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**Steven Holzner, PhD**  
Author, Quantum Physics For Dummies

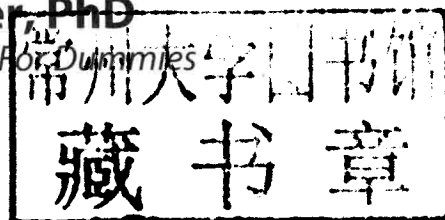


# Quantum Physics Workbook

FOR  
DUMMIES®

by Steven Holzner, PhD

Author of *Quantum Physics For Dummies*



Wiley Publishing, Inc.

## Quantum Physics Workbook For Dummies®

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## ***About the Author***

**Steven Holzner** is the award-winning writer of many books, including *Physics For Dummies*, *Differential Equations For Dummies*, *Quantum Physics For Dummies*, and many others. He graduated from MIT and got his PhD at Cornell University. He's been in the faculty of both MIT and Cornell.

## ***Dedication***

To Nancy, of course.

## ***Author's Acknowledgments***

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# Introduction

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**W**hen you make the leap from classical physics to the small, quantum world, you enter the realm of probability. Quantum physics is an exciting field with lots of impressive results if you know your way around — and this workbook is designed to make sure you do know your way around.

I designed this workbook to be your guided tour through the thicket of quantum physics problem-solving. Quantum physics includes more math than you can shake a stick at, and this workbook helps you become proficient at it.

## About This Book

Quantum physics, the study of the very small world, is actually a very big topic. To cover those topics, quantum physics is broken up into many different areas — harmonic oscillators, angular momentum, scattered particles, and more. I provide a good overview of those topics in this workbook, which maps to a college course.

For each topic, you find a short introduction and an example problem; then I set you loose on some practice problems, which you can solve in the white space provided. At the end of the chapter, you find the answers and detailed explanations that tell you how to get those answers.

You can page through this book as you like instead of having to read it from beginning to end — just jump in and start on your topic of choice. If you need to know concepts that I've introduced elsewhere in the book to solve a problem, just follow the cross-references.

## Conventions Used in This Book

Here are some conventions I follow to make this book easier to follow:

- ✓ The answers to problems, the action part of numbered steps, and vectors appear in **bold**.
- ✓ I write new terms in *italics* and then define them. Variables also appear in italics.
- ✓ Web addresses appear in monospace.

## *Foolish Assumptions*

Here's what I assume about you, my dear reader:

- ✓ **You've had some exposure to quantum physics, perhaps in a class.** You now want just enough explanation to help you solve problems and sharpen your skills. If you want a more in-depth discussion on how all these quantum physics concepts work, you may want to pick up the companion book, *Quantum Physics For Dummies* (Wiley). You don't have to be a whiz at quantum physics, just have a glancing familiarity.
- ✓ **You're willing to invest some time and effort in doing these practice problems.** If you're taking a class in the subject and are using this workbook as a companion to the course to help you put the pieces together, that's perfect.
- ✓ **You know some calculus.** In particular, you should be able to do differentiation and integration and work with differential equations. If you need a refresher, I suggest you check out *Differential Equations For Dummies* (Wiley).

## *How This Book Is Organized*

I divide this workbook into five parts. Each part is broken down into chapters discussing a key topic in quantum physics. Here's an overview of what I cover.

### *Part I: Getting Started with Quantum Physics*

This part covers the basics. You get started with state vectors and with the entire power of quantum physics. You also see how to work with free particles, with particles bound in square wells, and with harmonic oscillators here.

### *Part II: Round and Round with Angular Momentum and Spin*

Quantum physics lets you work with the micro world in terms of the angular momentum of particles as well as the spin of electrons. Many famous experiments — such as the Stern-Gerlach experiment, in which beams of particles split in magnetic fields — are understandable only in terms of quantum physics. You see how to handle problems that deal with these topics right here.

### *Part III: Quantum Physics in Three Dimensions*

Up to this point, the quantum physics problems you solve all take place in one dimension. But the world is a three-dimensional kind of place. This part rectifies that by taking quantum physics to three dimensions, where square wells become cubic wells and so on. You also take a look at the two main coordinate systems used for three-dimensional work: rectangular and spherical coordinates. You work with the hydrogen atom as well.

## *Part IV: Acting on Impulse — Impacts in Quantum Physics*

This part is on perturbation theory and scattering. Perturbation theory is all about giving systems a little shove and seeing what happens — like applying an electric field to particles in harmonic oscillation. Scattering theory has to do with smashing one particle against another and predicting what's going to happen. You see some good collisions here.

## *Part V: The Part of Tens*

The Part of Tens is a common element of all *For Dummies* books. In this part, you see ten tips for problem-solving, a discussion of quantum physics's ten greatest solved problems, and ten ways to avoid common errors when doing the math.

## *Icons Used in This Book*

You find a few icons in this book, and here's what they mean:



This icon points out example problems that show the techniques for solving a problem before you dive into the practice problems.



This icon gives you extra help (including shortcuts and strategies) when solving a problem.



This icon marks something to remember, such as a law of physics or a particularly juicy equation.

## *Where to Go from Here*

If you're ready, you can do the following:

- ✓ **Jump right into the material in Chapter 1.** You don't have to start there, though; you can jump in anywhere you like. I wrote this book to allow you to take a stab at any chapter that piques your interest. However, if you need a touchup on the foundations of quantum physics, Chapter 1 is where all the action starts.
- ✓ **Head to the table of contents or index.** Search for a topic that interests you and start practicing problems. (*Note:* I do suggest that you don't choose the answer key as your first "topic of interest" — looking up the solutions before attempting the problems kind of defeats the purpose of a workbook! I promise you're not being graded here, so just relax and try to understand the processes.)

- ✓ **Check out *Quantum Physics For Dummies*.** My companion book provides a more comprehensive discussion. With both books by your side, you can further strengthen your knowledge of quantum physics.
- ✓ **Go on vacation.** After reading about quantum physics, you may be ready for a relaxing trip to a beach where you can sip fruity cocktails, be waited on hand and foot, and read some light fiction on parallel universes. Or maybe you can visit Fermilab (the Fermi National Accelerator Laboratory), west of Chicago, to tour the magnet factory and just hang out with their herd of bison for a while.

# Part I

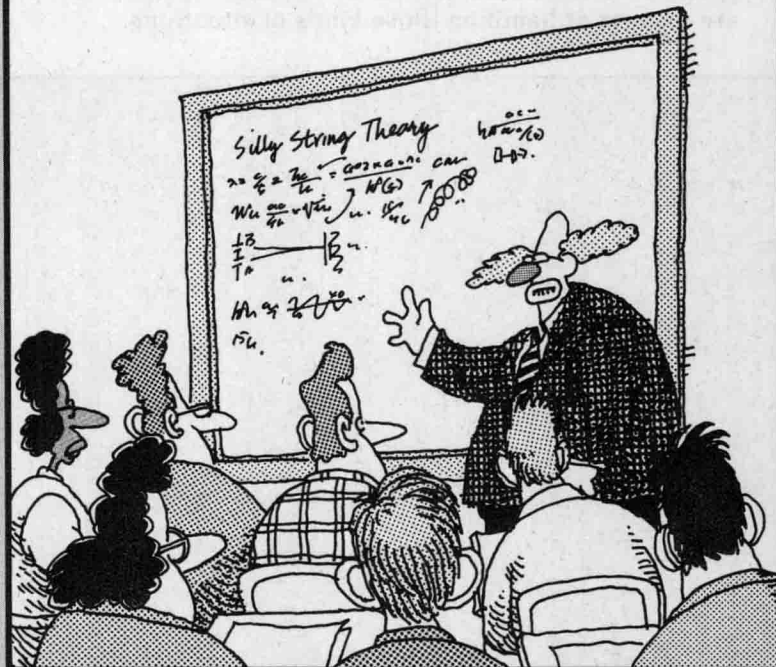
# Getting Started with Quantum Physics

## The 5<sup>th</sup> Wave

By Rich Tennant

After the circus, Bozo the Physicist went on to distinguish himself for his work on the Wave/Particle Joy Buzzer, squirting quarks, and Quantum Pratfall Theory.

©RICH TENNANT





### ***In this part . . .***

**T**his part gets you started in solving problems in quantum physics. Here, you find an introduction to the conventions and principles necessary to solve quantum physics problems. This part is where you see one of quantum physics's most powerful topics: solving the energy levels and wave functions for particles trapped in various bound states. You also see particles in harmonic oscillation. Quantum physicists are experts at handling those kinds of situations.

## Chapter 1

# The Basics of Quantum Physics: Introducing State Vectors

---

### *In This Chapter*

- ▶ Creating state vectors
  - ▶ Using quantum physics operators
  - ▶ Finding expectation values for operators
  - ▶ Simplifying operations with eigenvalues and eigenvectors
- 

**I**f you want to hang out with the cool quantum physics crowd, you have to speak the lingo. And in this field, that's the language of mathematics. Quantum physics often involves representing probabilities in matrices, but when the matrix math becomes unwieldy, you can translate those matrices into the bra and ket notation and perform a whole slew of operations.

This chapter gets you started with the basic ideas behind quantum physics, such as the state vector, which is what you use to describe a multistate system. I also cover using operators, making predictions, understanding properties such as commutation, and simplifying problems by using eigenvectors. Here you can also find several problems to help you become more acquainted with these concepts.

## *Describing the States of a System*

The beginnings of quantum physics include explaining what a system's *states* can be (such as whether a particle's spin is up or down, or what orbital a hydrogen atom's electron is in). The word *quantum* refers to the fact that the states are *discrete* — that is, no state is a mix of any other states. A quantum number or a set of quantum numbers specifies a particular state. If you want to break quantum physics down to its most basic form, you can say that it's all about working with multistate systems.

Don't let the terminology scare you (which can be a constant struggle in quantum physics). A *multistate system* is just a system that can exist in multiple states; in other words, it has different energy levels. For example, a pair of dice is a multistate system. When you roll a pair of dice, you can get a sum of 2, 3, 5, all the way up to 12. Each one of those values represents a different state of the pair of dice.