曾唯尧 著

A Collection of Mathematic Papers By Zeng Weiyao

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# 序

# PREFACE

湖南科学技术出版社决定出版《曾唯尧数学论文选》，唯尧夫人百友同志请我作序，不禁使我百感交集。唯尧教授风华正茂，却英年早逝。

曾唯尧教授于1964年10月9日出生于益阳县一个农民家庭。1981年7月于益阳县一中高中毕业后考入湖南师范大学数学系;1985年7月本科毕业并以优异成绩考入福州大学数学系攻读硕士学位，师从林[illegible]先生;1988年7月获硕士学位并在湖南省轻工业高等专科学校从事应用数学的教学与科研工作。1993年破格晋升为副教授;1995年又破格晋升为教授,1996年7月调长沙铁道学院工作。他先后担任过湖南省轻工业高等专科学校的系主任、校长助理及长沙铁道学院数理力学系副系主任等职。他曾被评为湖南省轻工系统的优秀教师、全国优秀教师并被授予“人民教师奖章”,还荣获铁道部詹天佑青年人才奖等多项奖励。

曾唯尧教授的研究兴趣在微分方程与动力系统，主要研究概周期微分方程、高维动力系统的分支与混沌理论。他先后主持了一项国家自然科学基金研究课题，作为主要

成员参加了两项国家自然科学基金研究课题，并应邀去美国、意大利等国访问与讲学；在《中国科学》、《数学学报》、《J. Dyna. Diff. Eqs.》等国内外权威学术刊物上发表论文50多篇，取得了一系列具有国际先进水平的研究成果，受到国内外同行专家的赞誉。这不由使人想起唯尧教授追悼会上的那副挽联：“一身才气，满腹经伦，道德文章均上品；学海泛舟，书山攻玉，教学科研皆楷模。”

曾唯尧教授证明了：如果概周期系统的任何两个解在 $R_+$ 上指数型收敛，则该系统具有唯一的概周期解。他一举解决了美国著名概周期微分方程专家 A. M. Fink 在1974年提出的一个猜想，一位法国数学家在美国《数学评论》上对此结果给予了很高的评价。

1993年，他发表了“无穷时滞微分积分方程的概周期解的存在性”(《数学物理学报》)，解决了一位美国数学家的猜想并证明了最佳估计。后来美国有人在《J. Diffential Eguations》上发表了类似的结果，但其结果所需要的条件比唯尧教授的强。

众所周知，概周期函数空间在一致收敛拓扑下不像周期函数空间那样具有紧性，因而研究周期解存在性的许多不动点定理不能用来研究概周期解的存在性。为此，唯尧教授另辟蹊径，把 Liapunov 函数法、指数二分性理论和不动点定理结合起来，探索出了一种研究概周期解存在性的新方法。利用此法他得到了一系列类似于周期解存在性的结果。美国《数学评论》对此作了高度评价。

指数二分性理论中的一个重要问题是指数二分性的粗糙度估计。著名概周期微分方程专家 Coppel 花了很大精力

来处理这个问题,但不尽人意。唯尧教授竟用相当简单的方法就统一地处理了 Coppel 的几个问题并改进了他们的结果,得到了粗糙度的最佳估计,而且这一结果几乎不能再改进了。

唯尧教授才思敏捷，善于发现，精于创新，他在研究概周期微分方程的同时，又着力研究动力系统的分支与混沌理论，而且在很短的时间内，做出令人瞩目的成就。

在研究非线性动力系统的复杂性理论中，Melnikov 创造并经 Holmes 等发展建立了一种把微分方程化为可微映射，然后通过寻求横截同宿轨道（点）来研究是否产生混沌的 Melnikov 方法，其关键是要建立相应系统的 Melnikov 函数。

在动力系统的同（异）宿分支理论中，唯尧教授建立了判断系统从退化同宿轨道分支出横截同宿轨道的 Melnikov 函数，“非常巧妙”地解决了著名数学家 J. Morse 和 Palmer 提出的关于退化情形下同宿分支的一个著名猜想。

他还得到了一般的判断高维非线性系统从同宿于非双曲平衡点的同宿轨道分支出横截同宿轨道的 Melnikov 函数，推广了 S. N. Chow，X. B. Lin 等人在该领域的有关工作。

