

国外计算机科学教材系列

数值方法 (MATLAB版)

(第三版)

Numerical Methods Using MATLAB
Third Edition

英文原版

[美] John H. Mathews 著
Kurtis D. Fink



电子工业出版社
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内 容 简 介

本书介绍了数值方法的理论及实用知识,讲述了如何利用 MATLAB 软件实现各种数值算法,以便为读者今后的学习打下坚实的数值分析与科学计算基础。本书内容丰富、翔实,可以根据不同的学习对象和学习目的选择相应的章节,形成理论与实践相结合的学习策略。书中的每个概念均以实例说明,同时还包含大量的习题,范围涉及多个不同的领域。通过这些实例,进一步说明数值方法是如何被实际应用的。本书的突出特点是强调利用 MATLAB 进行数值方法的程序设计,可提高读者的实践能力和加深对数值方法理论的理解;同时它的覆盖范围广,包含数据方法的众多研究领域,可以满足不同专业和不同层次学生的需求。

本书概念清晰、逻辑性强,可作为大专院校计算机、工程和应用数学专业的教材和参考书。

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此外,我们还将与国外著名出版公司合作,提供一些教材的教学支持资料,希望能为授课老师提供帮助。今后,我们将继续加强与各高校教师的密切联系,为广大师生引进更多的国外优秀教材和参考书,为我国计算机科学教学体系与国际教学体系的接轨做出努力。

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Preface

This book provides a fundamental introduction to numerical analysis suitable for undergraduate students in mathematics, computer science, physical sciences, and engineering. It is assumed that the reader is familiar with calculus and has taken a structured programming course. The text has enough material fitted modularly for either a single-term course or a year sequence. In short, the book contains enough material so instructors will be able to select topics appropriate to their needs.

Students of various backgrounds should find numerical methods quite interesting and useful, and this is kept in mind throughout the book. Thus, there is a wide variety of examples and problems that help to sharpen one's skill in both the theory and practice of numerical analysis. Computer calculations are presented in the form of tables and graphs whenever possible so that the resulting numerical approximations are easier to visualize and interpret. MATLAB programs are the vehicle for presenting the underlying numerical algorithms.

Emphasis is placed on understanding why numerical methods work and their limitations. This is challenging and involves a balance between theory, error analysis, and readability. An error analysis for each method is presented in a fashion that is appropriate for the method at hand, yet does not turn off the reader. A mathematical derivation for each method is given that uses elementary results and builds the student's understanding of calculus. Computer assignments using MATLAB give students an opportunity to practice their skills at scientific programming.

Shorter numerical exercises can be carried out with a pocket calculator/computer, and the longer ones can be done using MATLAB subroutines. It is left for the instructor to guide the students regarding the pedagogical use of numerical computations. Each instructor can make assignments that are appropriate to the available comput-

ing resources. Experimentation with the MATLAB subroutine libraries is encouraged. These materials can be used to assist students in the completion of the numerical analysis component of computer laboratory exercises.

This Third Edition grows out of much polishing of the narrative for the Second Edition. For example, the QR method has been added to the chapter on Eigenvalues and Eigenvectors. New to this edition is the explicit use of the software MATLAB. An appendix gives an introduction to MATLAB syntax. Examples have been added throughout the text with MATLAB and complete MATLAB programs are given in each section. An instructor's disk is available upon request from the publisher.

Previously we took the attitude that any software program that students mastered would work fine. However, many students entering this course have yet to master a programming language (computer science students excepted). MATLAB has become the tool of nearly all engineers and applied mathematicians, and its newest versions have improved the programming aspects. So we think that students will have an easier and more productive time in this MATLAB version of our text.

Acknowledgments

We would like to express my gratitude to all the people whose efforts contributed to the various editions of this book. I (John Mathews) thank the students at California Statue University, Fullerton. I thank my colleagues Stephen Goode, Mathew Koshy, Edward Sabotka, Harris Schultz, and Soo Tang Tan for their support in the first edition; additionally, I thank Russell Egbert, William Gearhart, Ronald Miller, and Greg Pierce for their suggestions for the second edition. I also thank James Friel, Chairman of the Mathematics Department at CSUF, for his encouragement.

Reviewers who made useful recommendations for the first edition are Walter M. Patterson, III, Lander College; George B. Miller, Central Connecticut State University; Peter J. Gingo, The University of Akron; Michael A. Freedman, The University of Alaska, Fairbanks; and Kenneth P. Bube, University of California, Los Angeles. For the second edition, we thank Richard Bumby, Rutgers University; Robert L. Curry, U.S. Army; Bruce Edwards, University of Florida; and David R. Hill, Temple University.

