浑沌经济学理论及其应用研究

宋学锋 著

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内 容 提 要

本书系统研究了浑沌经济学的基本理论，总结了浑沌的定量特征及其判别方法，并编制了相应的计算机程序，同时还概括出了浑沌的定性判别准则；提出了“区间分析法”和“浑沌度”的概念；另外，还总结了浑沌经济学的各种研究方法，概述了浑沌控制理论的最新成果；并运用浑沌经济学理论对农产品供求波动和煤炭供需失衡问题进行了研究和分析；综述了浑沌经济学理论和方法在研究股票价格波动方面的成果。

本书适合于对浑沌经济学有兴趣的具有一定数学和经济学基础的各类读者选读，也可以用作大学高年级和研究生课程的教材或参考书。

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The Theory of Chaotic Economics And Its Applications

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SUMMARY

1 Introduction

As a new branch of economics, chaotic economics was born at the first of 1980s. It has been rapidly developed since then. R. H. Day, Stutzer, Benhabib, Shafer, Woof, Woodford, Deneekerre, Poliman and others have done a lot of research work in the field of chaotic economics. However, I have not seen any monograph on chaotic economics so far, and indeed there are still many open questions about basic theory of chaotic economics. For example, how can we define economic chaos? what are the qualitative and quantitative features of economic chaos? how can we find the border point of chaos in the economic data? How can we describe the complex degree of chaotic economic systems? how can we control the chaotic economic systems? So in this book we just try to answer these questions and summarize systematically the achievements on the fields obtained in near ten years.

In this book, firstly, a various of chaos definitions, the qualitative and quantitative features of chaos, the mechanism of chaos, the qualitative and quantitative methods to determine chaos, the methods to estimate border point and the behavior of chaotic economic system are analyzed and reviewed systematically. Secondly we develop a new definition of chaos that is suited to economics, present a new method to search the border point of chaos by economic data directly, propose the concept of the chaos degree. Thirdly, we not only systematically summarize the theories and methods of researching the chaotic economics, but also the theories and methods to control the chaotic economic systems. Finally, on the basis of the theory and methods of chaotic economics, we study the unbalance of coal demand and supply in China as well as the other applications of chaotic economics.

2 The Definition and Mechanism of Chaos

At present, there are several different definitions of chaos, but those definitions seem quit difficult to understand through the senses for socialists and economists. So we present another definition of economic chaos below.

Definition 1 The economic chaos is the state of non — linear economic system in which the system is disorder in macrocosm and order in microcosm. The chaotic economic systems have the following main qualitative features:

① Non—additional;

② Anti—direct perception (non—cycle);

③ Accumulate feature;

④ Fractal feature (irregularity);

⑤ Resemble in its construction.

As to the mechanism of the chaotic economic systems, we can understand that by the below a simple model. Consider a simple Logistic function:

$$X_{n+1} = KX_n(1 - X_n), K \in [0,4], X_n \in [0,1], n = 1,2,3\cdots \tag{1}$$

We have the following conclusions:

① If $0 < K < 1$, then (1) has one fixed point: $x = 0$ on [0, 1];② If $1 < K < 3$, then (1) has two fixed points: $x = 0$ and $1 - 1/K$;③ If $3 < K < 3.57$, then (1) begins to bifurcate; ④ If $3.57 < K < 4$, then (1) is chaotic on [0, 1].

3 The Quantitative Features of Chaotic Systems

From the discussion in reference [120], we can conclude the quantitative features of chaos are as follows:

(1) Positive Liapunov Exponent:

$$\sigma(x_0, w_0) = \left[\frac{1}{t}\ln\frac{\| w(t) \|}{\| w_0 \|}\right]$$

Every chaotic economic system has positive Liapunov index.

(2) Continuous Power Spectral:

Chaotic economic systems are always of continuous power spectral, which can be defined by the below formulae.

$$C(w) = \| X(w) \|^2,$$

$$X(w) = \lim_{T\to\infty}\frac{1}{T}\int_0^T X(t)e^{iwt}dt$$

(3) Fractal Dimension:

Fractal dimension is also a basic feature of chaotic economic systems, which is defined as followings:

$$D_F = \lim_{\varepsilon\to 0}\frac{M(\varepsilon)}{\text{lm}(1/\varepsilon)}$$

(4) Positive Metric Entropy:

The another basic feature of chaotic economic systems is positive metric entropy, which can be described by the following formula.

$$H_\mu(\alpha) = \sum_{i=1}^{p} \mu(\alpha_i) \ln \mu(\alpha_i)$$

(5) Feigenbaum Constant Number:

