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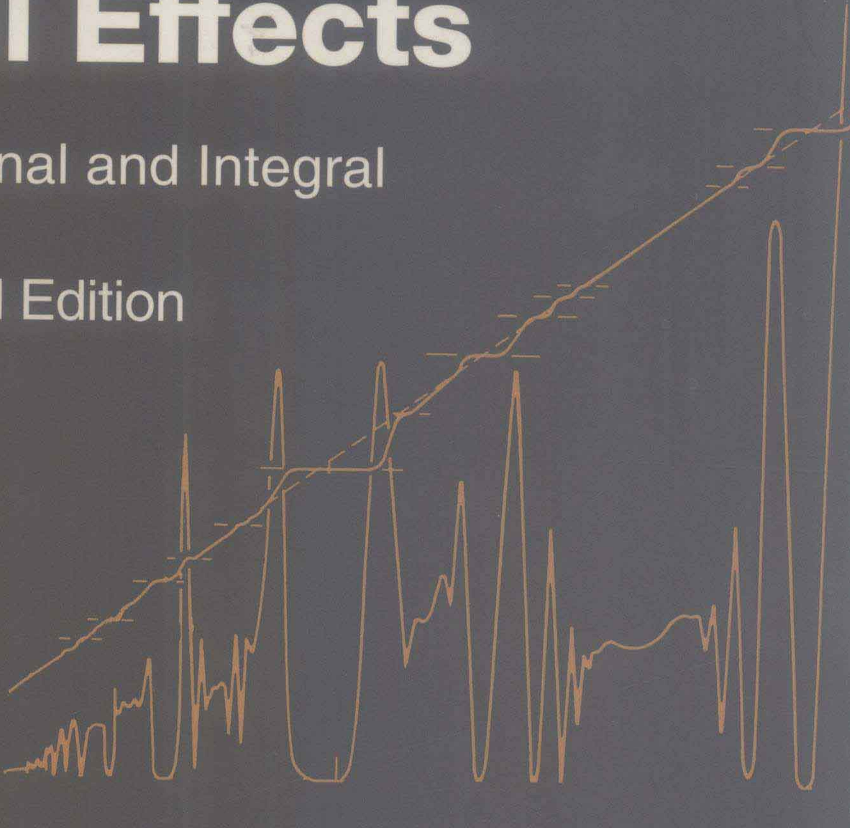
P. Pietiläinen

The Quantum Hall Effects

Fractional and Integral

Second Edition



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Tapash Chakraborty
Pekka Pietiläinen

The Quantum Hall Effects

Integral and Fractional

Second Enlarged and Updated Edition

With 129 Figures

Springer

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Foreword

The experimental discovery of the fractional quantum Hall effect (FQHE) at the end of 1981 by Tsui, Störmer and Gossard was absolutely unexpected since, at this time, no theoretical work existed that could predict new structures in the magnetotransport coefficients under conditions representing the extreme quantum limit. It is more than thirty years since investigations of bulk semiconductors in very strong magnetic fields were begun. Under these conditions, only the lowest Landau level is occupied and the theory predicted a monotonic variation of the resistivity with increasing magnetic field, depending sensitively on the scattering mechanism. However, the experimental data could not be analyzed accurately since magnetic freeze-out effects and the transitions from a degenerate to a nondegenerate system complicated the interpretation of the data. For a two-dimensional electron gas, where the positive background charge is well separated from the two-dimensional system, magnetic freeze-out effects are barely visible and an analysis of the data in the extreme quantum limit seems to be easier. First measurements in this magnetic field region on silicon field-effect transistors were not successful because the disorder in these devices was so large that all electrons in the lowest Landau level were localized. Consequently, models of a spin glass and finally of a Wigner solid were developed and much effort was put into developing the technology for improving the quality of semiconductor materials and devices, especially in the field of two-dimensional electron systems.

The formation of a Wigner lattice has been observed for the two-dimensional electron gas at the helium surface with the consequence that all sorts of unexpected results on two-dimensional systems in semiconductors were assigned to some kind of charge-density-wave or Wigner crystallization. First attempts to explain the FQHE were therefore guided by the picture of a Wigner solid with triangular crystal symmetry. However, a critical analysis of the data demonstrated that the idea of the formation of an incompressible quantum fluid introduced by Laughlin seems to be the most likely explanation.

The theoretical work collected in this book demonstrates that the Laughlin wave function forms a very good basis for a discussion of the FQHE. Even though many questions in the field of FQHE remain unanswered, this book offers a valuable source of information and is the first general review of the work of different groups in this field. The intense activity in the field of high-T_c superconductivity also calls for a book about the FQHE since certain similarities seem to be emerging in the theoretical treatment of the quantum Hall effect and that of high-T_c superconductivity.

I hope that this book will inspire scientists to new ideas.

Stuttgart
June 1988

Klaus von Klitzing

Preface

In the field of the fractional quantum Hall effect, we have witnessed tremendous theoretical and experimental developments in recent years. Our intention here is to present a general survey of most of the theoretical work in this area. In doing so, we have also tried to provide the details of formal steps, which, in many cases, are avoided in the literature. Our effort is motivated by the hope that the present compilation of theoretical work will encourage a nonexpert to explore this fascinating field, and at the same time, that it will provide guidelines for further study in this field, in particular on many of the open problems highlighted in this review. Although the focus is on the theoretical investigations, to see these in their right perspective, a brief review of the experimental results on the excitation gap is also presented. This review is of course, by no means complete; the field continues to present new surprises, and more theoretical work is still emerging. However we hope that the compilation in its present form will to some extent satisfy the need of the experts, nonexperts and the curious.

Stuttgart, Oulu, January, 1988

Tapash Chakraborty
Pekka Pietiläinen

* * *

After the first edition of the book was published, there were several interesting developments in the field of fractional quantum Hall effect. Most notably, the experimental evidence of the spin-reversed ground state and quasiparticles which had been predicted earlier in the theoretical studies. Similarly, experimental verification of the fractional charge of the quasiparticles is also a siginificant achievement. Magnetoluminescence experiments are rapidly opening up an entirely new route to study the quantum Hall effect, and the elusive phase transition of the incompressible quantum liquid state to Wigner crystal is becoming more and more transparent in experiments.

On the theoretical side, fractional statistics objects – the *anyons*, have fired the imagination of several researchers investigating the phenomenon of high-temperature superconductivity. Although their relevance in that field has not been proven, anyons gained credibility at a very early stage as the elementary excitations in the fractional quantum Hall state. Recent theoretical studies have indicated that going from electrons to fermions with a Chern-Simons field results in a very useful approach to the understanding of the behavior when the lowest Landau level is half filled by electrons.

All of these issues and more, have been discussed here to make the present edition more up to date. A major addition in the second edition is a *brief* survey of the integer quantum Hall effect which is intended to make the book more self-contained. The emphasis is however, still as in the first edition, to provide a complete, comprehensive review of the exciting field of the fractional quantum Hall effect.

Madras, Oulu, July, 1994

Tapash Chakraborty
Pekka Pietiläinen

Acknowledgments

We wish to express our gratitude to Professor Peter Fulde and Professor Klaus von Klitzing for offering us the opportunity to write this review and for their encouragement and support during the course of this work. We are very much indebted to Professor von Klitzing for writing a foreword for this publication.

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The other (P. P.) would like to thank Professor Fulde for arranging a visit to the Max-Planck-Institute, Stuttgart, during the final stage of preparation of the first edition. He also thanks the Department of Theoretical Physics, University of Oulu for support.

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Last but not the least, we would like to thank our families for their patience and understanding.

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Contents

1. Quantum Hall Effect: The Basics

The quantization of the Hall effect discovered by *von Klitzing* et al. [1.1] in 1980 is a remarkable macroscopic quantum phenomenon which occurs in two-dimensional electron systems at low temperatures and strong perpendicular magnetic fields. Under these conditions, the Hall-conductivity exhibits plateaus at integral multiples of e^2/h (a universal constant). The striking result is the accuracy of the quantization (better than a part in ten million) which is totally indifferent to impurities or geometric details of the two-dimensional system. Each plateau is accompanied by a deep minimum in the diagonal resistivity, indicating a dissipationless flow of current. In 1982, there was yet another surprise in this field. Working with much higher mobility samples, *Tsui* et al. [1.2] discovered the fractional quantization of the Hall conductivity. The physical mechanisms responsible for the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE) are quite different, despite the apparent similarity of the experimental results. In the former case, the role of the random impurity potential is quite decisive, while in the latter case, electron-electron interaction plays a predominant role resulting in a unique collective phenomenon.

In the following chapters, we shall briefly describe the theoretical and experimental developments in the QHE. It should be mentioned, however, that the QHE has been one of the most active fields of research in condensed matter physics for over a decade. It is therefore, quite impossible to describe here all the details of the major developments. Our aim here is to touch upon the most significant theoretical and experimental work to construct a reasonably consistent picture of the QHE. For more details on the topics discussed, the reader is encouraged to read the original work cited here and some of the reviews available in the literature [1.3–11].

1.1 Two-Dimensional Electron Gas

The major impetus in the studies of the QHE is due to experimental realization of almost ideal two-dimensional electron systems. The electrons are dynamically two-dimensional because they are free to move in only two spatial dimensions. In the third dimension, they have quantized energy levels (in reality, the wave functions have a finite spatial extent in the third dimension [1.12]). In the following, we provide a very brief discussion on the systems where the electron layers are created. For details see the reviews by *Ando* et al. [1.12] and *Störmer* [1.13].

Electron layers have been created in many different systems. Electrons on the surface of liquid helium provides an almost ideal two-dimensional system [1.14,15]. They are trapped on the surface by a combination of an external field and an image potential. The electron concentration in this system is, however, very low ($10^5 - 10^9$ cm^{-2}) and the system behaves classically. The high-density electron systems where the QHE is usually observed are typically created in the Metal-Oxide-Semiconductor Field Effect Transistor (the MOSFET) and in semiconductor heterojunctions.

A schematic picture of an n-channel Si-MOSFET is shown in Fig. 1.1(a). The system consists of a semiconductor (p-Si) which has a plane interface with a thin film of insulator (SiO_2), the opposite side of which carries a metal gate electrode. Application of a voltage (gate voltage V_G) between the gate and the Si/SiO_2 interface results in bending of the electron energy bands. For a strong enough electric field, as the bottom of the conduction band is pushed down below the Fermi energy E_F, electrons accumulate in a two-dimensional quasi-triangular potential well close to the interface [Fig. 1.1(b)]. As the width of the well is small (~ 50Å), electron motion perpendicular to the interface is quantized but the electrons move freely parallel to the interface. In the plane, the energy spectrum is

$$\varepsilon_i(\boldsymbol{k}) = \varepsilon_i^0 + \frac{\hbar^2 k_\parallel^2}{2m^*}$$

where m^* is the effective mass of the electrons, $k_\parallel$ is the two-dimensional wave vector and ε_i^0 is the bottom of the corresponding subband. The system is called an inversion layer because here the charge carriers are the electrons while the semiconductor is p-type.

At low temperatures ($kT \ll \Delta E$, the subband spacing) the electrons are trapped in the lowest subband and the system is purely two-dimensional. The MOSFET is quite useful in the present study because by varying the gate voltage the electron concentration can be varied within a wide range ($n_0 \sim 0 - 10^{13}$cm^{-2}).

Two-dimensional electron layers are also created in semiconductor heterostructures at a nearly perfectly lattice-matched semiconductor/semiconductor interface. One such widely used system is the $GaAs/Al_xGa_{1-x}As$ ($0 <$

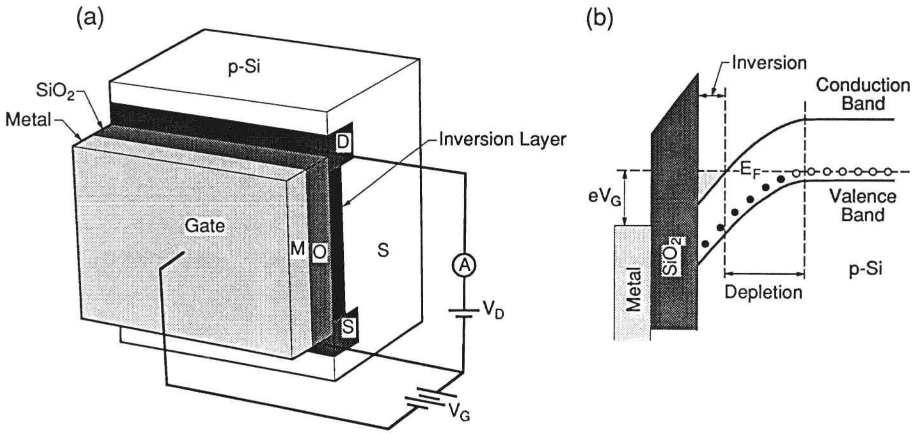

Fig. 1.1. (a) Schematic view of a Si-MOSFET and (b) energy level diagram

$x \leq 1$) heterostructure. The lattice constants of GaAs and $Al_xGa_{1-x}As$ are almost the same so that the interface is nearly free from any disorder. The band gap of the alloy is wider than that of GaAs and it increases with the aluminum concentration x. Carriers in the neighborhood of the hetero-

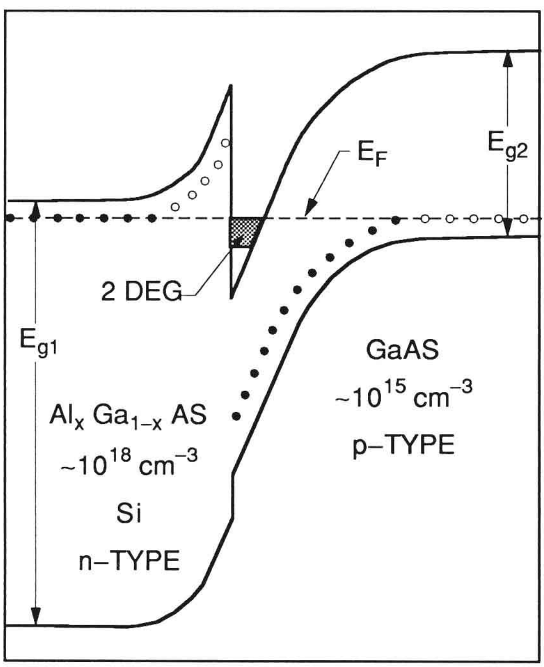

Fig. 1.2. Energy diagram at a GaAs-heterostructure interface

junction transfer from the doped AlGaAs alloy across the interface to the low-lying band edge states of the narrow band gap material (GaAs). The electric field due to the charge transfer bends the energy bands as shown in Fig. 1.2. A quasi-triangular potential well (~ 100Å) formed in the GaAs traps the electrons as two-dimensional carriers.

The mobile carriers are spatially separated from their parent ionized impurities via modulation doping. This leads to very high carrier mobilities and, in fact, the FQHE was discovered in these high-mobility GaAs-heterostructures [1.13]. However, unlike MOSFETs the electron concentration in heterostructures can be varied only within a very narrow range. Carrier densities in these systems typically range from $1 \times 10^{11}\text{cm}^{-2}$ to $1 \times 10^{12}\text{cm}^{-2}$.

