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A Journey into CDOs, Copulas, Correlations and Dynamic Models

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Preface

RANDOMIZED BY FOOLISHNESS

This is a book that has been written very rapidly in order to respond to a perceived need for clarity in the quantitative world. The second section of this preface, "How I learned to stop worrying and love the CDOs", is obviously ironic in referring to the hysteria that has often characterized accounts of modelling and mathematical finance in part of the press and the media, and the demonization of part of the market products related to the crisis, such as CDOs and derivatives more generally (Brigo *et al.*, 2009b). Let us be clear and avoid any misunderstanding: the crisis is very real; it has caused suffering to many individuals, families and companies. However, it does not help to look for a scapegoat without looking at the whole picture with a critical eye. Accounts that try to convince the public that the crisis is due mainly to modelling and to the trading of sophisticated and obscure products are necessarily partial, and this book is devoted to rectifying this perceptive bias.

Indeed, public opinion has been bombarded with so many clichés on derivatives, modelling and quantitative analysis that we feel that a book offering a little clarity is needed. And while we are aware that this sounds a little Don Quixotesque, we hope the book will help to change the situation. In trying to do so, we need to balance carefully the perspectives of different readerships. We would like our book to be attractive to the relatively general industry and academic public without disappointing the scientific and technically minded specialists, and at the same time we do not want our book to be a best-seller type of publication full of bashing and negatively provocative ideas with very little actual technical content. All of this, while keeping windmills at large.[1] Hence we will be walking the razor's hedge in trying to maintain a balance between the popular account and the scientific discourse.

We are not alone in our attempt to bring clarity.[2] This book, however, does so in an extensive and technical way, showing past and present research that is quite relevant in disproving a number of misconceptions on the role of mathematics and quantitative analysis in relation to the crisis. This book takes an extensive technical path, starting with static copulas and ending with dynamic loss models. Even though our book is short, we follow a long path for credit derivatives and multi-name credit derivatives in particular, focusing on Collateralized Debt Obligations (CDOs). What are CDOs? To describe the simplest possible CDO, say a synthetic CDO on the corporate market, we can proceed as follows.

We are given a portfolio of, say, 125 names. The names may default thus generating losses to investors exposed to those names. In a CDO tranche there are two parties: a protection buyer and a protection seller. A tranche is a portion of the loss of the portfolio between two percentages. For example, the 3–6% tranche focuses on the losses between 3% (attachment point) and 6% (detachment point). Roughly speaking, the protection seller agrees to pay to the buyer all notional default losses (minus the recoveries) in the portfolio whenever they occur due to one or more defaults of the entities, within 3% and 6% of the total pool loss. In exchange for this, the buyer pays the seller a periodic fee on the notional given by the portion of the tranche that is still "alive" in each relevant period.

[1] *The Ingenious Hidalgo Don Quixote of La Mancha*, Miguel de Cervantes, 1605 and 1615.

[2] See primarily Shreve (2008) and also, for example, Donnelly and Embrechts (2009). Szego (2009) gives a much broader overview of the crisis, with some critical insight that is helpful in clarifying the more common misconceptions.

In a sense, CDOs look like contracts selling (or buying) insurance on portions of the loss of a portfolio. The valuation problem is trying to determine the fair price of this insurance.

The crucial observation here is that "tranching" is a non-linear operation. When computing the price (mark to market) of a tranche at a point in time, one has to take the expectation of the future tranche losses under the pricing measure. Since the tranche is a non-linear function of the loss, the expectation will depend on all moments of the loss and not just on the expected loss. If we look at the single names in the portfolio, the loss distribution of the portfolio is characterized by the marginal distributions of the single-name defaults and by the dependency among the defaults of different names. Dependence is commonly called, with an abuse of language, "correlation". This is an abuse of language because correlation is a complete description of dependence for jointly Gaussian random variables, although more generally it is not. In general, the term "dependence" should be used in lieu of "correlation". The complete description is either the whole multivariate distribution or the so-called "copula function" – that is, the multivariate distribution once the marginal distributions have been standardized to uniform distributions.

