国外电子与通信教材系列

小波与傅里叶分析基础

A First Course in Wavelets with Fourier Analysis

英文版

[美] Albert Boggess
Francis J. Narcowich 著

Prentice Hall

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内 容 简 介

本书的目的主要是向读者展示傅里叶分析和小波的许多基础知识以及在信号分析方面的应用。全书分为8章和2个附录，前言部分是学习第1章至第7章的准备知识，即内积空间；第1章讲解傅里叶系列的基础知识；第2章讲解傅里叶变换；第3章介绍离散傅里叶变换以及快速傅里叶变换；第4章～第7章讨论小波，重点在于正交小波的构建；附录部分则介绍稍微复杂的一些技术主题以及演示概念或产生图形的MATLAB代码。

小波分析的应用领域十分广泛，它包括：数学领域的许多学科；信号分析、图像处理；量子力学、理论物理；军事电子对抗与武器的智能化；计算机分类与识别；音乐与语言的人工合成；医学成像与诊断；地质勘探数据处理；大型机械的故障诊断等方面。

许多关于小波的文章和参考书籍均要求读者具有复杂的数学背景知识，本书则只要求学生具有较好的微积分知识以及线性代数知识，通俗易懂，是数学、计算机、电子、通信、地质、医学、机械等专业高年级本科生及研究生的基础教科书，也可作为相关技术人员的参考书。

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序

2001年7月间，电子工业出版社的领导同志邀请各高校十几位通信领域方面的老师，商量引进国外教材问题。与会同志对出版社提出的计划十分赞同，大家认为，这对我国通信事业、特别是对高等院校通信学科的教学工作会很有好处。

教材建设是高校教学建设的主要内容之一。编写、出版一本好的教材，意味着开设了一门好的课程，甚至可能预示着一个崭新学科的诞生。20世纪40年代MIT林肯实验室出版的一套28本雷达丛书，对近代电子学科、特别是对雷达技术的推动作用，就是一个很好的例子。

我国领导部门对教材建设一直非常重视。20世纪80年代，在原教委教材编审委员会的领导下，汇集了高等院校几百位富有教学经验的专家，编写、出版了一大批教材；很多院校还根据学校的特点和需要，陆续编写了大量的讲义和参考书。这些教材对高校的教学工作发挥了极好的作用。近年来，随着教学改革不断深入和科学技术的飞速进步，有的教材内容已比较陈旧、落后，难以适应教学的要求，特别是在电子学和通信技术发展神速、可以讲是日新月异的今天，如何适应这种情况，更是一个必须认真考虑的问题。解决这个问题，除了依靠高校的老师和专家撰写新的符合要求的教科书外，引进和出版一些国外优秀电子与通信教材，尤其是有选择地引进一批英文原版教材，是会有好处的。

一年多来，电子工业出版社为此做了很多工作。他们成立了一个“国外电子与通信教材系列”项目组，选派了富有经验的业务骨干负责有关工作，收集了230余种通信教材和参考书的详细资料，调来了100余种原版教材样书，依靠由20余位专家组成的出版委员会，从中精选了40多种，内容丰富，覆盖了电路理论与应用、信号与系统、数字信号处理、微电子、通信系统、电磁场与微波等方面，既可作为通信专业本科生和研究生的教学用书，也可作为有关专业人员的参考材料。此外，这批教材，有的翻译为中文，还有部分教材直接影印出版，以供教师用英语直接授课。希望这些教材的引进和出版对高校通信教学和教材改革能起一定作用。

在这里，我还要感谢参加工作的各位教授、专家、老师与参加翻译、编辑和出版的同志们。各位专家认真负责、严谨细致、不辞辛劳、不怕琐碎和精益求精的态度，充分体现了中国教育工作者和出版工作者的良好美德。

