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*SYSTEM DYNAMICS:
A UNIFIED APPROACH*



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PREFACE

The principal contribution of this book is its unified approach to the modeling and manipulation of dynamic engineering systems, an approach made possible by the use of bond graphs. The basic physics involved in the description of mechanical, electrical, hydraulic, thermal, magnetic, and fluid dynamic systems is phrased in terms of four types of generalized variables—effort, flow, displacement, and momentum. Bond-graph models can be manipulated systematically to yield state-space equations of standard form. Furthermore, the ENPORT computer program can provide dynamic responses directly from a suitable bond-graph model and specified excitations, without requiring the explicit prior formulation of state equations. We believe that bond-graph methods offer the most unified and understandable way to proceed from the basic physical modeling of components, devices, and their connections to analytical and computational results for complex systems involving a variety of types of energy flow.

This book was written with several goals in mind. First, despite the range of coverage, we desired the book to be reasonably compact. Second, we wanted to enable a student to use part of the material to begin a study of the simpler mechanical, electrical, hydraulic, thermal, and transducer systems, but in a style that would generalize suitably if he or she should continue more deeply into the areas of system dynamics, control, and computation. Third, we wished to provide ample advanced material to demonstrate the power of the approach for complex systems including transduction and modulation, and for large-scale systems. We hope the inclusion of this material will stimulate interest in the earlier parts of the book and, at the same time, allow the book to be used for more advanced courses.

Naturally, nothing is without its price, and the most difficult decision for us has been which mathematical methods to include and which to

omit. The book falls naturally into two parts. In the first part, Chapters 1 through 6, the level of mathematical sophistication has been kept low. In fact, the first part can be studied by a student who is taking a first course in differential equations concurrently. All formulations are organized in a state-space-compatible form, but the actual formalism of vectors and matrices has been omitted. In the second part of the book, which deals with large-scale and nonlinear systems and with a richer variety of physical systems, matrix notation is used, and some familiarity with differential equations is assumed.

The existence of various excellent computer programs for performing many phases of systems analysis once a model is made has enabled us to count on the student being able to examine the dynamics of a variety of systems without having to program at a language level like FORTRAN. This generally has increased the student's incentive and interest and has permitted us to consider a greater variety of engineering examples than otherwise might have been possible. We strongly recommend the use of automated computational aids in conjunction with this book.

The first part has been used in a number of engineering schools across the country in a preliminary version for several years, in courses at both the under- and upper-division levels. The second part has been used in senior-graduate courses, where the students have had some prior systems experience, whether bond-graph style or not. Various instructors have chosen to shape their own special variations by the material they add and omit. At some point, students must become conversant with block diagrams, sinusoidal frequency response, transfer functions, Laplace transforms, and other topics that we have omitted or treated in a somewhat abbreviated fashion. Experience has shown that additional topics may be fitted in very nicely to the flow of the book. In that sense the book serves as a unifying basis on which to build one's understanding of system dynamics, rather than a complete statement of all the techniques useful in attaining this understanding.

Because this is the first system dynamics textbook to adopt the bond-graph approach as its basis, we would like to express our appreciation to those who have helped us in our efforts to bring it to publication. To Professor Henry Paynter, who started the whole thing in the late 1950s, and to the many intrepid investigators who were willing to use novel methods in a hard-headed business like engineering, go our thanks for advancing the state-of-the-art to its present condition. To the many engineering instructors who found themselves explaining subtleties of a method they had only recently learned themselves go our thanks for their professorial courage. To the College of Engineering at Michigan State University, which made available the preliminary edition of the material,

on which so many of us have relied for so long, go our prepublication thanks. And to our wives, who have been with us somewhat longer than bond graphs, go our thanks and probably our royalties. As a final note we would like to add that flashes of insight and brilliance detectable in the text should be credited to the authors, while errors of substance should be attributed to faulty interpretation of our intended meaning.

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Davis, California
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1

INTRODUCTION

This book is concerned with the development of an understanding of the dynamic physical systems that engineers are called upon to design. Methods for modeling real systems will be presented, ways of analyzing systems in order to shed light on system behavior will be shown, and techniques for using computers to simulate the dynamic response of systems to external stimuli will be developed. Before beginning the study of physical systems it is worthwhile to reflect a moment on the nature of the discipline that is usually called "system dynamics" in engineering.

