ISBN 7-5053-0079-2/O1

PREFACE

The purpose of this book is to treat matrix algebra and calculus in an elementary and self-taught manner, with a view to quickly applying them in the numerical solution of engineering problems. Set theory along with the terminology of modern linear algebra is not assumed. The prerequisites for this text are simply calculus through differential equations, computer programming in Fortran or Basic, and a degree of maturity in the basic engineering sciences.

The book contains enough materials for a semester course at the undergraduate senior or beginning graduate level. For a shorter course some topics may be left out at the discretion of the instructor. Whenever appropriate, an effort has been made to provide problems with a practical flavor to motivate the student. Answers to problems are provided, except those requiring analytical proofs.

A number of computer programs in Fortran and Basic are provided in the appendices. These have been developed and tried out by students at California State Polytechnic University, Pomona. The use of these programs results in the many illustrative examples throughout the text. The authors have drawn materials from different fields of engineering, so the book is intended to be interdisciplinary. For students already with some background in linear algebra, the first three chapters may be omitted. They are included for those who need an introduction to the subject or a quick review.

Chapters One through Three deal with determinants and elementary matrix operations. Chapters Four through Six cover the eigenvalue problem, its applications, and numerical solutions. General matrix functions are described in Chapter Seven. Chapters Eight and Nine concentrate on differential equations, while applications of systems of simultaneous equations are deferred to Chapter Ten, which may of course be taken up right after Chapter Three if desired. Chapter Eleven deals with vector spaces and linear transformations, as preparation to the study of linear systems. Chapter Twelve concludes with an introduction to Linear Programming and its diverse applications.

The expected outcome of a course using this text is the ability to readily solve numerically eigenvalue problems, problems involving systems of simultaneous equations and ordinary differential equations, and optimization of linear functions subject to linear constraints. Computational speed and error analysis are not treated in this elementary text. For this the appropriate literature should be consulted.

The authors hope that the publication of this text will help in a small way the academic exchange between the People's Republic of China and the United States.

序

本书以便于读者自学的方式讲述矩阵代数和矩阵微积分的基本内容，使之能迅速将其应用于以数值方法解工程问题。书中没有涉及集合论和使用现代线性代数的术语。本书的先修课程仅有微积分、微分方程、用 FORTRAN 或 BASIC 编计算机程序，并要求读者在一定程度上熟悉基础工程科学。

本书包含的材料可供大学四年级或研究生初入学水平的一学期课程使用。对于学时较少的课程，教师可自行略去某些论题。书中在适当地方尽量给出了一些联系实际的问题，以引导学生。除了那些要求分析证明的习题外，书末给出了全部答案。

附录中提供了多个 FORTRAN 和 BASIC 计算机程序。这些程序是由加州州立综合技术大学（波蒙那校区）的学生们编写并运行过的。使用这些程序得出了本书中许多示例。著者取材自不同的工程领域，使本书适用于多学科。前三章对于已经具有线性代数基础的学生可以略过，它是为那些需要对此主题有一初步概念或简短复习的读者准备的。

第一章至第三章讨论行列式和基本矩阵运算。第四章至第六章包括特征值问题及其应用与数值解法。普通矩阵函数在第七章中讲述。第八和九章集中讨论微分方程，而联立方程组的应用则推后至第十章。当然，如果需要的话，它可以放在第三章之后学。第十一章讨论矢量空间和线性变换，以作为学习线性系统的准备。第十二章对线性规划及其在各方面的应用作了简要介绍。

著者希望本书的出版将会多少有益于中美两国之间的学术交流。

CONTENTS

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CHAPTER 1

Matrices——Basic Definitions and Operations

1.1 Notations and Definitions

A matrix $\boldsymbol{A}$ is a rectangular array of elements denoted by

$$\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix}$$

The element, a_{ij}, is located in the i^{th} row and j^{th} column. If $m=n$, $\boldsymbol{A}$ becomes a square matrix of order n; otherwise in general it is of order (m, n) or "m by n". Some particular types of matrices and special terms are defined as follows:

(i) Matrix equality: Matrix $\boldsymbol{A}$ is equal to matrix $\boldsymbol{B}$ if and only if they are of the same order (m,n) and each element in $\boldsymbol{A}$ is equal to the corresponding element in $\boldsymbol{B}$, or, $a_{ij}=b_{ij}$.