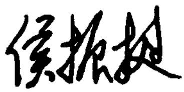
在奇异摄动系统大范围分支理论中,唯尧教授构造出了这类系统存在横截同宿轨道的 Melnikov 函数。著名动力系统专家 Palmer 称赞由此“所得到的结果非常重要,推广了一系列重要结果。特别是利用指数二分性来证明奇异摄动系统的同宿轨道的横截性是一个非常重要的方法,而且

这个结果还纠正了意大利数学家 Battelli 的一个错误。”

在利用解析方法研究大范围分支理论时，小参数线性系统的指数二分性显得非常重要，Palmer 对此作过许多研究。唯尧教授对这类系统作了一个重要变换，就使这个问题的处理大大简化了，把 Palmer 的结果推广到了一般情形，而且这个方法在“同、异宿分支理论中将有重要应用。”

目前，学者们已经知道，一个未扰动系统如果具有 inclination－flip 同宿轨道、orbit－flip 同宿轨道或者具有共振（resonance）特征值的同宿轨道，则该系统可能产生倍同宿分支、Smale 马蹄（混沌性态）和奇怪吸引子。但除此之外，还有没有其他类型的同宿轨道也能产生这些复杂现象呢？从逐次倍同宿分支进入到混沌状态的中间机制是什么呢？能否将传统的几何方法和解析方法结合起来，并利用现代计算技术，以建立起研究处理这类问题的统一方法呢……对这些问题，人们知之甚少，有些还没有人去研究。为了探讨这些问题，唯尧教授收集了这方面的几乎所有最新文献，并于去年向国家自然科学基金申报了“由同宿和异宿现象所引起的动力系统的复杂性研究”课题。但壮志未酬身先折，撇下这么多没有解决的问题，唯尧教授匆匆地走了。

唯尧教授的一生是短暂的一生，却是非常成功的一生。他将激励更多的人去献身于我们神圣的科研教学事业。



1999 年 5 月

# 目录

# CONTENTS

# 1. PERIODIC SOLUTION AND ALMOST PERIODIC SOLUTION OF DIFFERENTIAL EQUATION IN CRITICAL CASE*


## Abstract

In this paper, we investigate the existence of periodic solution and almost periodic solution of the differential equation system:

$$x' = A(t)x + g(t, x)$$

in critical case.


## § 1 Introduction

Let us consider systems:

$$x' = A(t)x \tag{1}$$

and

$$x' = A(t)x + g(t, x) \tag{2}$$

where $x \in R^n$, $A(t)$ is $n \times n$ almost periodic matrix on $R = (-\infty, +\infty)$, $g(t, x)$: $R \times R^n \to R^n$ is uniformly almost periodic function in t with respect to x on any compact sets of $R^n$. It is well known that when system (1) admits exponential dichotomy and $g(t, x)$ satisfies:

$$|g(t, x) - g(t, y)| < L|x - y| \tag{*}$$

with $x, y \in R^n$ and L enough small, then almost periodic system (2)

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* 该文1987年发表于《Ann. of Diff. Eqs.》(第3卷第4期)

has unique almost periodic solution [1],[2]. In this paper condition ( * ) is replaced by $\lim_{|x|\to\infty} |g(t,x)|/|x| = 0$ uniformly with respect to $t \in R$, we extend Favard theorem to system (2). We also discuss the existence of almost fperiodic solution of system (2) when $g(t,x)$ satisfies:

$$\lim_{n\to\infty}\inf (1/n) \left\{ \int_{-\infty}^{t} \exp(-\alpha(t-s)) \sup_{\|u\|\leqslant n} |g(s,u)| \, ds + \int_{t}^{\infty} \exp(-\alpha(s-t)) \sup_{\|u\|\leqslant n} |g(s,u)| \, ds \right\} = 0$$

uniformly in t and system (1) admits exponential dichotomy. Our tools used here are fixed point theorems.