For this third edition we wish to thank Tim Sauer, George Mason University; Gerald M. Pitstick, University of Oklahoma; Victor De Brunner, University of Oklahoma; George Trapp, West Virginia University; Tad Jarik, University of Alabama, Huntsville; Jeffrey S. Scroggs, North Carolina State University; Kurt Georg, Colorado State University; and James N. Craddock, Southern Illinois University at Carbondale.

Suggestions for improvements and additions to the book are always welcome and can be made by corresponding directly with the authors.

John H. Mathews
Mathematics Department
California State University
Fullerton, CA 92634
mathews@fullerton.edu

Kurtis D. Fink
Department of Mathematics
Northwest Missouri State University
Maryville, MO 64468
kfink@mail.nwmissouri.edu

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1

Preliminaries

Consider the function $f(x) = \cos(x)$, its derivative $f'(x) = -\sin(x)$, and its antiderivative $F(x) = \sin(x) + C$. These formulas were studied in calculus. The former is used to determine the slope $m = f'(x_0)$ of the curve $y = f(x)$ at a point $(x_0, f(x_0))$, and the latter is used to compute the area under the curve for $a \leq x \leq b$.

The slope at the point $(\pi/2, 0)$ is $m = f'(\pi/2) = -1$ and can be used to find the tangent line at this point (see Figure 1.1(a)):

$$y_{\text{tan}} = m \left(x - \frac{\pi}{2} \right) + 0 = f' \left(\frac{\pi}{2} \right) \left(x - \frac{\pi}{2} \right) = -x + \frac{\pi}{2}.$$

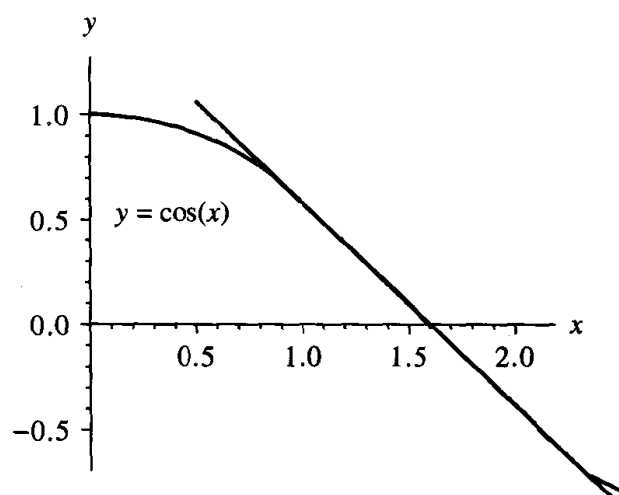


Figure 1.1 (a) The tangent line to the curve $y = \cos(x)$ at the point $(\pi/2, 0)$.

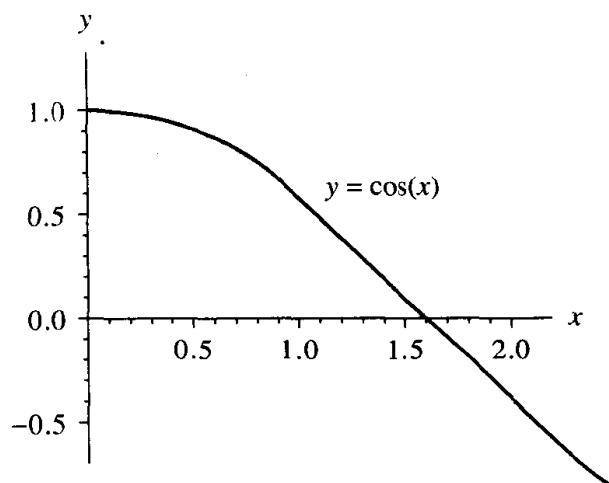


Figure 1.1 (b) The area under the curve $y = \cos(x)$ over the interval $[0, \pi/2]$.

The area under the curve for $0 \leq x \leq \pi/2$ is computed using an integral (see Figure 1.1(b)):

$$\text{area} = \int_0^{\pi/2} \cos(x) dx = F\left(\frac{\pi}{2}\right) - F(0) = \sin\left(\frac{\pi}{2}\right) - 0 = 1.$$

These are some of the results that we will need to use from calculus.

Review of Calculus

It is assumed that the reader is familiar with the notation and subject matter covered in the undergraduate calculus sequence. This should have included the topics of limits, continuity, differentiation, integration, sequences, and series. Throughout the book we refer to the following results.