(ii) Column matrix $(m,1)$ has m rows and one column.

$$\text{e.g. } \boldsymbol{A}(3,1)=\begin{bmatrix} 1 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

(iii) Row matrix $(1,n)$ has one row and n columns.

$$\text{e.g. } \boldsymbol{A}(1,3) = [1 \ \sqrt{3} \ 0]$$

(iv) Real matrix (m, n) has only real elements.

$$\therefore \quad (a_{ij})^* = a_{ij}.$$

(v) Transposition: when a matrix is transposed the rows and columns are interchanged.

$$\text{e.g. } \boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \xrightarrow{\text{transposed}} \boldsymbol{A}' = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

It is seen that in $\boldsymbol{A}$ above, the first column becomes the first row, and the second column the second row in $\boldsymbol{A}'$. In fact the mechanics of transposing a square matrix is to hold the "diagonal" elements in place (like 1 and 4 above) and then "reflect" the "off-diagonal" elements (like 2 and 3) across the diagonal. Mathematically, such operation is denoted by: $(a_{ij})' = a_{ji}$. This means that the element $(a_{ij})'$ in $\boldsymbol{A}'$ is the element a_{ji} in $\boldsymbol{A}$.

(vi) Non-square matrices can also be transposed. For example, a row matrix when transposed becomes a column matrix, and vice versa.

(vii) Symmetric matrix is one that is equal to its own transpose.

$$\boldsymbol{A} = \boldsymbol{A}' \text{ or } a_{ij} = a_{ji}$$

(viii) Skew-symmetric matrix is one that has $a_{ij} = -a_{ji}$. In other words, the off-diagonal elements are the negative of their images across the diagonal.

$$\text{e.g. } \boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \xrightarrow{\text{transposed}} \boldsymbol{A}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -\boldsymbol{A}$$

(Since we have not yet defined matrix arithmetic operations at this point, we assume that if *every* element a_{ij} in $\boldsymbol{A}$ has its sign changed, then the new matrix is represented as $-\boldsymbol{A}$.)

(ix) Complex conjugate of a matrix $\boldsymbol{A}$ is a matrix derived from $\boldsymbol{A}$ by taking the complex conjugate of all the elements in $\boldsymbol{A}$.

$$\text{e.g. } \boldsymbol{A} = \begin{bmatrix} 2+i & 3 \\ i & 1-i \end{bmatrix}, \quad \boldsymbol{A}^* = \begin{bmatrix} 2-i & 3 \\ -i & 1+i \end{bmatrix}$$

(x) Hermitian matrix is one which is equal to the transpose of its own complex conjugate.

$$\text{e.g. } \boldsymbol{A} = \begin{bmatrix} 4 & 3+i \\ 3-i & 2 \end{bmatrix}, \quad \boldsymbol{A}^* = \begin{bmatrix} 4 & 3-i \\ 3+i & 2 \end{bmatrix}$$

$$(\boldsymbol{A}^*)' = \begin{bmatrix} 4 & 3+i \\ 3-i & 2 \end{bmatrix} = \boldsymbol{A}$$

(xi) Skew-Hermitian matrix is one which is equal to the negative of the transpose of its complex conjugate; or $\boldsymbol{A} = -(\boldsymbol{A}^*)'$.

$$\text{e.g. } \boldsymbol{A} = \begin{bmatrix} 0 & -2-i \\ 2-i & 0 \end{bmatrix}$$ can be easily shown to be Skew-Hermitian.