随着我国经济建设的发展和科学技术的不断进步，对高校教学工作会不断提出新的要求和希望。我想，无论如何，要做好引进国外教材的工作，一定要联系我国的实际。教材和学术专著不同，既要注意科学性、学术性，也要重视可读性，要深入浅出，便于读者自学；引进的教材要适应高校教学改革的需要，针对目前一些教材内容较为陈旧的问题，有目的地引进一些先进的和正在发展中的交叉学科的参考书；要与国内出版的教材相配套，安排好出版英文原版教材和翻译教材的比例。我们努力使这套教材能尽量满足上述要求，希望它们能放在学生们的课桌上，发挥一定的作用。

最后，预祝“国外电子与通信教材系列”项目取得成功，为我国电子与通信教学和通信产业的发展培土施肥。也恳切希望读者能对这些书籍的不足之处、特别是翻译中存在的问题，提出意见和建议，以便再版时更正。

吴佑寿

中国工程院院士、清华大学教授

“国外电子与通信教材系列”出版委员会主任

出 版 说 明

进入21世纪以来，我国信息产业在生产和科研方面都大大加快了发展速度，并已成为国民经济发展的支柱产业之一。但是，与世界上其他信息产业发达的国家相比，我国在技术开发、教育培训等方面都还存在着较大的差距。特别是在加入WTO后的今天，我国信息产业面临着国外竞争对手的严峻挑战。

作为我国信息产业的专业科技出版社，我们始终关注着全球电子信息技术的发展方向，始终把引进国外优秀电子与通信信息技术教材和专业书籍放在我们工作的重要位置上。在2000年至2001年间，我社先后从世界著名出版公司引进出版了40余种教材，形成了一套“国外计算机科学教材系列”，在全国高校以及科研部门中受到了欢迎和好评，得到了计算机领域的广大教师与科研工作者的充分肯定。

引进和出版一些国外优秀电子与通信教材，尤其是有选择地引进一批英文原版教材，将有助于我国信息产业培养具有国际竞争能力的技术人才，也将有助于我国国内在电子与通信教学工作中掌握和跟踪国际发展水平。根据国内信息产业的现状、教育部《关于“十五”期间普通高等教育教材建设与改革的意见》的指示精神以及高等院校老师们反映的各种意见，我们决定引进“国外电子与通信教材系列”，并随后开展了大量准备工作。此次引进的国外电子与通信教材均来自国际著名出版商，其中影印教材约占一半。教材内容涉及的学科方向包括电路理论与应用、信号与系统、数字信号处理、微电子、通信系统、电磁场与微波等，其中既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。我们还将与国外出版商一起，陆续推出一些教材的教学支持资料，为授课教师提供帮助。

此外，“国外电子与通信教材系列”的引进和出版工作得到了教育部高等教育司的大力支持和帮助，其中的部分引进教材已通过“教育部高等学校电子信息科学与工程类专业教学指导委员会”的审核，并得到教育部高等教育司的批准，纳入了“教育部高等教育司推荐——国外优秀信息科学与技术系列教学用书”。

为做好该系列教材的翻译工作，我们聘请了清华大学、北京大学、北京邮电大学、东南大学、西安交通大学、天津大学、西安电子科技大学、电子科技大学等著名高校的教授和骨干教师参与教材的翻译和审校工作。许多教授在国内电子与通信专业领域享有较高的声望，具有丰富的教学经验，他们的渊博学识从根本上保证了教材的翻译质量和专业学术方面的严格与准确。我们在此对他们的辛勤工作与贡献表示衷心的感谢。此外，对于编辑的选择，我们达到了专业对口；对于从英文原书中发现的错误，我们通过与作者联络、从网上下载勘误表等方式，逐一进行了修订；同时，我们对审校、排版、印制质量进行了严格把关。

今后，我们将进一步加强同各高校教师的密切关系，努力引进更多的国外优秀教材和教学参考书，为我国电子与通信教材达到世界先进水平而努力。由于我们对国内外电子与通信教育的发展仍存在一些认识上的不足，在选题、翻译、出版等方面的工作中还有许多需要改进的地方，恳请广大师生和读者提出批评及建议。