## § 2 Definitions and Notations

For convenience, we initially give some definitions and notations as follow:

**Definition 1** $F(t,x)$: $R\times R^n \to R^n$, which is continuous in $t$, and $x$ is said to be uniformly almost periodic in $t$ with respect to $x$ on $R$ (for short u. a. p) if, for any $\varepsilon>0$, and compact subset $D$ of $R^n$, it is possible to find a real number $1(\varepsilon, D)$ such that for any interval with length $1(\varepsilon, D)$, there exists a number in this interval such that:

$$|F(t+\tau,x)-F(t,x)|<\varepsilon \text{ for } t\in R,\ x\in D.$$

We use $E(\varepsilon, F)$ to denote the set:

$$\{\tau \mid \tau\in R, |F(t+\tau,x)-F(t,x)|<\varepsilon \text{ for } t\in R,\ x\in D\}.$$

**Definition 2** Let $f(t,x)$ be uniformly almost periodic in $t$ with respect to $x$, then the module of $f(t,x)$, mod $(f)$, is the smallest addive group of real number that contains the exponents of $f(t,x)$.

**Definition 3** System (1) is said to admit an exponential dichotomy if there exist a projection $P$, positive constant $K$, a and fundamental solution matrix $X(t)$ satisfing:

$$|X(t)PX^{-1}(s)| \leqslant K\exp(-\alpha(t-s)),\ t \geqslant s,$$
$$|X(t)(I-P)X^{-1}(s)| \leqslant K\exp(-\alpha(s-t)),\ s \geqslant t. \quad (3)$$

## § 3 Main Results and Proofs

In order to prove our result, we first prove the following lemmas:

**Lemma 1** Let $f(t)$, $g(t)$ be a. p, then (Ⅰ), (Ⅱ) and (Ⅲ) are equivalence:

(Ⅰ) $T\alpha f$ converges uniformly on R, so does $T\alpha g$.

(Ⅱ) $E(\varepsilon, f) \subset E(\varepsilon, g)$.

(Ⅲ) mod $(g) \subset$ mod $(f)$.

The proof may be found in [3].

**Lemma 2** Let $f(t,x)$ be u. a. p, $g(t)$ and $\psi(t)$ be a. p. If mod $(f) \subset$ mod $(g)$ and mod $(\psi) \subset$ mod$(g)$ then mod $(f(t,\psi(t)) \subset$ mod$(g(t))$.

**Proof** Since mod $(f) \subset$ mod $(g)$ and mod $(\psi) \subset$ mod $(g)$, if $T\alpha g$ converges uniformly on $R$ then $T\alpha f$ and $T\alpha g$ converge uniformly on $R$ by Lemma 1. Since $f(t,x)$ is u. a. p. again, $T\alpha f(t,\psi(t))$ converges uniformly on R, therefore mod $(f(t,\psi(t)) \subset$ mod $(g)$. This proves lemma.

**Lemma 3** Let $\{f_n(t)\}$ be an almost periodic sequence such that it converges locally uniformly on $R$. If mod $(f_n) \subset$ mod $(f)$, $n=1, 2, \cdots$, where $f(t)$ is a. p then $\{f_n(t)\}$ converges uniformly on $R$.

**Proof** Since $f(t)$ is a. p, for $\varepsilon > 0$, there exists real number 1 $(\varepsilon) > 0$ which is inclusion length. Because $\{f_n(t)\}$ converges locally uniformly on $R$, there exists $N(\varepsilon) > 0$, when $n > N(\varepsilon)$ we have:

$$|f_n(t) - f_{n+p}(t)| < \varepsilon \qquad t \in [0, 1(\varepsilon)].$$

If $t \notin [0, 1(\varepsilon)]$, there exists $\tau \in E\{\varepsilon, f\} \subset E(f_n, \varepsilon)$ such that $t + \tau \in [0, 1(\varepsilon)]$ by lemma 1 and conditions of lemma 3. Hence.

$$|f_n(t) - f_{n+p}(t)| \leqslant |f_n(t) - f_n(t+\tau)| + |f_{n+p}(t+\tau) - f_n(t+\tau)| \leqslant 3\varepsilon,$$

therefore we show that $\{f_n(t)\}$ converges uniformly on $R$.

**Lemma 4** Let $C$ be a normed space, $B$ the closed convex subset with boundary $\partial B$ of $C$. If $T$ is a continuous compact mapping of $C$ into $C$ such that $T(\partial B) \subset B$, then $T$ has a fixed point.

This lemma can be found in [4].