1.2 Electrons in a Strong Magnetic Field

Let us begin with the problem of a free electron (with effective mass m^*) in a uniform magnetic field B. The Hamiltonian is then written

$$\mathcal{H}_0 = \left(\Pi_x^2 + \Pi_y^2\right)/2m^* \tag{1.1}$$

where, $\boldsymbol{\Pi} = -\mathrm{i}\hbar\nabla + \frac{e}{c}\boldsymbol{A}$ is the kinetic momentum and $\boldsymbol{A}$ is the vector potential which is related to the magnetic field in the manner, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$. Following *Kubo* et al. [1.16] we introduce the center coordinates of the cyclotron motion (X, Y) as

$$X = x - \xi, \qquad Y = y - \eta \tag{1.2}$$

where

$$\xi = (c/eB)\Pi_y, \qquad \eta = -(c/eB)\Pi_x \tag{1.3}$$

are the relative coordinates. It can be easily seen that (ξ, η) represents a cyclotron motion with frequency

$$\omega_\mathrm{c} = \frac{eB}{m^*c}, \tag{1.4}$$

(cyclotron frequency). Defining the magnetic length

$$\ell_0 \equiv \left(\frac{\hbar c}{eB}\right)^{\frac{1}{2}} \tag{1.5}$$

(cyclotron radius) and from the commutation relation

$$[\xi, \eta] = -\mathrm{i}\ell_0^2$$

it is clear that ξ and η are subject to an uncertainty of order ℓ_0. The Hamiltonian (1.1) is now rewritten in terms of (ξ, η) as

$$\mathcal{H}_0 = \frac{\hbar\omega_\mathrm{c}}{2\ell_0^2}\left(\xi^2 + \eta^2\right) \tag{1.6}$$

whose eigenenergies are the discrete *Landau levels* [1.17,18]

$$E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega_\mathrm{c}, \qquad n = 0, 1, 2, \ldots \tag{1.7}$$

The Hamiltonian (1.6) does not contain (X, Y) which means that electrons in cyclotron motion with different center coordinates have the same energy. The center coordinates also follow the commutation rule, $[X, Y] = \mathrm{i}\ell_0^2$.

Choosing now the Landau gauge[1] such that the vector potential $\boldsymbol{A}$ has only one nonvanishing component, say, $A_y = Bx$, the Hamiltonian is

$$\mathcal{H}_0 = \frac{1}{2m^*}\left[p_x^2 + \left(p_y + \frac{eB}{c}x\right)^2\right]. \tag{1.8}$$

The variables are easily separable and an eigenfunction is written in the form

$$\phi = \mathrm{e}^{\mathrm{i}k_y y}\chi(x) \tag{1.9}$$

where the usual identification is made, $p_y = -\mathrm{i}\hbar\partial/\partial y \to \hbar k_y$. The function $\chi(x)$ is the eigenfunction of the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m^*}\chi'' + \tfrac{1}{2}m^*\omega_\mathrm{c}^2\left(x - X\right)^2\chi = E\chi(x) \tag{1.10}$$

where $X = -k_y\ell_0^2$. The above equation is easily recognized as the Schrödinger equation corresponding to a *harmonic oscillator* of spring constant $\hbar\omega_\mathrm{c} = \hbar^2/m^*\ell_0^2$, with equilibrium point at X.

The eigenfunction (ignoring the normalization factor) is now written

$$\phi_{nX} = \mathrm{e}^{\mathrm{i}k_y y}\exp\left[-(x - X)^2/2\ell_0^2\right] H_n\left[(x - X)/\ell_0\right] \tag{1.11}$$

with H_n the Hermite polynomial. The functions are extended in y and localized in x. The localization remains unaffected under a gauge transformation. When the system is confined in a rectangular cell with sides L_x

[1] The other choice of gauge viz. the symmetric gauge is discussed in Appendix A