Limits and Continuity

Definition 1.1. Assume that $f(x)$ is defined on a set S of real numbers. Then f is said to have the *limit* L at $x = x_0$, and we write

$$(1) \quad \lim_{x \rightarrow x_0} f(x) = L,$$

if, given any $\epsilon > 0$, there exists a $\delta > 0$ such that, whenever $x \in S$, $0 < |x - x_0| < \delta$ implies that $|f(x) - L| < \epsilon$. When the h -increment notation $x = x_0 + h$ is used, equation (1) becomes

$$(2) \quad \lim_{h \rightarrow 0} f(x_0 + h) = L. \quad \blacktriangle$$

Definition 1.2. Assume that $f(x)$ is defined on a set S of real numbers and let $x_0 \in S$. Then f is said to be *continuous at $x = x_0$* if

$$(3) \quad \lim_{x \rightarrow x_0} f(x) = f(x_0).$$

The function f is said to be continuous on S if it is continuous at each point $x \in S$. The notation $C^n(S)$ stands for the set of all functions f such that f and its first n derivatives are continuous on S . When S is an interval, say $[a, b]$, then the notation $C^n[a, b]$ is used. As an example, consider the function $f(x) = x^{4/3}$ on the interval $[-1, 1]$. Clearly, $f(x)$ and $f'(x) = (4/3)x^{1/3}$ are continuous on $[-1, 1]$, while $f''(x) = (4/9)x^{-2/3}$ is not continuous at $x = 0$. ▲

Definition 1.3. Suppose that $\{x_n\}_{n=1}^{\infty}$ is an infinite sequence. Then the sequence is said to have the limit L , and we write

$$(4) \quad \lim_{n \rightarrow \infty} x_n = L,$$

if, given any $\epsilon > 0$, there exists a positive integer $N = N(\epsilon)$ such that $n > N$ implies that $|x_n - L| < \epsilon$. ▲

When a sequence has a limit, we say that it is a *convergent sequence*. Another commonly used notation is " $x_n \rightarrow L$ as $n \rightarrow \infty$." Equation (4) is equivalent to

$$(5) \quad \lim_{n \rightarrow \infty} (x_n - L) = 0.$$

Thus we can view the sequence $\{\epsilon_n\}_{n=1}^{\infty} = \{x_n - L\}_{n=1}^{\infty}$ as an *error sequence*. The following theorem relates the concepts of continuity and convergent sequence.

Theorem 1.1. Assume that $f(x)$ is defined on the set S and $x_0 \in S$. The following statements are equivalent:

- (6) (a) The function f is continuous at x_0 .
 (b) If $\lim_{n \rightarrow \infty} x_n = x_0$, then $\lim_{n \rightarrow \infty} f(x_n) = f(x_0)$.

Theorem 1.2 (Intermediate Value Theorem). Assume that $f \in C[a, b]$ and L is any number between $f(a)$ and $f(b)$. Then there exists a number c , with $c \in (a, b)$, such that $f(c) = L$.

Example 1.1. The function $f(x) = \cos(x - 1)$ is continuous over $[0, 1]$, and the constant $L = 0.8 \in (\cos(0), \cos(1))$. The solution to $f(x) = 0.8$ over $[0, 1]$ is $c_1 = 0.356499$. Similarly, $f(x)$ is continuous over $[1, 2.5]$, and $L = 0.8 \in (\cos(2.5), \cos(1))$. The solution to $f(x) = 0.8$ over $[1, 2.5]$ is $c_2 = 1.643502$. These two cases are shown in Figure 1.2. ■

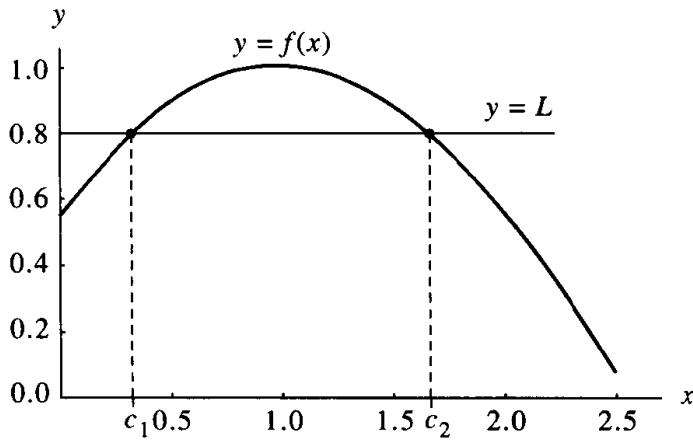


Figure 1.2 The intermediate value theorem applied to the function $f(x) = \cos(x - 1)$ over $[0, 1]$ and over the interval $[1, 2.5]$.

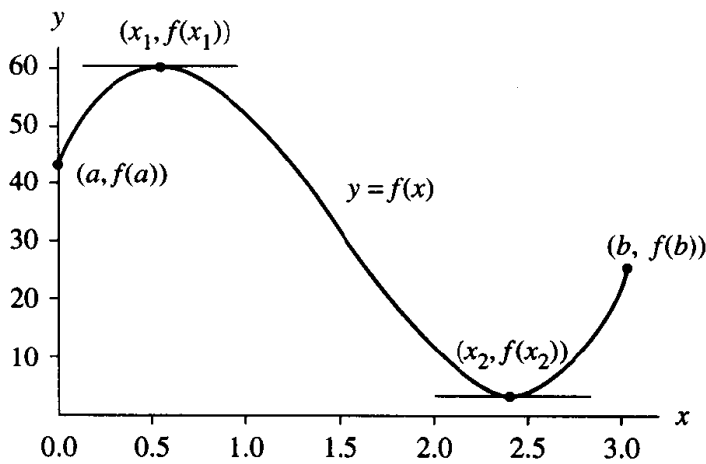


Figure 1.3 The extreme value theorem applied to the function $f(x) = 35 + 59.5x - 66.5x^2 + 15x^3$ over the interval $[0, 3]$.

Theorem 1.3 (Extreme Value Theorem for a Continuous Function). Assume that $f \in C[a, b]$. Then there exists a lower bound M_1 , an upper bound M_2 , and two numbers $x_1, x_2 \in [a, b]$ such that

$$(7) \quad M_1 = f(x_1) \leq f(x) \leq f(x_2) = M_2 \quad \text{whenever } x \in [a, b].$$

We sometimes express this by writing

$$(8) \quad M_1 = f(x_1) = \min_{a \leq x \leq b} \{f(x)\} \quad \text{and} \quad M_2 = f(x_2) = \max_{a \leq x \leq b} \{f(x)\}.$$

Differentiable Functions

Definition 1.4. Assume that $f(x)$ is defined on an open interval containing x_0 . Then f is said to be differentiable at x_0 if

$$(9) \quad \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists. When this limit exists, it is denoted by $f'(x_0)$ and is called the *derivative* of f at x_0 . An equivalent way to express this limit is to use the h -increment notation:

$$(10) \quad \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} = f'(x_0).$$

A function that has a derivative at each point in a set S is said to be *differentiable* on S . Note that, the number $m = f'(x_0)$ is the slope of the tangent line to the graph of the function $y = f(x)$ at the point $(x_0, f(x_0))$. ▲

Theorem 1.4. If $f(x)$ is differentiable at $x = x_0$, then $f(x)$ is continuous at $x = x_0$.

It follows from Theorem 1.3 that, if a function f is differentiable on a closed interval $[a, b]$, then its extreme values occur at the end points of the interval or at the critical points (solutions of $f'(x) = 0$) in the open interval (a, b) .

Example 1.2. The function $f(x) = 15x^3 - 66.5x^2 + 59.5x + 35$ is differentiable on $[0, 3]$. The solutions to $f'(x) = 45x^2 - 123x + 59.5 = 0$ are $x_1 = 0.54955$ and $x_2 = 2.40601$. The maximum and minimum values of f on $[0, 3]$ are:

$$\min\{f(0), f(3), f(x_1), f(x_2)\} = \min\{35, 20, 50.10438, 2.11850\} = 2.11850$$

and

$$\max\{f(0), f(3), f(x_1), f(x_2)\} = \max\{35, 20, 50.10438, 2.11850\} = 50.10438. \quad \blacksquare$$

Theorem 1.5 (Rolle's Theorem). Assume that $f \in C[a, b]$ and that $f'(x)$ exists for all $x \in (a, b)$. If $f(a) = f(b) = 0$, then there exists a number c , with $c \in (a, b)$, such that $f'(c) = 0$.

Theorem 1.6 (Mean Value Theorem). Assume that $f \in C[a, b]$ and that $f'(x)$ exists for all $x \in (a, b)$. Then there exists a number c , with $c \in (a, b)$, such that

$$(11) \quad f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Geometrically, the Mean Value Theorem says that there is at least one number $c \in (a, b)$ such that the slope of the tangent line to the graph of $y = f(x)$ at the point $(c, f(c))$ equals the slope of the secant line through the points $(a, f(a))$ and $(b, f(b))$.

Example 1.3. The function $f(x) = \sin(x)$ is continuous on the closed interval $[0.1, 2.1]$ and differentiable on the open interval $(0.1, 2.1)$. Thus, by the Mean Value Theorem, there is a number c such that

$$f'(c) = \frac{f(2.1) - f(0.1)}{2.1 - 0.1} = \frac{0.863209 - 0.099833}{2.1 - 0.1} = 0.381688.$$

The solution to $f'(c) = \cos(c) = 0.381688$ in the interval $(0.1, 2.1)$ is $c = 1.179174$. The graphs of $f(x)$, the secant line $y = 0.381688x + 0.099833$, and the tangent line $y = 0.381688x + 0.474215$ are shown in Figure 1.4. ■