**Lemma 5** Let us consider the system:

$$x' = A(t)x + f(t). \tag{4}$$

where $A(t)$ and $f(t)$ are a.p. If system (1) admits Favard condition and bounded solution, then system (4) has unigue minmal solution which is a.p with mod $(x(t)) \subset$ mod $(A(t), f(t))$.

**Proof** Refer to [5].

**Lemma 6** Suppose system (1) admits an exponential dichotomy (3), then almost periodic system (4) has unique almost periodic solution $x(t)$ with mod $(x(t)) \subset$ mod $(A(t), f(t))$.

**Proof** Refer to [1],[2].

Using Lemma 1~6, we can prove our theorem.

**Theorem 1** Consider the differential system (2), where $A(t)$ is a.p, $g(t,x)$ is u.a.p such that.

$$\lim_{|x| \to \infty} |g(t,x)| / |x| = 0 \quad \text{uniformly in } t \in R.$$

Assume that system (1) satisfies Favard condition and has no nontrivial alomost periodic solution $x(t)$ with mod $(x(t)) \subset$ mod $(A(t), g(t,x))$. If for any $u(t) \in C = \{u(t) \mid u(t)$ is a.p and mod $(u) \subset$ mod $(A(t), g(t,x))\}$ the system

$$x' = A(t)x + g(t, u(t)) \tag{5}$$

has bounded solution, then system (2) has almost periodic solution $x(t)$ *with mod* $(x(t))\subset\text{mod}\ (A(t), g\ (t,x))$.

**Proof** We prove our theorem by the following steps:

**Step 1** We difine a Banach space and an operator. Let C be a set $C=\{u(t)\mid u(t)$ is a.p with mod$(u(t))\subset\text{mod}\ (A(t),\ g(t,x))\}$ with norm $\| u(t)\| = \sup_{t\in R}|u(t)|$. It is obvious that C is a Banach space. For any $u\ (t)\in C$, then system (5) has unique almost periodic solution $x_u(t)$ with mod $(x_u(t))\subset\text{mod}\ (A(t),\ g(t,u(t))\subset\text{mod}\ (A(t),\ g(t,x))$ by lemma 5 and conditions of theorem. Hence we can define operator $T$:

$$T: C\to C.$$

$$u(t)\to x_u(t)=Tu(t).$$

**Step 2** We show $T$ is a compact operator. It is sufficient to show that for any sequence $\{u_n(t)\}$ of C such that $\| u_n(t)\|\leqslant M$, $n=1,2,\cdots$, there exists *a* subsequence $\{Tu_{n_k}(t)\}=\{x_{u_{nk}}(t)\}$ which converges in Banach space $C$.

Frist we show $\{x_n(t)=Tu_n(t)\}$ has bounded. Suppose that $\{x_n(t)\}$ has no bounded, we may assume:

$$\| x_n\| = \sup_{t\in R}|x_n(t)|\to\infty \text{ as} n\to\infty.$$

From (2) we have

$$(x_n/\| x_n\|)' = A(t)x_n/\| x_n\| + g(t,u_n(t))/\| x_n\|. \quad (7)$$

Let $Z_n(t)=x_n/\| x_n\|$, then $\{Z'_n(t)\}$ has bounded, hence $\{Z_n(t)\}$ is equicontinuous and uniformly bounded. Applying Ascoli theorem, we obtain that $\{Z_n(t)\}$ converges locally uniformly on $R$. Since mod $(Z_n(t))\subset\text{mod}\ (A(t), g\ (t,x))$, by Lemma 3, $\{Z_n(t)\}$ converges uniformly on $R$, therefore $\{Z_n(t)\}$ converges to $Z(t)$ which is a.p with mod $(Z(t))\subset\text{mod}\ (A(t),\ g\ (t,x))$ because of almost periodicity of $\{Z_n(t)\}$. Let $n\to\infty$ in (7), we obtain:

$$Z'(t)=A(t)Z(t).$$

Hence system (1) has an almost periodic solution $Z(t)$ with mod $(Z(t))\subset$mod $(A(t), g(t,x))$, which is contradictory to assumption of theorem. Therefore $\{x_n(t)=Tu_n(t)\}$ is uniformly bounded.