The word "system" is used so often and so loosely to describe a variety of concepts that it is hard to give a meaningful definition of the word or even to see the basic concept that unites its diverse meanings. When the word "system" is used in this book, two basic assumptions are being made: (1) A system is assumed to be an entity separable from the rest of the universe (the environment of the system) by means of a physical or conceptual boundary. An animal, for example, can be thought of as a system that reacts to its environment (the temperature of the air, for example) and that interchanges energy and information with its environment. In this case the boundary is physical or spatial. An air traffic control system, on the other hand, is a complex, man-made system, the environment of which is not only the physical surroundings but also the fluctuating demands for air traffic which ultimately come from human decisions about travel and the shipping of goods. The unifying element in these two disparate systems is the ability to decide what belongs in the system and what represents an external disturbance or command originating from outside the system. (2) A system is composed of interacting parts. In an animal we recognize organs with specific functions, nerves that transmit information, and so on. The air traffic control system is

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composed of men and machines with communication links between them. Clearly the *reticulation* of a system into its component parts is something that requires skill and art since most systems could be broken up into so many parts that any analysis would be swamped with largely irrelevant detail.

These two aspects of systems can be recognized in everyday situations as well as in the more specific and technical applications that form the subject matter of most of this book. For example, when one hears a complaint that the transportation system in this country does not work well, one may see that there is some logic in using the word system. First of all, the transportation system is roughly identifiable as an entity. It consists of air, land, and sea vehicles and the men, machines, and decision rules by which they are operated. In addition, many parts of the system can be identified—cars, planes, ships, baggage handling equipment, computers, and the like. Each part of the transportation system could be further reticulated into parts (i.e., each component part is itself a system), but for obvious reasons we must exercise restraint in this division process.

The essence of what may be called the “systems viewpoint” is to concern oneself with the operation of a complete system rather than with just the operation of the component parts. Complaints about the transportation system are often real “system” complaints. It is possible to start a trip in a private car that functions just as its designers had hoped it would, transfer to an airplane that can fly at its design speed with no failures, and end in a taxi that does what a taxi is supposed to do and yet have a terrible trip because of traffic jams, air traffic delays, and the like. Perfectly good components can be assembled into an unsatisfactory system.

In engineering, as indeed in virtually all other types of human endeavor, tasks associated with the design or operation of a system are broken up into parts which can be worked on in isolation to some extent. In a power plant, for example, the generator, turbine, boiler, and feed water pumps typically will be designed by separate groups. Furthermore, heat transfer, stress analysis, fluid dynamic, and electrical studies will be undertaken by subsets of these groups. In the same way, the bureaucracy of the federal government represents a splitting up of the various functions of government. All the separate groups working on an overall task must interact in some manner to make sure that not only will the parts of the system work but also the system as a whole will perform its intended function. Many times, however, oversimplified assumptions about how the system will operate are made by those working on a small part of the system. When this happens the results can be disappointing. The power plant may undergo damage during a full load rejection or the economy of a country

may collapse because of the unfavorable interaction of segments of government each of which assiduously pursues seemingly reasonable policies.

In this book, the main emphasis will be on studying system aspects of behavior as distinct from component aspects. This requires a knowledge of the component parts of the systems of interest and hence some knowledge in certain areas of engineering that are taught and sometimes even practiced in splendid isolation from other areas. In the engineering systems of primary interest in this book, topics from vibrations, strength of materials, dynamics, fluid mechanics, thermodynamics, automatic control, and electrical circuits will be used. It is possible, and perhaps even common, for an engineer to spend a major part of his professional career in just one of these disciplines, despite the fact that few significant engineering projects concern a single discipline. Systems engineers, on the other hand, must have a reasonable command of several of the engineering sciences as well as knowledge pertinent to the study of systems per se.

Although many systems may be successfully designed by careful attention to static or steady-state operation in which the system variables are assumed to remain constant in time, in this book the main concern will be with *dynamic* systems, that is, those systems whose behavior as a function of time is important. For a transport aircraft that will spend most of its flight time at a nearly steady speed, the fuel economy at constant speed is important. For the same plane, the stress in the wing spars during steady flight is probably less important than the time varying stress during flight through turbulent air, during emergency maneuvers, or during hard landings. In studying the fuel economy of the aircraft, a static system analysis might suffice. For stress prediction, a dynamic system analysis would be required.

Generally, of course, no system can operate in a truly static or steady state, and both slow evolutionary changes in the system and shorter time transient effects associated, for example, with start up and shut down, are important. In this book, despite the importance of steady-state analysis in design studies, the emphasis will be on dynamic systems. Dynamic system analysis is more complex than static analysis but is extremely important since decisions based on static analyses can be misleading. Systems may never actually achieve a possible steady state, because of external disturbances or instabilities that appear when the system dynamics are taken into account. Also, systems of all kinds can exhibit counter-intuitive behavior when considered statically. A change in a system or a control policy may appear beneficial in the short run based on static considerations, but may have long-run repercussions opposite to

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the initial effect. The history of social systems abounds with sometimes tragic examples and there is hope that dynamic system analysis can help avoid some of the errors in “static thinking” [1]. However, even in engineering with rather simple systems, one must have some understanding of the dynamic response of a system before one can reasonably study the system on a static basis.

A simple example of a counter-intuitive system in engineering is the case of a hydraulic power generating plant. In order to reduce power, wicket gates just before the turbine are moved toward the closed position. Temporarily, however, the power actually increases as the inertia of the water in the penstock forces the flow through the gates to remain almost constant, resulting in a higher velocity of flow through the smaller gate area. Ultimately, the water in the penstock slows down and power is reduced. Without an understanding of the dynamics of this system, one would be led to open the gates to *reduce* power. If this were done, the immediate result would be a gratifying decrease in power followed by a surprising and inevitable increase in power. Clearly a good understanding of dynamic response is crucial to the design of a control policy for dynamic systems.

1.1 MODELS OF SYSTEMS

A central idea involved in the study of the dynamics of real systems is the idea of a *model* of the system. Models of systems are simplified, abstracted constructs used to predict the behavior of systems of interest. Scaled, physical models are well known in engineering. In this category fall the wind tunnel models of aircraft, ship hull models used in towing tanks, structural models used in civil engineering, plastic models of metal parts used in photoelastic stress analysis, and the “breadboard” models used in the design of electric circuits.

The characteristic feature of these models is that some, but not all, of the features of the real system are reflected in the model. In the wind tunnel aircraft model, for example, no attempt is made to reproduce the color or interior seating arrangement of the real aircraft. Aeronautical engineers assume that some aspects of a real craft are unimportant in determining the aerodynamic forces on the craft and thus the model contains only those aspects of the real system that are supposed to be important to the characteristics of the system under study.

In this book, another type of model, often called a *mathematical model*, is considered. Although this type of model may seem much more abstract than the physical model, there are strong similarities between physical and mathematical models. The mathematical model is used to predict only

certain aspects of the system response to inputs. For example, a mathematical model might be used to predict how a proposed aircraft would respond to pilot input command signals during test maneuvers. But such a model would not have the capability of predicting every aspect of the real aircraft response. The model might not contain any information on changes in aerodynamic heating during maneuvers nor about high-frequency vibrations of the aircraft structure, for example.

Because a model must be a simplification of reality, there is a great deal of art in the construction of models. An overly complex and detailed model may contain parameters virtually impossible to estimate, may be practically impossible to analyze, and may cloud important results in a welter of irrelevant detail if it can be analyzed. An overly simplified model will not be capable of exhibiting important effects. It is important, then, to realize that *no system can be modeled exactly* and that any competent system designer needs to have a procedure for constructing a variety of system models of varying complexity so that he can find the simplest model capable of answering the questions he has about the system under study.

The remainder of this book deals with models of systems and with the procedures for constructing models and for extracting system characteristics from models. The models will be mathematical models in the usual meaning of the term even though the models may be represented by stylized graphs and computer printouts rather than the more conventional sets of differential equations.

System models will be constructed using a uniform notation for all types of physical systems. It is a remarkable fact that models based on apparently diverse branches of engineering science all can be expressed using the notation of *bond graphs* based on energy and information flow. This allows one to study the *structure* of a system model. The nature of the parts of the model and the manner in which the parts interact can be made evident in a graphical format. In this way, analogies between various types of systems are made evident and experience in one field can be extended to other fields.