Any chaotic system has the Feigenbaum number during it bifurcates.

$$\delta = \lim_{n\to\infty} \frac{\alpha_n - \alpha_{n-1}}{\alpha_{n+1} - \alpha_n} = 4.6992016910\cdots$$

4 Find the Chaos Border Point for Continuous Economic Systems

The chaos border for continuous economic systems can be determined by use of the Feigenbaum number δ. Consider a general continuous economic system:

$$f:[a,b] \to [a,b],(x,\alpha) \to f(x,\alpha)$$

Suppose the function f has one maximum value $M = f(x^*, \alpha)$ on $[a,b]$, and in the neighborhood of the value M it can be unfolded as following series.

$$f(x,\alpha) = M - a(x - x^*)^k + \cdots$$

Let α_n be the parameter of the system while it bifurcates from 2^{n-1} period point to 2^n period point, then similar to $f(x,\alpha) = \alpha x(1-x)$, α_n is convergent to α_∞ by the following way:

$$\alpha_n = \alpha_\infty - c\delta^{-n} \tag{2}$$

where c is a constant number, and $\delta = \lim \frac{\alpha_n - \alpha_{n-1}}{\alpha_{n+1} - \alpha_n}$. Specially for the system with $k=2$, we have $\delta = 4.6692\cdots$, and for $k=4,6,8$, the value of δ will be 7.284, 9.296, 10.048 respectively. For economic systems, $k=2$ is most popular because it is very difficult to model economic systems by the functions with exponent more than two.

Following the below steps, we can find the chaos border point for continuous systems. Firstly, we determine the two bifurcate points of the double—period by solving the following equations:

$$f(x,\alpha) = x$$

$$f(f(x,\alpha)) = x$$

From (2), we have:

$$\alpha_0 = \alpha_\infty - c, \quad \alpha_1 = \alpha_\infty - c\delta^{-1} \tag{3}$$

Solving them, we get:

$$\alpha_\infty = \alpha_0 + \frac{\alpha_1 - \alpha_0}{1 - \delta^{-1}}, \quad c = \frac{\alpha_1 - \alpha_0}{1 - \delta^{-1}} \tag{4}$$

(4) implies that we can find the chaos border point α_∞ by use of α_0 and α_1. Besides, we can also get the other bifurcate points:

$$\begin{aligned} \alpha_m &= \alpha_\infty - c\delta^{-m} = c\delta^{-1} + \alpha_1 - c\delta^{-m} \\ &= \frac{\alpha_1 - \alpha_0}{1 - \delta} + \alpha_1 - \frac{\alpha_1 - \alpha_0}{1 - \delta^{-1}}\delta^{-m} \\ &= \alpha_0 \frac{\delta^{1-m} - 1}{\delta - 1} + \alpha_1 \frac{\delta - \delta^{1-m}}{\delta - 1} \end{aligned} \tag{5}$$

For example, consider $f(x,\alpha) = \alpha x(1-x)$, if $\alpha = 1$, we can get $\alpha_0 = 1$, and if $\alpha = 3$, then we can calculate out the bifurcate point $\alpha_1 = 3$, thus from (4) we get: $\alpha_\infty = 3.57$, also from (5), we get the other bifurcate points, see $\alpha_2 = 3.43$.

5 Find the Chaos Border Point for Discrete Economic Systems

If we only have some time series data for an economic system, we shall be impossible to find the chaos border point by the methods above, unless we can set up a model by these data. Unfortunately, it is very difficult to construct a valid model by some time series data because of the complexity of the chaotic economic systems. Therefore, we now try to explore the way of calculating the chaos border point by the time series data directly.

Suppose we have a time series data of an economic system:

$$x_1, x_2, \cdots, x_n \qquad (6)$$

Assume the precise prescribe is $\varepsilon > 0$, then we give the following interval algorithm:

Step 1: Judge whether the system (6) is chaotic or not by mean of the methods in section 3 of the paper. If it is not chaotic, then stop; if not, take the step 2.

Step 2: Delete the data belongs to the first interval with ε length, if there is not any data left, then the system (6) is chaotic fully, if not take the next step.

Step 3: Judge the data left, if it is chaotic, then go to step 2, if not, then the interval deleted last step is just the chaotic interval we want to find.