Since $\{x_n(t)\}$ satisfies:

$$x'_n(t)=A(t)x_n(t)+g(t,u_n(t)),$$

$\{x_n'(t)\}$ is uniformly bounded. Using Ascoli theorem again, we obtain that $\{x_n(t)\}$ converges locally uniformly on $R$. Because mod $(x_n(t))\subset$mod $(A(t), g(t,x))$, by applyting Lemma 3, we show that $\{x_n(t)\}$ converges uniformly on $R$. So $T$ is a compact operator.

**Step 3** We show that $T$ is continuous.

Let $u(t)\in C$, for any $\{u_n(t)\}\subset C$ such that $\{u_n(t)\}$ converges to $u(t)$ in $C$. We show that $Tu_n(t)$ converges to $Tu(t)$ as $n\to\infty$. Since $\{u_n(t)\}$ converges to $u(t)$ in $C$. $\{u_n(t)\}$ is bounded. By the same methods as above we show that $Tu_n(t)$ converges to $Z(t)$ uniformly on $R$ which is an almost periodic solution of the system:

$$x'=A(t)x+g(t,u(t)).$$

Since system (1) has no nontrivial almost periodic solution, $Z(t)=Tu(t)$. that is, $Tu_n(t)\to Tu(t)$ in $C$ as $n\to\infty$. so far, we have shown that T is a compact continuous operator.

**Step 4** We show that there exists $R$ such that $T\partial B\subset B$, where $B=\{u(t)\mid u(t)\in C\ \ \|u(t)\|<\text{R}\}$. We prove

$$\lim_{\|u\|\to\infty}\|Tu\|/\|u\|=0$$

holds. Suppose this doesn't hold, then there exist a number $C>0$, a sequence $\{u_n\}$ in $C$ such that $\|u_n\|\to\infty$, *as* $n\to\infty$, $\|Tu_n\|\geqslant C\cdot\|u_n\|$. *Let* $Z_n=Tu_n/\|Tu_n\|$, we have:

$$Z'_n(t)=A(t)Z_n(t)+g(t,u_n(t))/\|Tu_n\|. \qquad (8)$$

Since $\lim_{n\to\infty} |g(t,u_n(t))|/\|Tu_n\| =$

$\lim_{n\to\infty}(|g(t,u_n(t))|/|u_n(t)|)\times(|v_n(t)|)/\|Tv_n\|) =$

$0.$

uniformly for $t\in R$, let $n\to\infty$ in (8), by the same method as above we can show that $\{Z_n(t)\}$ uniformly converges to $Z(t)$ with is almost periodic solution of system (1), which is contradictory to our assumption. Therefore $\lim_{\|u\|\to\infty} \|Tu\|/\|u\| = 0$ holds. Let $R$ so large that: $\|Tu\|/\|u\| < 1$, if $\|u\| = R$ and $B_R = \{u(t)|u(t) \in C, \|u\| \leqslant R\}$, then $T\partial B_R \subset B_R$ By Lemma 4, T has fixed point $u(t)$, a.e, $Tu(t) = u(t)$. Hence $u(t)$ is an almost periodic solution of system (2). This completes the proof of Theorem 1.

From theorem 1 and lemma 6, we immediately obtain the following theorem 2.

**Theorem 2** If $A(t)$ is *a.p*, $g(t,x)$ is u.a.p. Suppose (1) admits exponential dichotomy and $\lim_{|x|\to\infty} |g(t,x)|/|x| = 0$ uniformly in $t$ then system (2) has almost periodic solution $x(t)$ with mod $(x(t)) \subset$ mod $(A(t), g(t,x))$.

The method used to prove Theorem 1 also can be applied to periodic system (2). We have:

**Theorem 3** If $A(t)$ and $g(t,x)$ is periodic in $t$ with period $T$, if periodic system (1) has no nontrival periodic solution with period $T$ and $\lim_{|x|\to\infty} |g(t,x)|/|x| = 0$ uniformly in $t$, then periodic system (2) has T-periodic solution.

Now we discuss the existence of almost periodic solution of other type. We have the following result:

**Theorem 4** If $A(t)$ is a.p, $g(t,x)$ u.a.p and system (1) admits an exponential dichotomy. Suppose $g(t,x)$ satisfies: