Studies of Vortex Dominated Flows

Edited by
M.Y. Hussaini and M.D. Salas

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Introduction

From the astrophysical scale of a swirling spiral galaxy, through the geophysical scale of a hurricane, down to the subatomic scale of elementary particles, vortical motion and vortex dynamics have played a profound role in our understanding of the physical world. Kuchemann referred to vortex dynamics as "the sinews and muscles of fluid motion." In order to update our understanding of vortex dominated flows, NASA Langley Research Center and the Institute for Computer Applications in Science and Engineering (ICASE) conducted a workshop during July 9–11, 1985. The subject was broadly divided into five overlapping topics—vortex dynamics, vortex breakdown, massive separation, vortex shedding from sharp leading edges and conically separated flows. Some of the experts in each of these areas were invited to provide an overview of the subject. This volume is the proceedings of the workshop and contains the latest, theoretical, numerical, and experimental work in the above-mentioned areas.

Leibovich, Widnall, Moore and Sirovich discussed topics on the fundamentals of vortex dynamics, while Keller and Hafez treated the problem of vortex breakdown phenomena; the contributions of Smith, Davis and LeBalleur were in the area of massive separation and inviscid-viscous interactions, while those of Cheng, Hoeijmakers and Murman dealt with sharp-leading-edge vortex flows; and Fiddes and Marconi represented the category of conical separated flows.

The opening article of this volume by Leibovich deals with the principal features of weakly nonlinear bending waves (in the form of solitons) on infinitely long, initially straight vortex filaments. Such studies, while of interest in their own right, also provide possible insight into the highly nonlinear vortex breakdown phenomena. In this article, in outline form, Leibovich treats the global bifurcation of axially symmetric, steady, inviscid vortex flows, and suggests the connection with Benjamin's vortex breakdown theory.

Widnall gives a brief review of the linear stability theory for concentrated vortex structures. She distinguishes between three types of instability—two-dimensional, three-dimensional long-wave and three-dimensional short-wave instability. Then she analyzes the three-dimensional instability of a single simple vortex shedding from a cylinder and the Foppl vortices modelling the flow behind the cylinder. Such instability mechanisms are proposed as sources of three-dimensionality in separated and turbulent flows.

Sirovich and Lim provide a historical background for Karman vortex street, and they interpret it afresh in the light of a recent experiment on the flow behind a circular cylinder. This experiment relates the flow structure to that of a low order nonlinear dynamical system. They give a fairly complete treatment to the initial value problem for the linear evolution of an initial perturbation to the Karman vortex trail. They find "encouraging" similarities between their theoretical results and some experiments for the rotation number (the ratio of the second frequency to the shedding frequency) behavior with Reynolds number. Such a similarity is also found for the phenomenon of wave propagation along the trail.

The relevance of vortex sheets to separated flows and the origin of turbulent shear flows is very well established. In their article, de Bernardinis and Moore discuss the ring-vortex representation of an axi-symmetric vortex sheet for practical purposes. This is an extension of Van der Vooren's procedure for two-dimensional vortex sheets. This representation loses accuracy if the vortex sheet intersects the axis of symmetry. This loss of accuracy is discussed in the case of an instantaneously spherical vortex sheet.

The work of Keller et al. extends Benjamin's variational principle for axi-symmetric flows to include free-surface flows. They apply it to a Rankine vortex, and show a loss-free transition between two vortex states, and discuss its relevance to a certain type of vortex breakdown.

Hafez and Salas present results (based on two entirely different numerical methods) which show that the pertinent equations governing a steady axi-symmetric inviscid flow with swirl yield solutions with closed streamlines. They also present solutions to the steady axi-symmetric Navier–Stokes equations, but the Reynolds numbers calculated are too low to allow any conclusions on whether the inviscid solutions obtained are limiting solutions of the viscous problem.