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Preface

Fourier series and the Fourier transform have been around since the nineteenth century and many research articles and books (at both the graduate and undergraduate levels) have been written about these topics. By contrast, the development of wavelets has been much more recent. While its origins go back many decades, the subject of wavelets has become a popular tool in signal analysis and other areas of applications only within the last two decades or so partly as a result of Ingrid Daubechies's celebrated work on the construction of compactly supported, orthonormal wavelets. Consequently, most of the articles and reference materials on wavelets require a sophisticated mathematical background (a good first-year real analysis course at the graduate level). Our goal with this book is to present many of the essential ideas behind Fourier analysis and wavelets, along with some of their applications to signal analysis, to an audience of advanced undergraduate science, engineering, and mathematics majors. The only prerequisites are a good calculus background and some exposure to linear algebra (a course that covers matrices, vector spaces, linear independence, linear maps, and inner product spaces should suffice). The applications to signal processing are kept elementary, without much use of the technical jargon of the subject, in order for this material to be accessible to a wide audience.

Fourier Analysis

The basic goal of Fourier series is to take a signal, which will be considered as a function of the time variable t, and decompose it into its various frequency components. The basic building blocks are the sine and cosine functions:

$$\sin(nt) \qquad \cos(nt),$$

which vibrate at a frequency of n times per 2π interval. As an example, consider the following function:

$$f(t) = \sin(t) + 2\cos(3t) + 0.3\sin(50t).$$

This function has three components that vibrate at frequency 1 (the $\sin t$ part), at frequency 3 [the $2\cos(3t)$ part], and at frequency 50 [the $0.3\sin(50t)$ part]. The graph of f is given in Figure 1.

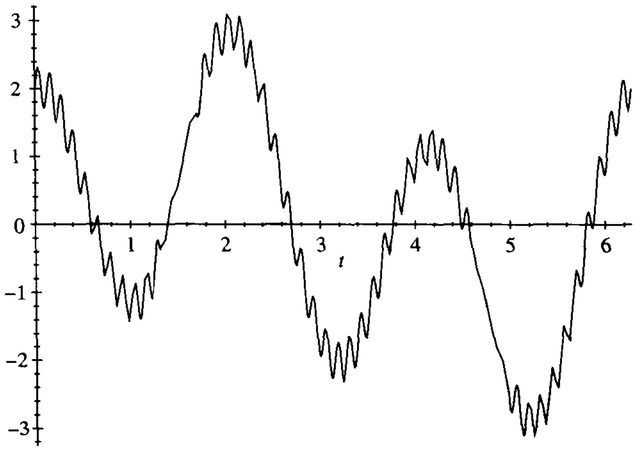

Figure 1 Plot of $f(t) = \sin(t) + 2\cos(3t) + 0.3\sin(50t)$

A common problem in signal analysis is to filter out unwanted noise. The background hiss on a cassette tape is an example of high-frequency (audio) noise that various devices (Dolby filters) try to filter out. In the preceding example, the component, $0.3\sin(50t)$, contributes the high-frequency wiggles to the graph of f in Figure 1. By setting the coefficient 0.3 equal to zero, the resulting function is

$$\tilde{f}(t) = \sin(t) + 2\cos(3t)$$

whose graph (given in Figure 2) is the same as the one for f but without the high-frequency wiggles.

The preceding example shows that one approach to the problem of filtering out unwanted noise is to express a given signal, $f(t)$, in terms of sines and cosines:

$$f(t) = \sum_n a_n \cos(nt) + b_n \sin(nt)$$

and then to eliminate (i.e., set equal to zero) the coefficients (the a_n and b_n) that correspond to the unwanted frequencies. In the case of the signal f just presented, this process is easy since the signal is already presented as a sum of sines and cosines. Most signals, however, are not presented in this manner. The subject of Fourier series, in part, is the study of how to efficiently decompose a function into a sum of cosine and sine components so that various types of filtering can be accomplished easily.