(xii) Zero or null matrix has all its elements zero and is denoted by $\boldsymbol{0}$, or, $a_{ij} = 0$ for all i and j.

(xiii) Diagonal matrix is a square matrix whose off-diagonal elements are zeros.

(ix) Identity or unit matrix $\boldsymbol{I}$ is a square matrix with all diagonal elements equal to one and all off-diagonal elements zero;

$$\text{or} \quad a_{ij} = \delta_{ij} = 1 \quad \text{if } i = j$$
$$= 0 \quad \text{if } i \neq j$$

1.2 Matrix Addition

Matrix $\boldsymbol{A}$ can be added to matrix $\boldsymbol{B}$ if both matrices are of the same order. The result is a matrix $\boldsymbol{C}$ whose elements c_{ij} is the sum of the corresponding elements a_{ij} and b_{ij}.

$$\text{e.g. } \boldsymbol{A}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \quad \boldsymbol{B}=\begin{bmatrix}5 & 6\\ 0 & -3\end{bmatrix},$$

$$\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{C}=\begin{bmatrix}6 & 8\\ 3 & 1\end{bmatrix}$$

It is clear, by definition, that matrix addition is associative, i.e., $\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})=(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}$.

1.3 Matrix Mutiplication by a Scalar

When matrix $\boldsymbol{A}$ is multiplied by a scalar k, the result is written as $k\boldsymbol{A}$ whose elements are ka_{ij}.

$$\text{e.g. } k=2, \quad \boldsymbol{A}=\begin{bmatrix}1 & 2\\ 3 & 4\\ 0 & 1\end{bmatrix}, \quad k\boldsymbol{A}=\begin{bmatrix}2 & 4\\ 6 & 8\\ 0 & 2\end{bmatrix}$$

$$k=-1, \quad \boldsymbol{A}=\begin{bmatrix}0 & 1\\ -1 & 2\end{bmatrix}, \quad k\boldsymbol{A}=\begin{bmatrix}0 & -1\\ 1 & -2\end{bmatrix}$$

1.4 Matrix Subtraction

Matrix subtraction, like addition, is defined only for matrices of the same order. Making use of the concept of scalar multiplication, matrix subtraction can be defined as:

$$\boldsymbol{C}\equiv\boldsymbol{A}-\boldsymbol{B}=\boldsymbol{A}+(-1)\boldsymbol{B} \quad \text{or} \quad c_{ij}=a_{ij}-b_{ij}$$

$$\text{e.g. } \begin{bmatrix}2 & -6\\ 1 & 3\end{bmatrix}-\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}=\begin{bmatrix}1 & -7\\ 2 & 3\end{bmatrix}$$

Again, the associative law applies; $\boldsymbol{A}-(\boldsymbol{B}-\boldsymbol{C})=(\boldsymbol{A}-\boldsymbol{B})+\boldsymbol{C}$.

PROBLEMS

$$\boldsymbol{A} = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 3 & -2 \\ -2 & 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \qquad \boldsymbol{D} = \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix}$$

$$\boldsymbol{E} = \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \qquad \boldsymbol{F} = \begin{bmatrix} 0 & -1-i \\ 1-i & 0 \end{bmatrix}$$

(1) Find $\boldsymbol{A} - 2\boldsymbol{B}$.

(2) Find $\boldsymbol{C} + \boldsymbol{D} + \boldsymbol{E}$.

(3) Find $\boldsymbol{E} + i\boldsymbol{F}$.

(4) Find $(1/3)(\boldsymbol{A} + \boldsymbol{B})$. Identify the resulting matrix.

(5) Which of the above matrices are real and symmetric.

(6) Identify any Hermitian or Skew-Hermitian matrices.

(7) Find $\boldsymbol{E} + \boldsymbol{E}'$. Is this skew-symmetric?