The high-Reynolds-number, large-scale separated flows have been one of the central problems of fluid dynamics. The theory is far from complete, and the full-scale computations are not absolutely convincing. Smith presents a succinct review of the available theory for two-dimensional incompressible flows with massive separation and discusses their properties and numerical solutions. He adopts the view that since small-scale separated flows can be described completely within the framework of triple deck theory, they can be used to get an insight into the physical mechanisms involved in massive separations. The work of Rothmayer is such an attempt. The paper of LeBalleur is a fairly comprehensive review of the status of inviscid-viscous interaction procedures for the computation of massively separated flows.

Hoeijmakers reviewed the prevalent computational methods for the simulation of aerodynamic flow configurations involving a leading-edge vortex. These methods are categorized into rigid-vortex methods, fitted-vortex methods and captured-vortex methods. The rigid-vortex methods consist of classical potential flow techniques (such as vortex-lattice methods) which incorporate empirical concepts to account for vortical interactions without resolving the vortical flow details. Fitted-vortex methods fix vortex sheets in potential flow models to allow for vortical interactions and include discrete vortex methods and nonlinear vortex lattice methods. Captured-vortex methods are based on Euler equations or Navier–Stokes equations. A very useful comparative study of these methods is given. Cheng et al. employ two types of fitted-vortex methods to simulate the vortex dynamics of a leading edge flap, while Murman and Powell study the leading edge vortex shed from a delta wing. The latter solve the Euler equations by Jameson's finite-volume technique on a rather fine grid. They obtain good agreement with the measured pitot pressures.

Fiddes and Marconi both consider separated flows about cones at incidence. Fiddes focuses on modelling the vortical flow by line vortex models and vortex sheet models. A major interest of this work is in understanding the development of asymmetry in the flow field. Marconi solves the Euler equations by Moretti's lambda-scheme, and fits both the bow shock and the cross-flow shock. This work is one of the most systematic studies of the conically separated flows on record.

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Contents

Section I

Vortex Dynamics

Waves and Bifurcations in Vortex Filaments

SIDNEY LEIBOVICH

ABSTRACT

Weakly nonlinear bending waves on vortex filaments are briefly described. These take the form of solitons governed by the cubically nonlinear Schrodinger equation. In addition, conditions leading to the global bifurcation of inviscid, axially-symmetric vortices are outlined.

1. Introduction

Concentrated vortices, such as those commonly found in flows of aerodynamical interest, allow waves to propagate along their cores much like water waves on a running stream. In addition, vortices often are generated which, due to instabilities, cannot persist without change of form. In this paper, we consider inviscid phenomena that may happen in infinitely long, initially straight vortex filaments having vortex cores with prescribed structures. These idealized vortices are assumed to exist in an unbounded incompressible fluid.

This work is motivated in part by the phenomenon of vortex breakdown (see Leibovich (1984)). These coherent, highly nonlinear, disruptions of nearly columnnar vortex flows (that is, flows with cylindrical streamsurfaces) do not yet have a satisfactory theoretical description. At their best, existing theories for waves on vortices provide a description of weakly nonlinear motions, and therefore cannot be expected to amount to a fully acceptable theory for vortex breakdown. Despite this, it has been argued (Faler and Leibovich, 1977; Leibovich, 1983, 1984; Maxworthy et al., 1983; Escudier et al., 1982) that wave theories have provided useful insights concerning vortex breakdown events. Those wave theories which have been heretofore invoked in relation to this phenomenon admit only axially symmetric disturbances, even though it is known that vortex breakdown does not have this symmetry. In particular, "bending waves", those which cause the vortex centerline to move radially, are known to exist in all forms of vortex breakdown (Leibovich, 1984). The first part of this paper concerns weakly nonlinear bending waves: these, of course, are of interest beyond their possible connections to vortex breakdown, as the recent work of Hopfinger et al. (1984) and Maxworthy et al. (1985) attest.