Another related problem in signal analysis is that of data compression. Imagine that the graph of the signal $f(t)$ in Figure 1 represents a telephone conversation. The horizontal axis is time, perhaps measured in milliseconds, and the

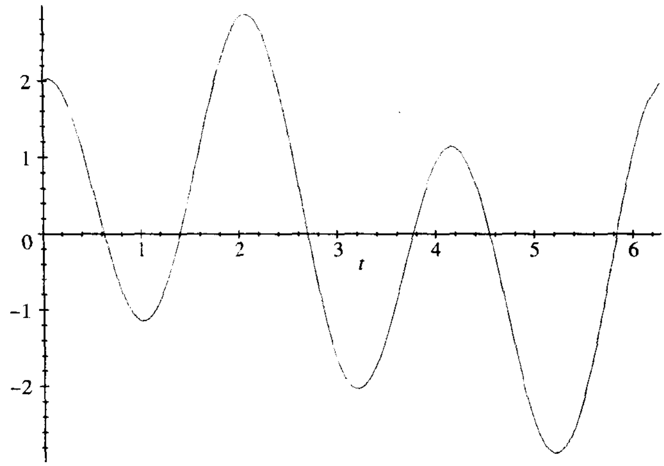

Figure 2 Plot of $f(t) = \sin(t) + 2\cos(3t)$

vertical axis represents the electric voltage of a sound signal generated by someone's voice. Suppose this signal is to be digitized and sent via satellite overseas from America to Europe. One naive approach is to sample the signal every millisecond or so and send these data bits across the Atlantic. However, this would result in thousands of data bits per second for just one phone conversation. Since there will be many such conversations between the two continents, the phone company would like to compress this signal into as few digital bits as possible without significantly distorting the signal. A more efficient approach is to express the signal into its Fourier series: $f(t) = \sum_n a_n \cos(nt) + b_n \sin(nt)$ and then discard those coefficients, a_n and b_n, that are smaller than some tolerance for error. Only those coefficients that are above this tolerance need to be sent across the Atlantic, where the signal can then be reconstructed. For most signals, the number of significant coefficients in its Fourier series is relatively small.

Wavelets

One disadvantage of Fourier series is that its building blocks, sines and cosines, are periodic waves that continue forever. While this approach may be appropriate for filtering or compressing signals that have time-independent wavelike features (as in Figure 1), other signals may have more localized features for which sines and cosines do not model very well. As an example, consider the graph given in Figure 3. This may represent a sound signal with two isolated noisy pops that need to be filtered out. Since these pops are isolated, sines and cosines do not model this signal very well. A different set of building blocks, called *wavelets*, is designed to model these types of signals. In a rough sense, a wavelet looks like a wave that travels for one or more periods and is nonzero only