It is necessary to state that the popular half—and half method is invalid here. If we divide the system (6) into two subsystems:

$$x_1, x_2, \cdots x_{n/2} \qquad (7)$$

and

$$x_{n/2+1}, x_{n/2+2}, \cdots, x_n \qquad (8)$$

then we can calculate out their Liapunov exponent σ_1, σ_2, respectively, and there is at lest one of the following three cases true:

(i) $\sigma_1\sigma_2 < 0$

(ii) $\sigma_1 > 0$, $\sigma_2 > 0$

(iii) $\sigma_1 < 0$, $\sigma_2 < 0$

Even in case (i), the algorithm can go ahead, but we still can not delete the case (ii) or (iii) because of the non—additional feature of the chaotic system. Once the case (ii) or (iii) is true, the algorithm will not be continued. So the half—and—half method is not feasible for above problem.

In addition, the interval algorithm is obviously convergent.

6 The Degree of Chaos and Its Importance

Definition 2 The degree of chaos is irregular and complex degree of the chaotic systems. The irregular degree means the non—periodic degree and the disorder degree of the systems. The complex degree means the sensitivity to initial value and the fractal of the structure.

In order to express the concept of chaotic degree, it is necessary to quantify it. Thus, we need to determine which quantitative index can describe the complex degree of the chaotic economic systems. From the analysis in section 3 above, we can conclude the main quantitative features of chaos are as follows:

(1) Positive Liapunov Exponent;

(2) Fractal Dimension;

(3) Positive Metric Entropy;

(4) Continuous Power Spectral.

Besides, for economic system, the maximum range of the wave of the economic system is of special importance because the excessive range of the wave making the economic system go out. For this, it is necessary to look the maximum range of the wave upon as one of the measures of chaos. From [120], we know that the metric entropy I (ε) and the positive Liapunov exponent have the following relationship:

$$I(\varepsilon) = \sum \sigma_i, \ \sigma_i > 0$$

That implies that the metric entropy is equal to Liapunov exponent in some sense. On the other hand, both the continuous powerful spectral and fractal dimension are geometry feature. So we can select one of the them as a representative.

Therefore, we can use Liapunov exponent fractal dimension, and the maximum range of the wave of economic chaotic systems as chaotic degree of economic systems.

Definition 3 Let (f, I) be chaotic economic systems, σ_i, $i=1, 2, \cdots, n$ be Liapunov exponent, m be the maximum orange of the wave of economic chaotic systems, that is $d = \text{Max } f(x) - \text{Min } f(x)$, then we call the following vector:

$$\beta = (\sigma, m, d)$$

the vector of chaotic degree of the systems, where $\sigma = \text{Max}\{\sigma_i\}$.

The vector of chaotic degree β can be looked upon as a magnanimity of the chaotic economic systems. Different system may has different vector of chaotic degree. In order to compare different chaotic economic systems, we analyze their structure in order.

consider one dimension system: $X_{n+1} = f(X_n)$, $n = 0, 1, 2, \cdots$, the Liapunov exponent can be derived from the following formula:

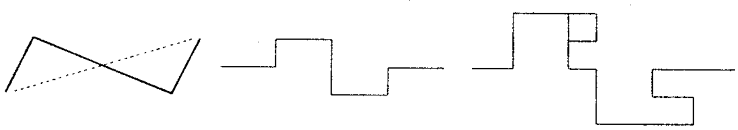
$$\begin{aligned} \Delta X_n &= f^n(X_0 + \Delta X_0) - f^n(X_0) \\ &\approx \frac{df^n(X_0)}{dX} \Delta X_0 \\ &= e^{\sigma n} \Delta X_0 \end{aligned} \tag{9}$$

where is the initial value of the system, n is the times of iterating, ΔX_0 is the error of the initial value. According to (9), the more the σ is, the higher the chaotic degree of the system is.

With respect to fractal dimension m, it is more than the topology dimension of the system but less than the dimension of the space that is occupied by the system. i. e. :

$$n < m < n + 1$$

For example, Cantor set consists of a lot of line sections from a line. Obviously, its dimension must be more than zero (one point) but less than one because it can not occupy the line fully. In fact, its dimension m is equal to 0. 631. In general, the more the fractal dimension is, the complex the system is. For instant, we consider three kind of irregular curves as following chart $(a)-(c)$:



$r=1/5$, $N=3$ (a) $r=1/4$, $N=8$ (b) $r=1/6$, $n=18$ (c)

Their fractal dimensions are:

$m = \ln 3/\ln 5 = 1.363$

$m = \ln 8/\ln 4 = 1.5$

$m = \ln 18/\ln 6 = 1.613$

So the more the irregular of the system is, the more the fractal dimension is. It is necessary to mention that the fractal dimension changes with r the scale of measuring the curve, even for the same curve, that is because if r is too big, the wave less than or would be overlooked. That is also true for economic systems. For example, if we want to study the daily state of systems, then we should use their daily data, and the hour data is neglected. However, instead of daily data, we use monthly data, then we could not know the daily state of the system. On the other hand, economic curves are almost fractal, but the are not all chaotic. In general, only is the fractal dimension of economic curve bigger than 1. 5 [32], would the economic system be chaotic.