A second topic addressed here concerns the bifurcation of a class of fully nonlinear vortex flows. We ask when multiple solutions can occur for steady, axially symmetric vortex flows with specified boundary data. The method used is global in the sense that the existence of multiple solutions, and the location of the bifurcation criterion are established without use of local expansions.

The treatment of both topics of this paper will be in outline form only; more detailed presentations of the subject matter will be given elsewhere.

2. Bending waves and spiral solitons

We deal with the Euler equations and use cylindrical (r,θ,z) coordinates. Any columnar flow with velocity vector

$$\underset{\sim}{U}(r) = (0, V(r), W(r)) \tag{1}$$

as a possible inviscid motion. Perturbations to this flow are considered with velocity vectors

$$\underset{\sim}{v} = U(r) + \varepsilon \underset{\sim}{u}(r,\theta,z,t;\varepsilon);$$

$$\underset{\sim}{u} = (u,v,w) = \underset{\sim}{u}_0(r,\theta,z,t,\tau,Z) + \varepsilon \underset{\sim}{u}_1 + \varepsilon^2 \underset{\sim}{u}_2 + \cdots , \tag{2}$$

where ε is a small amplitude parameter and dots stand for higher order terms which will not be considered. The arguments τ and Z are slow time and space variables to be specified; they are included in anticipation of the need for a multiple scaling analysis in the accounting for nonlinearities.

2.1 Linear analysis

The assumed expansion is substituted into the Euler equations and terms higher order than those linear in ε are neglected, leaving a problem to be solved for $\underset{\sim}{u}_0$. All disturbance quantities are assumed to be in normal mode form:

$$\underset{\sim}{u}_0 = \underset{\sim}{u}_0(r) \exp i(kz + m\theta - \omega t) . \tag{3}$$

and we let the radial component of $\underset{\sim}{u}_0$ be f. The modal amplitude $f(r)$ satisfies a second order differential equation first given by Howard and Gupta (1962). This equation may be compactly written as follows

$$D(SD_* f) - \left(1 + \frac{a}{\gamma} + \frac{b}{\gamma^2}\right) f = 0 \tag{4}$$

where

$$\gamma = \underset{\sim}{k} \cdot \underset{\sim}{U} - \omega; \quad \underset{\sim}{k} = mr^{-1} \underset{\sim}{e}_\theta + k \underset{\sim}{e}_z; \quad S = 1/|\underset{\sim}{k}|^2;$$

$$a = r^2\nabla\cdot[r^{-2}(\underset{\sim}{k}\times\underset{\sim}{\zeta})/|\underset{\sim}{k}|^2]; \quad b = -2(\underset{\sim}{k}\cdot\underset{\sim}{\Omega})(\underset{\sim}{k}\cdot\underset{\sim}{\zeta})/|\underset{\sim}{k}|^2; \quad \underset{\sim}{\Omega} = \underset{\sim}{e}_z V r^{-1}$$

Here $\underset{\sim}{e}_\theta$ and $\underset{\sim}{e}_z$ are unit vectors and ζ is the unperturbed vorticity vector.

The function f(r) must vanish as r tends to infinity, and, for $|m| = 1$, single-valuedness requires $Df = 0$ at $r = 0$. We consider here only the bending modes $|m| = 1$.

The parameter k is regarded as given, so that the problem for f is an eigenvalue problem with the frequency ω as eigenvalue. For general velocity fields $\underset{\sim}{U}$ and arbitrary k, the eigenvalue problem must be solved numerically. For long waves ($k \ll 1$) numerical computation reveals at least two branches of the dispersion relation, $\omega = \omega(k)$. Both have zero frequency at $k = 0$; one has zero frequency and phase speed at $k = 0$, and we call this the "slow branch". The other(s) (generally more than one) branch(es) of the dispersion relation has nonzero frequency and infinite phase speed at $k = 0$; such a branch we call a "fast branch".