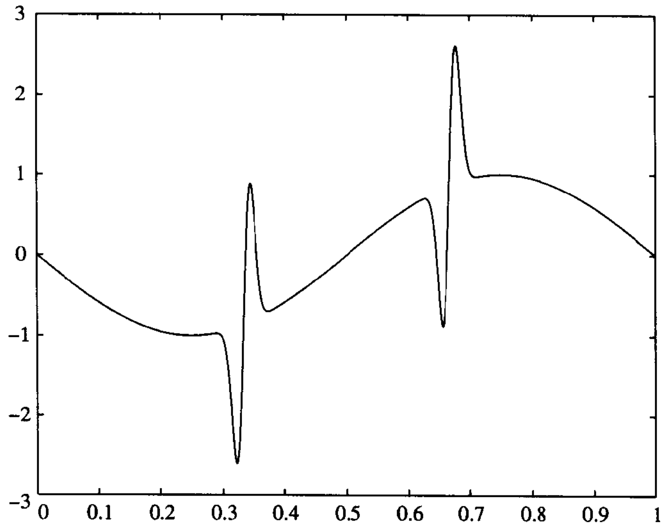

Figure 3 Graph of a signal with isolated noise

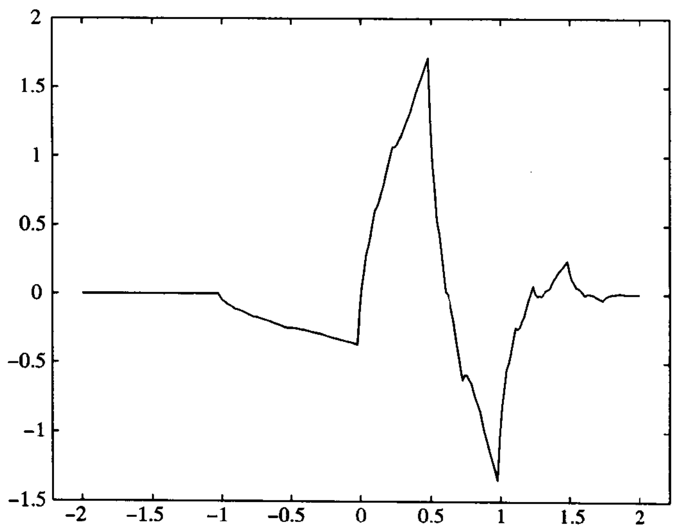

Figure 4 Graph of Daubechies wavelet

over a finite interval instead of propagating forever the way sines and cosines do [see Figure 4 for the graph of the Daubechies ($N = 2$) wavelet]. A wavelet can be translated forward or backward in time. It also can be stretched or compressed by scaling to obtain low- and high-frequency wavelets (see Figure 5). Once a wavelet function is constructed, it can be used to filter or compress signals in

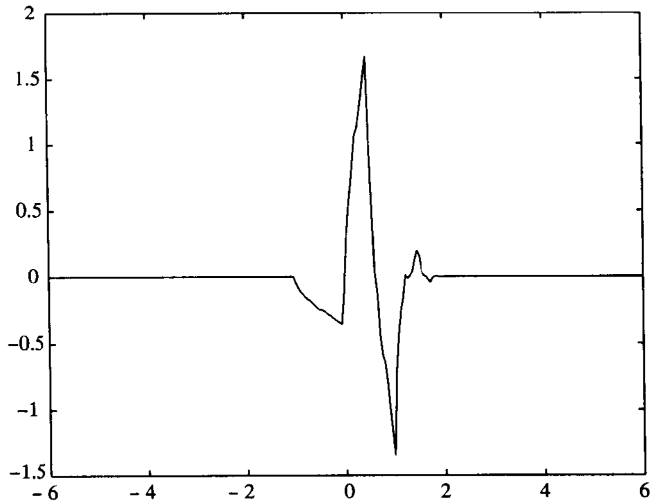

Figure 5 High-frequency Daubechies wavelet

much the same manner as Fourier series. A given signal is first expressed as a sum of translations and scalings of the wavelet. Then the coefficients corresponding to the unwanted terms are removed or modified.

In order to implement efficient algorithms for decomposing a signal into an expansion (either Fourier or wavelet based), the building blocks (sines, cosines or wavelets) should satisfy various properties. One convenient property is *orthogonality*, which for the sine function states

$$\frac{1}{\pi}\int_0^{2\pi} \sin(nt)\sin(mt)\,dt = \begin{cases} 0 & \text{if } n \neq m \\ 1 & \text{if } n = m. \end{cases}$$

The analogous properties hold for the cosine function as well. In addition, $\int_0^{2\pi} \sin(nt)\cos(mt)\,dt = 0$ for all n and m. We shall see that these orthogonality properties result in simple formulas for the Fourier coefficients (the a_n and b_n) and efficient algorithms (fast Fourier transform) for their computation.

One of the difficult tasks in the construction of a wavelet is to make sure that its translates and rescalings satisfy analogous orthogonality relationships, so that efficient algorithms for the computation of the wavelet coefficients of a given signal can be found. This is why we cannot construct a wavelet simply by truncating a sine or cosine wave by declaring it to be zero outside of one or more of its periods. Such a function, while satisfying the desired support feature of a wavelet, would not satisfy any reasonable orthogonality relationship with its translates and rescales and thus would not be as useful for signal analysis.