The dependence of the tranche on "correlation" is crucial. The market assumes that there is a Gaussian Copula connecting the defaults of the 125 names. This copula is parameterized by a matrix with 7750 entries of pairwise correlation parameters. However, when looking at a tranche, these 7750 parameters are all assumed to be equal to each other. So one has a unique parameter. This is such a drastic simplification that we need to make sure it is noticed:

$$7750 \text{ parameters} \longrightarrow 1 \text{ parameter.}$$

One then chooses the tranches that are liquid on the market for standardized portfolios, for which the market price is known as these tranches are quoted. The unique correlation parameter is then reverse-engineered to reproduce the price of the liquid tranche under examination. This is called *implied* correlation, and once obtained it is used to value related products. The problem is that whenever the tranche is changed, this implied correlation also changes. Therefore, if at a given time the 3–6% tranche for a 5-year maturity has a given implied correlation, the 6–9% tranche for the same maturity will have a different one. It follows that the two tranches on the same pool are priced with two models having different and inconsistent loss distributions, corresponding to the two different correlation values that have been implied.

This may sound negative, but as a matter of fact the situation is even worse. We will explain in detail that there are two possible implied correlation paradigms: compound correlation and base correlation. The second correlation is the one that is prevailing in the market. However, the base correlation is inconsistent even at a single tranche level, in that it prices the 3–6% tranche by decomposing it into the 0–3% tranche and 0–6% tranche and, for that tranche level, using two different correlations (and hence distributions). Therefore, the base correlation is already inconsistent at the single tranche level, and this inconsistency shows up occasionally in negative losses (i.e. in defaulted names resurrecting).

This is admittedly enough to spark a debate. Even before modelling enters the picture, some famous market protagonists have labelled the objects of modelling (i.e. derivatives) as being responsible for a lot of problems. Warren Buffett (2003), in a very interesting report, wrote:

> [. . .] Charlie and I are of one mind in how we feel about derivatives and the trading activities that go with them: We view them as time bombs, both for the parties that deal in them and the economic system. [. . .] In our view [. . .] derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal. [. . .] The range of derivatives contracts is limited only by the imagination of man (or sometimes, so it seems, madmen).

While, when hearing about products such as Constant Proportion Debt Obligations (CPDOs) or CDO squared, one may sympathize with Mr Buffett, this overgeneralization might be a little excessive. Derivatives, when used properly, can be quite useful. For example, swap contracts on several asset classes (interest rates, foreign exchange, oil and other commodities) and related options allow entities to trade risks and buy protection against adverse market moves. Without derivatives, companies could not protect themselves against adverse future movements in the prices of oil, exchange rates, interest rates, etc. This is not to say that derivatives cannot be abused. They certainly can, and we invite the interested-readers to reason on the case of CPDOs[3] as an example, and to read the whole report by Mr Buffett.

When moving beyond the products and entering the modelling issues, one may still find popular accounts resorting to quite colourful expressions such as "the formula that killed Wall Street". Indeed, if one looks at popular accounts such as Salmon (2009) or Jones (2009), just to take two examples, one may end up with the impression that the quantitative finance ("quant") community has been incredibly naive in accepting the Gaussian Copula and implied correlation without questioning it, possibly leading to what Warren Buffett calls "mark to myth" in his above-mentioned report, especially when applying the calibrated correlation to other non-quoted "bespoke tranches". In fact both articles have been written on the Gaussian Copula – a static model that is little more than a static multivariate distribution which is used in credit derivatives (and in particular CDOs) valuation and risk management. Can this simple static model have fooled everyone into believing it was an accurate representation of a quite dynamic reality, and further cause the downfall of Wall Street banks? While Salmon (2009) correctly reports that some of the deficiencies of the model have been known, Jones (2009) in the *Financial Times* wonders why no one seemed to have noticed the model's weaknesses. The crisis is considered to have been heavily affected by mathematical models, with the accent on "mathematical".

This is in line with more general criticism of anything quantitative appearing in the news. As an example, the news article "McCormick

[3] See, for example, Torresetti and Pallavicini (2007) and the Fitch Ratings report "First Generation CPDO: Case Study on Performance and Ratings", published before the crisis in April 2007, stating "[. . .] Fitch is of the opinion that the past 10 years by no means marked a high investment grade stress in the range of AAA or AA."