Long waves are of considerable interest, since many phenomena arising in vortex filaments occur on length scales large compared to the radius of the vortex core. It is therefore significant that the eigenfunction and dispersion relation on the slow branch in the limit $k \to 0$ can be found by a singular perturbation analysis of the Howard-Gupta equation for arbitrary vortex core structure. The analysis will be described elsewhere; here we cite only the final results. The composite expansion of the eigenfunction valid to $O(k^2)$ is

$$\begin{aligned} f(r) &= \Omega(r) + m|k|W(r) \\ &\quad - k^2\{\Omega r^2/2 + \int_0^r [x^3\Omega^2(x)]^{-1}dx \int_0^x [y^3\Omega^2(y) + 2y^2 WW'(y)]dy\} \\ &\quad - m\omega - \frac{\Gamma_0}{2\pi r^2} + \frac{\Gamma_0}{4\pi} k^2[\ln(|k|r/2) + \gamma_e + \tfrac{1}{2}] \\ &\quad - |k|\frac{\Gamma_0}{2\pi}\frac{d}{dr} K_1(|k|r) \end{aligned} \tag{5}$$

where K_1 is the modified Bessel function of the second kind, and the dispersion relation to this order is

$$\omega = -mk^2 \frac{\Gamma_0}{4\pi}\left(\beta + \ln(2/|k|)\right) \tag{6}$$

where

$$\beta = \int_0^1 \frac{\Gamma^2}{\Gamma_0^2}\frac{dr}{r} + \int_1^\infty \frac{\Gamma^2-\Gamma_0^2}{\Gamma_0^2}\frac{dr}{r} - \frac{8\pi^2}{\Gamma_0^2}\int_0^\infty r\,W^2\,dr - \gamma_e\,. \tag{7}$$

Here γ_e is Euler's constant, and

$$\Gamma_0 = 2\pi \lim_{r\to\infty} rV(r) \tag{8}$$

which is assumed nonzero.

This agrees with the long wave dispersion relation developed by Moore and Saffman (1972), although their derivation of it is ad hoc. They consider only the rigid rotational motion of an ideal helical vortex filament, using the Biot-Savart formula with a cutoff established by comparison with a steady calculation of Widnall et al. (1971). The connection between such a problem and the desired dispersion relation is obscure.

Numerical computations of the primary fast mode were given by Leibovich and Ma (1983) for the particular vortex

$$W(r) = 0, \quad V(r) = [1 - \exp(-r^2)]/r \; . \tag{9}$$

There are many fast branches of the dispersion relation for this example, all of which merge as k tends to zero, and all have zero group velocity there. Fast branches, such as these, also turn out to be amenable to analysis for general velocity fields in the limit of vanishing wavenumber. The details have been worked out by S. N. Brown, and we will report the details in a joint paper. It is, however, the slow branch which is of immediate interest, since Maxworthy et al. (1984) report the discovery of slow branch solitons in experiments on vortex filaments. We note that their attempts to compare with the soliton results of Leibovich and Ma (1983) were not generally successful. This presumably is so because the Leibovich and Ma soliton is on the (primary) fast branch.

We turn briefly now to consider solitons centered on the slow branch.

2.2 Solitons

To extend the analysis to higher order, modulation of the wave over long spatial and temporal scales must be allowed. This may be done by permitting the amplitude A in equation (1) to depend on a slow space variable X and a slow time variable τ where, to balance the nonlinearities, we take

$$X = \varepsilon(z - c_g t), \qquad \tau = \varepsilon^2 t \; ,$$

and c_g is the group velocity,

$$c_g = d\omega/dk \; .$$

Thus the "carrier wave" of frequency ω and wavenumber k is modulated by the wave envelope $A(X,\tau)$, and the motion of the wave packet is determined by the evolution of A, which is controlled by the nonlinear Schrodinger equation

$$i\,\partial A/\partial\tau + \mu\,\partial^2 A/\partial X^2 + \nu A|A|^2 = 0 \tag{10}$$

where

$$\mu = d^2\omega/dk^2 \; ,$$

and the constant ν is found from an orthogonality condition. Soliton solutions to