Outline

This text has eight chapters and two appendices. Chapter 0, on inner product spaces, contains the necessary prerequisites for Chapters 1 through 7. The primary inner product space of interest is the space of square integrable functions, which is presented in simplified form without the use of the Lebesgue integral. Depending on the audience, this chapter can be covered at the beginning of a course or can be folded into the course as the need arises. Chapter 1 contains the basics of Fourier series. Several convergence theorems are presented with simplifying hypothesis so that their proofs are manageable. The Fourier transform is presented in Chapter 2. Besides being of interest in its own right, much of this material is used in later chapters on wavelets. An informal proof of the Fourier inversion formula is presented in order to keep the exposition at an elementary level. A formal proof is given in the Appendix A. The discrete Fourier transform and fast Fourier transform are discussed in Chapter 3. This chapter also contains applications to signal analysis and to the identification of the natural vibrating frequency (or sway) of a building.

Wavelets are discussed in Chapters 4 through 7. Our presentation on wavelets starts with the case of the Haar wavelets in Chapter 4. The basic ideas behind a multiresolution analysis and the desired features of wavelets, such as orthogonality, are easy to describe with the explicitly defined Haar wavelets. However, the Haar wavelets are discontinuous and so they are of limited use in signal analysis. The concept of a multiresolution analysis in a general context is presented in Chapter 5. This gives a general framework that generalizes the structure of the wavelet spaces generated by the Haar wavelet. Chapter 6 contains the construction of the Daubechies wavelet, which is continuous and orthogonal. Prescriptions for smoother wavelets are also given. Chapter 7 contains more advanced topics, such as wavelets in higher dimensions and the wavelet transform.

The proofs of most theorems are given in the text. Some of the more technical theorems are discussed in a heuristic manner with complete proofs given in Appendix A. Some of these proofs require more advanced mathematics, such as some exposure to the Lebesgue integral.

MATLAB code that was used to generate figures or to illustrate concepts is found in Appendix B.

This text is not a treatise. The focus of the latter half of the book is on the construction of orthonormal wavelets. Little mention is made of bi-orthogonal wavelets using splines and other tools. There are ample references for these other types of wavelets (see, for example, [5]) and we want to keep the amount of material in this text manageable for a one-semester undergraduate course.

The basics of Fourier analysis and wavelets can be covered in a one semester undergraduate course using the following outline:

- Chapter 0, Sections 0.1 through 0.5 (Sections 0.6 and 0.7 on adjoints, least squares, and linear predictive coding are more topical in nature). This material can either be covered first or covered as needed throughout the rest of the course.

- Chapter 1 (Fourier Series), all sections.
- Chapter 2 (Fourier Transform), all sections except the ones on the adjoint of the Fourier transform, and the proof of the uncertainty principle, which are more topical in nature.
- Chapter 3 (Discrete Fourier Analysis), all sections except the Z-transform, which is more topical in nature.
- Chapter 4 (Haar Wavelet Analysis), all sections.
- Chapter 5 (Multiresolution Analysis), all sections.
- Chapter 6 (Daubechies Wavelets), all sections.

Acknowledgments

This book arose from lecture notes used by both authors for the Fourier Analysis and Wavelets course taught at Texas A&M. The authors would like to thank the many students in these classes that gave critical comments on the manuscript. We would especially like to thank Svenja Lowitzsch and Beng Ong who read the manuscript carefully and corrected many of our mistakes. The authors would also like to thank the editorial staff; in particular, George Lobell, for the kind encouragement during the development of this book. the following reviewers did an excellent job of making suggestions and pointing out errors, William Beckner, University of Texas; Joe Lakey, New Mexico State University; Edward A. Newburg, Rochester Institute of Technology; Oscar Rothaus, Cornell University; and David Weinberg, Texas Tech university. Of course, any mistakes remaining in the book are solely the fault of the authors. We also thank Steven S. Pawlowski and the rest of the staff at Prentice Hall for their professional production of the book. On a personal note, Fran Narcowich would also like to thank his wife Linda for support and encouragement.

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