In a word, for chaotic economic system, the more fractal dimension is, the higher the degree of chaos is.

In respect to the maximum range of the wave d, the bigger the d is, the unstable the economic system is.

Therefore, for different chaotic systems A and B, let their vectors of chaotic degree be β_1 and β_2 respectively, if $\beta_1 > \beta_2$ (or $\beta_1 < \beta_2$), then the chaotic degree of system A is more than or less than the chaotic degree. Of course, both A and B have the same chaotic degree if they have the same vector of chaotic degree.

Another situation should also be considered, that is: if the following vectors of chaotic degree

$\beta_1 = (\sigma_1, m_1, d_1)$ and $\beta_2 = (\sigma_2, m_2, d_2)$ satisfy:

$$\sigma_1 > \sigma_2,\ m_1 > m_2,\ d_1 < d_2$$

$$\sigma_1 > \sigma_2,\ m_1 < m_2,\ d_1 > d_2$$

etc. then we could not use the above method to judge $\beta_1 > \beta_2$ or $\beta_1 < \beta_2$. For this reason, we give the following definition.

Definition 4 Let the vector of chaotic degree of the economic system be $\beta = (\sigma, m, d)$, then we call $\gamma = P_1\sigma + P_2 m + P_3 d$ the chaotic index of the system, where $P_i > 0$, $i = 1,2,3$ and $\sum_{i=1}^{3} P_i = 1$.

Usually, if σ, m and d have the same importance, then we have $P_i = 1$, $i = 1,2,3$ and thus: $\gamma = 1/3(\sigma + m + d)$. Obviously, the bigger the (is, the bigger the chaotic degree is. Sometimes we can use $d/\mathrm{Max}(x)$ in place of the above d.

It is necessary to state that the definition 2 and definition 3 are not same because it is possible that the $\beta_1 \neq \beta_2$, $\gamma_1 = \gamma_2$ or $\beta_1 = \beta_2$, $\gamma_1 \neq \gamma_2$, in this case, we need to consider both of them.

On the basis of the both the vector of chaotic degree and the index of chaotic degree, it becomes possible for us to compare the complex degrees and the irregular degrees of different chaotic systems. Moreover, we can forecast and warn the chaotic economic system with β and γ. Besides, this method may become one of the ways to apply chaos theory to practice.

With the view of economics, is it good or bad when the economic system is chaotic? What is the difference between economic chaos and economic disorder? At present, Most of people is not clear for that. So, it is necessary to discuss these questions.

There are two points on the first question now, one is that chaotic economic system is good and heath system because the order comes from disorder; The other one is that chaotic economic system is bad because the system is unstable, irregular and difficult to control.

In our opinion, that chaotic economic system is whether good or bad can not be settled finally because it changes with different situations. According to the research conclusions of bionomy, the electronic picture of brain of healthy person is chaotic, on the contrary, the electronic picture of brain of ill person is regular and periodic. However, if the behavior of person is unstable and irregular, then we can not say that is good or healthy. As we know, that the stampede to buy things in our country in 1987 is the chaotic behaviors of consumers, but this chaos is not good for economic development. However, the chaotic wave of the index of bonds in normal stock market is good to absorb the investors. Generally speaking, the chaotic economic theory follows the principle of united microcosm and macrocosm as well as dynamic principle. Thus, we should use these both principles to evaluate the practice chaotic economic systems.

In a word, whether the chaotic economic system is good or bad depends on the practicing situation. As to the second question, we think that the chaos is not disorder and the disorder is not chaos either.

As a conclusion, we think that chaos is not equal to disorder, and disorder is certainly not chaos. We should not mix them up.

7 The Basic Methods of Researching Chaotic Economics

There are a few methods can be used to research chaotic economics. However the main basic methods of researching chaotic economics can be divided into the following four kind methods: the theoretic analysis methods, model analysis methods, numerical analysis methods and simulation methods.

The theoretic analysis means to analyze the all course of the researched economic systems by means of relative theories of economics and chaotic economics, which includes judging the chaotic features of the economic system, determining the border point of the systems, and presenting the ways to control or utilizing the chaotic economic systems. The theoretic analysis is the basic and necessary way to find the orderliness of the chaotic economic systems.

Model analysis method is to analyze the chaotic economic