Bad Dollars Derive From Deficits Model Beating Quants",[4] which is more focused on the currency markets, informs us that "[...] focus on the economic reasons for currency moves is gaining more traction after years when traders and investors relied on mathematical models of quantitative analysis." Then it continues with "These tools worked during times of global growth and declining volatility earlier this decade, yet failed to signal danger before the financial crisis sparked the biggest currency swings in more than 15 years. McCormick, using macroeconomic and quantitative analyses, detected growing stresses in the global economy before the meltdown." The reader, by looking at this, may understand that on one side "mathematical models of quantitative analysis" (a sentence that sounds quite redundant) fail in times of crises, whereas "macroeconomic and quantitative analyses" helped to predict some aspects of the crisis. One is left to ponder the different uses of "quantitative" between "mathematical models of quantitative analysis" and "macroeconomic and quantitative analyses". It is as if mathematics had all of a sudden become a bad word. Of course the article aims at saying that macroeconomic analysis and fundamentals are of increasing importance and should be taken more into account, although in our opinion it does not clearly distinguish valuation from prediction, but some of the sentences used to highlight this idea are quite symptomatic of the attitude we described above towards modelling and mathematics.

Another article that brings mathematics and mathematicians (provided that is what one means by "math wizards") into the "blaming" picture is Lohr (2009) – "Wall Street's Math Wizards Forgot a Few Variables" – which appeared in the *New York Times* of 12 September. Also, Turner (2009) has a section entitled "Misplaced reliance on sophisticated maths".

This overall hostility and blaming attitude towards mathematics and mathematicians, whether in the industry or in academia, is the reason why we feel it is important to point out the following: the notion that even more mathematically oriented quants have not been

[4] By Oliver Biggadike, 24 November 2009, Bloomberg.

aware of the Gaussian Copula model's limitations is simply false, as we are going to show, and you may quote us on this. The quant and academic communities have produced and witnessed a large body of research questioning the copula assumption. This is well documented: there is even a book based on a one-day conference hosted by Merrill Lynch in London in 2006, well before the crisis, and called *Credit Correlation: Life After Copulas* (Lipton and Rennie, 2007). This conference had been organized by practitioners. The *Life After Copulas* book contains several attempts to go beyond the Gaussian Copula and implied correlation, most of which come from practitioners (and a few from academics). But that book is only the tip of the iceberg. There are several publications that appeared pre-crisis that questioned the Gaussian Copula and implied correlation. For example, we warned against the dangers implicit in the use of implied correlation in the report "Implied Correlation: A paradigm to be handled with care", that we posted in SSRN in 2006, again well before the crisis (Torresetti *et al.*, 2006b).

Still, it seems that this is little appreciated by some market participants, commentators, journalists, critics, politicians and academics. There are still a number of people who think that a formula killed Wall Street (see also the discussion in Embrechts, 2009).

This book brings a little clarity by telling a true story of pre-crisis warnings and also of pre-crisis attempts to remedy the drawbacks of implied correlation. We do not document the whole body of research that has addressed the limits of base correlation and of the Gaussian Copula, but rather take a particular path inside this body, based on our past research, which we also update to see what our models tell us in-crisis.

To explain our book in a nutshell, we can say that it starts from the payoffs of CDOs, explaining how to write them and how they work. We then move to the introduction of the inconsistent Gaussian Copula model and the related implied correlations, both compound and base, moving then to the GPL model: an arbitrage-free dynamic loss model capable of consistently calibrating all the tranches across attachments and detachments for all the maturities at the same time.

En passant, we also illustrate the Implied Copula, a method that can consistently account for CDOs with different attachment and detachment points but not for different maturities, and the Expected Tranche Loss (ETL) surface, a model-independent approach to CDO price interpolation.

We will see that, already pre-crisis, both the Implied Copula and the dynamic loss model imply modes far down the right tail of the loss distribution. This means that there are default probability clusters corresponding to joint default of a large number of entities (sectors) of the economy.

The discussion is abundantly supported by market examples throughout history. We cannot stress enough that the dangers and critics we present to the use of the Gaussian Copula and of implied correlation, and the modes in the tail of the loss distribution obtained with consistent models, had all been published by us, among others, in 2006, well before the crisis.