(8) Find $\boldsymbol{F} + \boldsymbol{F}^*$. Is this skew-symmetric?

(9) Find $\boldsymbol{C} + (\boldsymbol{C}^*)'$. Identify the resulting matrix.

CHAPTER 2

Determinants——Definitions and Properties

2.1 Determinants

Each square matrix of order n has associated with it a scalar quantity D called the determinant of the matrix. Let us start with defining the determinant of matrices of order 2 and 3.

Let $\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then $D=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}$

Let $\boldsymbol{A}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then

$$D=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}$$

Note that in each of the above cases,

(a) there are $n!$ terms.

(b) each term is a product of n distinct elements.

(c) there are equal number of "plus" and "minus" terms.

It is of course expected that these are the features to be found in the evaluation of any determinant of order n. A formal definition is as follows:

$$D=\Sigma\pm(a_{1i}a_{2j}a_{3k}\cdot\cdots\cdots) \tag{2.1}$$

where

(i) the number of terms in the summation is $n!$.

(ii) the row suffices appear in normal order.

(iii) the column suffices appear in some permutation of normal order, and

(iv) the positive sign is taken if normal order can be derived by an even number of interchanges of adjacent suffices; otherwise the negative sign.

$$\text{e.g. } a_{12}a_{23}a_{31} \rightarrow a_{12}a_{21}a_{33} \rightarrow a_{11}a_{22}a_{33}$$

Two interchanges are required. ∴ positive.

$$\text{e.g. } a_{11}a_{23}a_{32} \rightarrow a_{11}a_{22}a_{33}$$

One interchange is required. ∴ negative.

2.2 Evaluation of Determinants

To evaluate a determinant D directly according to equation (2.1) can be difficult if the matrix is of order larger than three. To facilitate calculation, we will turn to a "systematic" procedure which is derived from the formal definition. First let us introduce two terms, the Minor (M_{ij}) and Cofactor (C_{ij}) of a determinant D.

2.3 Minor (M_{ij})

The minor M_{ij} of the element a_{ij} of the determinant D of order n is a new determinant of order $n-1$ obtained from D by deleting the i^{th} row and j^{th} column from D.

Example 2.1. $D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$;

$$M_{13} = \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}, \quad M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix}$$

2.4 Cofactor (C_{ij})

The cofactor C_{ij} of the element a_{ij} of D is given by:

$$C_{ij} = (-1)^{i+j} M_{ij} \tag{2.2}$$

e.g. $C_{12} = (-1)^{1+2} M_{12} = -M_{12}$

$C_{31} = (-1)^{3+1} M_{31} = M_{31}$

2.5 Evaluation of high-order Determinants

With the above definitions of minor and cofactor, the formal equation (2.1) for D of any order n can be shown to reduce to:

$$\begin{aligned} D &= \sum_{j=1}^{n} a_{ij} C_{ij} \quad \text{any row } i \\ &= \sum_{i=1}^{n} a_{ij} C_{ij} \quad \text{any column } j \end{aligned} \tag{2.3}$$

e.g. $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$; choosing row $i = 2$,

$$\begin{aligned} D &= \sum_{j=1}^{3} a_{2j} C_{2j} = a_{21} C_{21} + a_{22} C_{22} + a_{23} C_{23} \\ &= a_{21}(-1)^{2+1} M_{21} + a_{22}(-1)^{2+2} M_{22} + a_{23}(-1)^{2+3} M_{23} \\ &= -a_{21} M_{21} + a_{22} M_{22} - a_{23} M_{23} \\ &= -a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{22} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - a_{23} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ &= -a_{21}(a_{12} a_{33} - a_{13} a_{32}) + a_{22}(a_{11} a_{33} - a_{13} a_{31}) \\ &\quad - a_{23}(a_{11} a_{32} - a_{12} a_{31}) \end{aligned}$$

= results shown on the beginning page of this chapter.

2.6 Fundamental Properties of Determinants