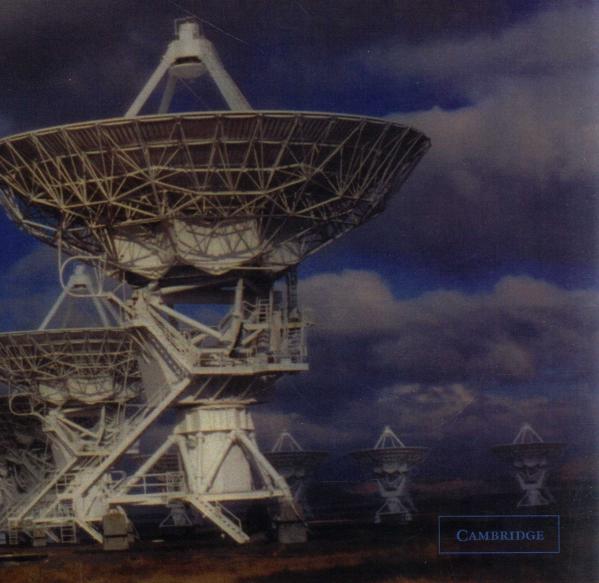


Electromagnetic Scintillation II. Weak Scattering



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Albert D. Wheelon

Environmental Technology Laboratory
National Oceanic and Atmospheric Administration
Boulder, Colorado, USA



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Electromagnetic scintillation describes the phase and amplitude fluctuations imposed on signals that travel through the atmosphere. These volumes provide a modern reference and comprehensive tutorial for this subject, treating both optical and microwave propagation. Measurements and predictions are integrated at each step of the development. The first volume dealt with phase and angle-of-arrival measurement errors, which are accurately described by geometrical optics.

This second volume concentrates on amplitude and intensity fluctuations of the received signal. Diffraction plays a dominant role in this aspect of scintillation and one must use a full-wave description. The Rytov approximation provides the basis for describing weak-scattering conditions that characterize a wide range of important measurements. Astronomical observations in the optical, infrared and microwave bands fall in this category. So also do microwave signals received from earth-orbiting satellites and planetary spacecraft. Weak scattering describes microwave communication near the surface. Level fluctuations induced by atmospheric irregularities both in the troposphere and in the ionosphere are estimated for these applications and compared with experimental results. Laser signals on terrestrial paths are described by this approach if the transmission distance is less than approximately 300 m. Experiments and applications using longer paths involve strong scattering, which will be discussed in Volume III.

This book will be of particular interest to astronomers, applied physicists and engineers developing instruments and systems at the frontier of technology. It also provides a unique reference for atmospheric scientists and scintillation specialists. It can be used as a graduate textbook and is designed for self study. Extensive references to original work in English and Russian are provided.

DR ALBERT D. WHEELON has been a visiting scientist at the Environmental Technology Laboratory of NOAA in Boulder, Colorado, for the past decade. He holds a BSc degree in engineering science from Stanford University and a PhD in physics from MIT, where he was a teaching fellow and a research associate in the Research Laboratory for Electronics. He has published thirty papers on radio physics and space technology in learned journals.

He has spent his entire career at the frontier of technology. He made important early contributions to ballistic missile and satellite technology at TRW, where he was director of the Radio Physics Laboratory. While in government service, he was responsible for the development and operation of satellite and aircraft reconnaissance systems. He later led the development of communication and scientific satellites at Hughes Aircraft. This firm was a world leader in high technology and he became its CEO in 1986.

He has been a visiting professor at MIT and UCLA. He is a Fellow of the American Physical Society, the IEEE and the AIAA. He is also a member of the National Academy of Engineering and has received several awards for his contributions to technology and national security including the R. V. Jones medal. He has been a trustee of Cal Tech and the RAND Corporation. He was a member of the Defense Science Board and the Presidential Commission on the Space Shuttle Challenger Accident. He has been an advisor to five national scientific laboratories in the USA.

These volumes are dedicated to Valerian Tatarskii who taught us all

Preface

History

Quivering of stellar images can be observed with the naked eye and was noted by ancient peoples. Aristotle tried but failed to explain it. A related phenomenon noted by early civilizations was the appearance of shadow bands on white walls just before solar eclipses. When telescopes were introduced, scintillation was observed for stars but not for large planets. Newton correctly identified these effects with atmospheric phenomena and recommended that observatories be located on the highest mountains practicable. Despite these occasional observations, the problem did not receive serious attention until modern times.

How It Began

Electromagnetic scintillation emerged as an important branch of physics following the Second World War. This interest developed primarily in response to the needs of astronomy, communication systems, military applications and atmospheric forecasting. The last fifty years have witnessed a growing, widespread interest in this field, with considerable resources being made available for measurement programs and theoretical research.

Radio signals coming from distant galaxies were detected as this era began, thereby creating the new field of radio astronomy. Microwave receivers developed by the military radar program were used with large apertures to detect these faint signals. Their amplitude varied randomly with time and it was initially suggested that the galactic sources themselves might be changing. Comparison of signals measured at widely separated receivers showed that the scintillation was uncorrelated, indicating that the random modulation was imposed by ionized layers high in the earth's atmosphere. Careful study of this scintillation now provides an important tool for examining ionospheric structures that influence reflected short-wave signals and transionospheric propagation.

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Vast networks of microwave relay links were established to provide wideband communications over long distances soon after the Second World War. The effect of scintillation on the quality of such signals was investigated and found not to be important for the initial systems. The same question arose later in connection with the development of communication satellites and gave rise to careful research. These questions are now being revisited as terrestrial and satellite links move to higher frequencies and more complicated modulation schemes.

Large optical telescopes were being designed after the war in order to refine astronomical images. It became clear that the terrestrial atmosphere places an unwelcome limit on the accuracy of position and velocity measurements. The same medium limits the collecting area for coherent signals to areas that are considerably smaller than the apertures of large telescopes. A concerted effort to understand the source of this optical noise was begun in the early 1950s. Temperature fluctuations in the lower atmosphere were identified as the source. When high-resolution earth-orbiting reconnaissance satellites were introduced in 1960 it was feared that the same mechanism might limit their resolution.

Development of long-range ballistic missiles began in 1953 and early versions relied on radio guidance. Astronomical experience suggested that microwave quivering would limit their accuracy. This concern encouraged numerous terrestrial experiments to measure phase and amplitude fluctuations induced by the lower atmosphere. The availability of controlled transmitters on earth-orbiting satellites after 1957 made possible a wide range of propagation experiments designed to investigate atmospheric structure.

The presence of refractive irregularities in the atmosphere suggested the possibility of scattering microwave signals to distances well beyond the optical horizon. This was confirmed experimentally in 1955 and became the basis for scatter-propagation communications links using turbulent eddies both in the troposphere and in the ionosphere. Because of its military importance, a considerable amount of research on the interaction of microwave signals with atmospheric turbulence was sponsored.

Understanding the Phenomenon

The measurement programs that explored these applications generated a large body of experimental data bearing directly on the scattering of electromagnetic waves in random media. There was an evident need to develop theoretical understandings that could explain these results. The first attempts used geometrical optics to describe the electromagnetic propagation combined with spatial correlation models for the turbulent atmosphere. Time variations of the field were included by assuming the existence of a frozen random medium carried past the propagation path on prevailing winds. These models were successful at describing phase and angle-of-arrival

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measurements, but failed to explain amplitude and intensity variations. The next step was to exploit the Rytov approximation to describe the influence of random media on electromagnetic waves. This technique includes diffraction effects and provides a reliable description for weak-scattering conditions.

Our understanding of refractive irregularities in the lower atmosphere benefited greatly from basic research on turbulent flow fields. Using dimensional arguments, Kolmogorov was able to explain the most important features of turbulent velocities. His approach was later used to describe the turbulent behavior of temperature and humidity, which directly influence electromagnetic waves. These models now provide a physical basis for describing many of the features observed at optical and microwave frequencies.

The Second Wave of Applications

The development of coherent light sources took scintillation research into a new and challenging regime. It is possible to form confined beams of optical radiation with laser sources. These beam waves find important applications in military target-location systems. The ability to deliver concentrated forms of optical energy onto targets at some distance soon led to laser weapons. Wave-front-tilt monitors and corrective mirror systems were combined to correct the angle-of-arrival errors experienced by such signals and later applied to large optical telescopes. Rapidly deformable mirrors were developed later in order to correct higher-order errors in the arriving wavefront. These applications stimulated research on many aspects of atmospheric structure and electromagnetic propagation.

Radio astronomy has moved from 100 MHz to over 100 GHz in the past forty years. Microwave interferometry has become a powerful technique for refining astronomical observations using phase comparison of signals received at separated antennas. The lower atmosphere defines the inherent limit of angular accuracy that can be achieved with earth-based arrays. Considerable effort has gone into programs to measure the phase correlation as a function of the separation between receivers and use it as a guide for the design of large interferometric arrays.

The development of precision navigation and location techniques using constellations of earth-orbiting satellites focused attention on phase fluctuations at 1550 MHz. Ionospheric errors are removed by scaling and subtracting the phase of two signals at nearby frequencies. The ultimate limit on position determination is thus set by phase fluctuations induced by the troposphere, which are the same errors that limit the resolution of interferometric arrays.

Coherent signals radiated by spacecraft sent to explore other planets have been used to examine the plasma distribution in our own solar system. Transmission of spacecraft signals through the atmospheres of planets and their moons provides a

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unique way to investigate the atmospheres of neighboring bodies. The discovery of microwave sources far out in the universe led to exploration of the interstellar plasma with scintillation techniques. Comparing different frequency components of pulsed signals that travel along the same path provides a unique tool with which to study this silent medium. One can use scintillation measurements to estimate the sizes of distant quasars with surprising accuracy.

An extension of scatter propagation occurred when radars became sensitive enough to measure backscattering by turbulent irregularities. The structure of atmospheric layers has been established from the troposphere to the ionosphere using high-power transmitters and large vertically pointed antennas. It was later found that scanning radars can detect turbulent conditions over considerable areas, thereby providing a valuable warning service to aircraft. The same phenomenon is now making an important contribution to meteorology. A network of phased-array radars has been installed in the USA to measure the vertical profiles of wind speeds and temperature by sensing the signal returned from irregularities and its Doppler shift.

Acoustic propagation is a complementary field to the one we will examine. Longrange acoustic detection programs sponsored by the military have supported important experimental and theoretical research. Controlled acoustic experiments that are not often possible with electromagnetic signals in the atmosphere can be done in the ocean. Theoretical descriptions of acoustic propagation have helped us to understand the strong scintillation observed at optical frequencies. Investigations of the acoustic and electromagnetic problems are mutually supporting endeavors.

These recent applications have stimulated further theoretical research. Laser systems often operate in the strong-scattering regime where the Rytov approximation is not valid. This encouraged a sustained effort to develop techniques that can describe saturation effects. It has taken three directions. The first is based on the Markov approximation, which results in differential equations for moments of the electric-field strength. The second approach is an adaptation of the path-integral method developed for quantum mechanics. The third approach relies on Monte Carlo simulation techniques in which the random medium is replaced by a succession of phase screens.

Resources for Learning About Scintillation

After fifty years of extensive experimental measurements and intense theoretical development this subject has become both deep and diverse. Many are asking "How can one learn about this expanding field?" Where does one go to find results that can be applied in the practical world? Despite its growing importance, one finds it difficult to establish a satisfactory understanding of the field without an enormous

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investment of time. That luxury is not available to most engineers, applied physicists and astronomers. They must find reliable results quickly and apply them.

There are few reference books in this field. A handful of early books were written in Russia where much of the basic work was done. These were influential in shaping research programs but subsequent developments now limit their utility. A later Russian series summarized the theory of strong scattering but made little contact with experimental data. Several books on special topics in random-medium propagation have appeared recently. Even with these references, it takes a great deal of time to establish a confident understanding of what is known and not known – even in small sectors of the field.

The Origin of this Series

This series on electromagnetic scintillation came about in a somewhat unusual way. It resulted from my return to a field in which I had worked as a young physicist. My life changed dramatically in 1962 and the demands of developing large radar, reconnaissance and communication systems at the frontier of technology took all of my energy for several decades. That experience convinced me how important research in this field has become. When I returned to scientific work in 1988, I resolved to explore the considerable progress that had been made during my absence.

I was immediately confronted with an enormous literature, scattered over many journals. Fortunately the Russian journals had been translated into English and were available at MIT where I was then teaching. As an aid to my exploration, I began to develop a set of notes with which I could navigate through the literature. My journal grew steadily as I added detail, made corrections and included new insights. It soon became several large notebooks. I was invited to work with the Environmental Technology Laboratories of NOAA in 1990. This coincided with the arrival from Moscow of several leaders in this field, which has made Boulder the premier center of research. I shared my notebooks with colleagues there who encouraged me to bring them into book form.

In reviewing the progress made over the past thirty years, I found a number of loose ends and apparent conflicts. To resolve these issues, I spent a good deal of time examining the field. Several areas needed clarification and this resulted in some original research, which is reported here for the first time.

Approach and Intended Users

The purpose of this series is to provide an understanding of the underlying principles of electromagnetic propagation through random media. We shall focus on

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transmission experiments in which small-angle forward scattering is the dominant mechanism. The elements common to different applications are emphasized by focusing on fundamental descriptions that transcend their boundaries. I hope that this approach will serve the needs of a diversified community of technologists who need such information. Measurements and theoretical descriptions are presented together in an effort to build confidence in the final results. In each application, I have tried to identify critical measurements that confirm the basic expressions. These experiments are often summarized in the form of tables that readily lead one to the original sources. Actual data is occasionally reproduced so that the reader can judge the agreement for himself. In some cases, I give priority to the early experiments to recognize pioneering work and to provide a sense of historical development. In other cases, I have used the most recent and accurate data for comparison.

It is important to explain how the series is organized. The goal is always to present the simplest description for a measured quantity. We advance to more sophisticated explanations only when the simpler models prove inadequate. The first volume explores the subject with the most elementary description of electromagnetic radiation – geometrical optics. We find that it gives a valid description for phase and angle-of-arrival fluctuations for almost any situation. In the second volume we introduce the Rytov approximation which includes diffraction effects and provides a significant improvement on geometrical optics. With it one can describe weak fluctuations of signal amplitude and intensity over a wide range of applications. The third volume is devoted to strong scattering, which is encountered at optical and millimeter-wave frequencies. That regime presents a greater analytical challenge and one must lean more heavily on experimental results to understand it.

This presentation emphasizes scaling laws that show how the measured quantities vary with the independent variables; namely frequency, distance, aperture size, inter-receiver separation, time delay, zenith angle and frequency separation. It is often possible to rely on these scaling laws without knowing the absolute value for a measured quantity. That is important because the level of turbulent activity changes diurnally and seasonally. The scaling laws are expressed in closed form wherever possible. Numerical computations are presented when it is not. Brief descriptions of the special functions needed for these analyses are given in appendices and are referenced in the text. This should allow those who have studied mathematical physics to proceed rapidly, while providing a convenient reference for those less familiar with such techniques. Problems are included at the end of each chapter. They are designed to develop additional insights and to explore related topics.

¹ The original plan for the series included a volume on wide-angle forward scattering and backscattering of electromagnetic waves. This plan was deferred as the material relating to line-of-sight propagation expanded rapidly.

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The turbulent medium itself is a vast subject about which much has been written. Each new work on propagation attempts to summarize the available information in order to lay a foundation for describing the electromagnetic response to it. I looked for ways to avoid that obligatory preamble. Alas, I could find no way out and a brief summary is included in the first volume, which identifies the basis on which we proceed. In doing so, I have tried to avoid promoting particular models of the turbulent medium. This is especially important for the very-large- and very-small-scale regions of the spectrum. Between these extremes, there is good reason to use the Kolmogorov model to characterize the inertial range. All too often, we find that large eddies or the dissipation range play an important – though subtle – role. We have no model based on physical understanding to describe these regions and they must be explored by experiment.

Any attempt to describe the real atmosphere must address the reality of anisotropy. We know that plasma irregularities in the ionosphere are elongated in the direction of the magnetic field. In the troposphere, irregularities near the surface are correlated over greater distances in the horizontal direction than they are in the vertical direction. That disparity increases rapidly with altitude. One cannot ignore the influence of anisotropy on signals that travel through the atmosphere and much of the new material included here is the result of recent attempts to include this effect.

Acknowledgments

This series is the result of many conversations with people who have contributed mightily to this field. Foremost among these is Valerian Tatarskii, who came to Boulder just as my exploration was taking on a life of its own. He has become my teacher and my friend. Valery Zavorotny also moved to Boulder and has been a generous advisor. Hal Yura reviewed the various drafts and suggested several ingenious derivations. Rich Lataitis has been a steadfast supporter, carefully reviewing my approach and suggesting important references. Reg Hill has subjected this work to searching examination for which I am truly grateful. Rod Frehlich helped by identifying important papers from the remarkable filing system that he maintains. Jim Churnside and Gerard Ochs have been generous reviewers and have paid special attention to the experiments I have cited. My friend Robert Lawrence did all the numerical computing and I owe him a special debt.

Steven Clifford extended the hospitality of the Environmental Technology Laboratory to me and has been a strong supporter from the outset. As a result of his initiative, I have enjoyed wonderful professional relationships with the people identified above. I cannot end without acknowledging the help of Mary Alice Wheelon who edited too many versions of the manuscript and double checked all the references. The drawings and figures were designed by Andrew Davies and Peter Wheelon.

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Jane Watterson and her colleagues at the NOAA Library in Boulder have provided prompt and continuing reference support for my research.

At the end of the day, however, the work presented here is mine. I alone bear the responsibility for the choice of topics and their accuracy. My reward is to have taken this journey with wonderful friends.

Albert D. Wheelon Santa Barbara, California

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