Despite these warnings, the Gaussian Copula model is still used in its base correlation formulation, although under some possible extensions such as random recovery. The reasons for this are complex. First, the difficulty of all the loss models, improving the consistency issues, in accounting for single-name data and to allow for single-name sensitivities. This is due to the fact that if we model the loss of the pool directly as an aggregate object, without taking into account single defaults, then the model sensitivities to single-name credit information are not in the picture. In other terms, while the aggregate loss is modelled so as to calibrate satisfactorily indices and tranches, the model does not see the single-name defaults but just the loss dynamics as an aggregate object. Therefore partial hedges with respect to single names are not possible. As these issues are crucial in many situations, the market practice remains with base correlation. Furthermore, even the few models achieving single-name consistency have not been developed and tested enough to become operational on a trading floor or on a large risk management platform. Indeed, a fully operational model with realistic run times and numerical stability across a large range of possible market inputs would be more than a prototype with some satisfactory properties that has been run in some "off-line" studies. Also, when one model has been coded in the libraries of a financial institution, changing the model implies a long path involving a number of issues that have little to do with modelling and more to do with IT problems, integration with other systems, etc. Therefore, unless a new model appears to be really promising and extremely convincing in all its aspects, there is reluctance to adopt it on the trading floor or for risk management systems.

Overall we conclude that the modelling effort in this area of the derivatives market is unfinished, partly for the lack of an operationally attractive single-name consistent dynamic loss model, and partly because of the diminished investment in this research area, but the fact that the modelling effort is unfinished does not mean that the quant community has been unaware of model limitations, as we abundantly document, and, although our narrative ends with an open finale, we still think it is an entertaining true story.

HOW I LEARNED TO STOP WORRYING AND LOVE THE CDOs

We cannot close this preface without going back to the large picture, and ask the more general question: Is the crisis due to poor modelling?

As we have seen, the market has been using simplistic approaches for credit derivatives, but it has also been trying to move beyond those approaches. However, we should also mention that CDOs are divided into two categories: cash and synthetics. Cash CDOs involve hundreds or even thousands of names and have complex path-dependent payouts ("waterfalls"). Even so, cash CDOs are typically valued by resorting to single homogeneous default-rate scenarios or very primitive assumptions, and very little research and literature is available on them. Hence, these are complex products with sophisticated and path-dependent payouts that are often valued with extremely simplistic models. Synthetic CDOs are the ones that we described in this preface and will be addressed in this book. They have more simple and standardized payouts than the cash CDOs but are typically valued with more sophisticated models, given the larger standardization and the ease in finding market quotes for their prices. Synthetic CDOs on corporates are epitomized by the quoted tranches of the standard pools DJ-iTraxx (Europe) and CDX (USA). However, CDOs, especially cash CDOs, are available on other asset classes, such as loans (CLO), Mortgage Portfolios, leading to Residential Mortgage Backed Portfolios (RMBS) and Commercial Mortgage Backed Securities (CMBS), and on and on. For many of these CDOs, and especially RMBSs which are quite related to the asset class that triggered the crisis, the problem is in the data rather than in the models. Bespoke corporate pools have no data from which to infer default "correlation" and dubious mapping methods are used. At times, data for valuation in mortgages CDOs (RMBSs and CDOs of RMBSs) are dubious and can be distorted by fraud.[5]

At times it is not even clear what is in the portfolio: the authors have seen offering circulars of a RMBS on a huge portfolio of residential mortgages where more than the 2% of properties in the portfolio were declared to be of unknown type. What inputs can we give to the models if we do not even know the kind of residential property underlying the derivative product?

All this is before modelling. Models obey a simple rule that is popularly summarized by the acronym GIGO (Garbage In → Garbage Out). As Charles Babbage (1791–1871) famously put it:

> On two occasions I have been asked [by members of Parliament], "Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?" I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.

So, in the end, is the crisis due to the inadequacy of the models? Is the crisis due to the pride of quantitative analysts and academics and an unawareness of the limitations of the models?

We show in this book that quants have been aware of the limitations and of extreme risks before the crisis. Lack of data or fraud-corrupted

[5] See, for example, the FBI Mortgage Fraud Report, 2007, www.fbi.gov/publications/fraud/mortgage_fraud07.htm.

data, the fragility in the "originate to distribute" system, liquidity and reserves policies, regulators' lack of uniformity, excessive leverage and concentration in real estate investment, poor liquidity risk management techniques, accounting rules and excessive reliance on credit rating agencies are often factors not to be underestimated. This crisis is a quite complex event that defies witch-hunts, folklore and superstition. Methodology certainly needs to be improved but blaming just the models for the crisis appears, in our opinion, to be the result of a very limited point of view.

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