Using the language of bond graphs, one may construct models of electrical, magnetic, mechanical, hydraulic, pneumatic, thermal, and other systems using only a rather small set of ideal elements. Standard techniques allow the models to be translated into differential equations or computer simulation schemes. Historically, diagrams for representing dynamic system models developed separately for each type of system. For example, parts *a*, *b*, and *c* of Figure 1.1 each represent a diagram of a typical model. Note that in each case the elements in the diagram seem to have evolved from sketches of devices, but in fact a photograph of the

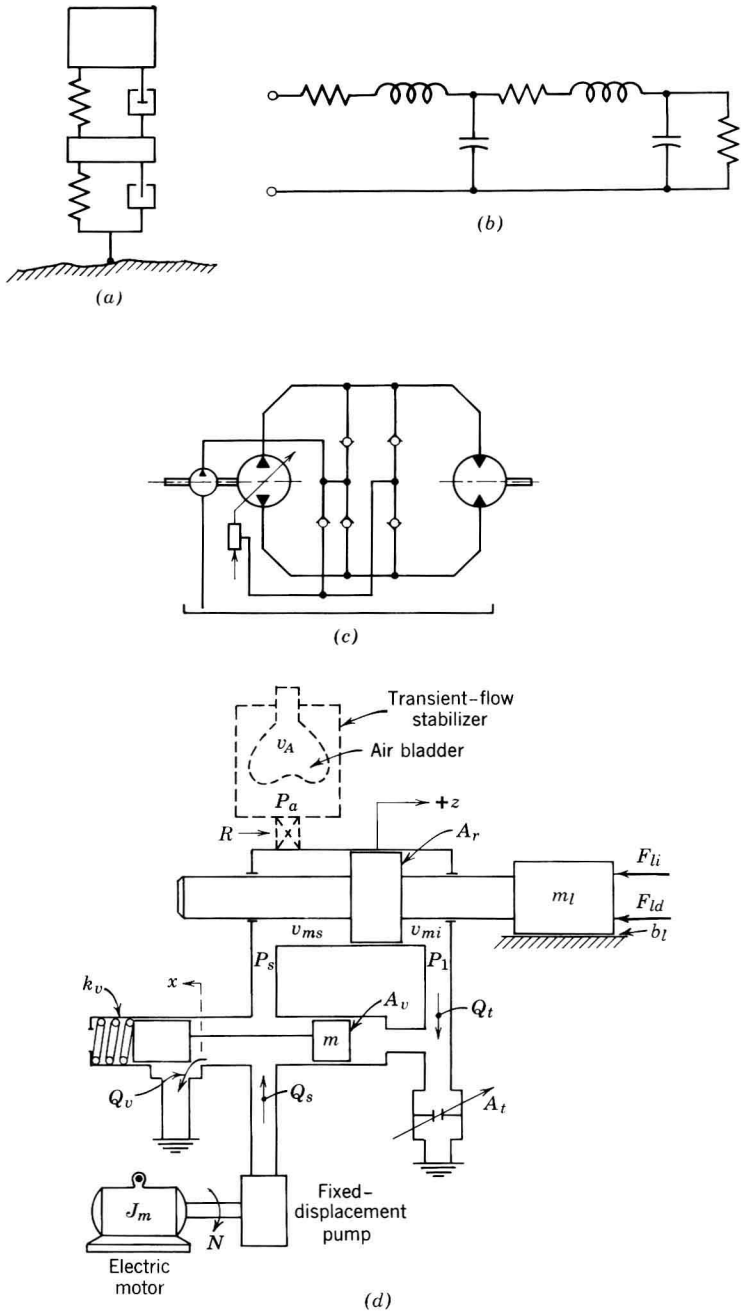


Figure 1.1. (a) Typical mechanical schematic diagram; (b) typical electric circuit diagram; (c) typical hydraulic diagram; (d) schematic diagram of system containing mechanical, electrical, and hydraulic components.

real system would not resemble the diagram at all. Figure 1.1a, might well represent the dynamics of heave motion of an automobile, but the masses, springs, and dampers of the model are not directly related to the parts of an automobile visible in a photograph. Similarly, symbols for resistors and inductors in diagrams such as Figure 1.1b may not correspond to separate physical elements called resistors and chokes but instead may correspond to the resistance and inductance effects present in a single physical device. Thus, even semipictorial diagrams are often a good deal more abstract than they might at first appear.

When mixed systems such as that shown in Figure 1.1d are to be studied, the conventional means of displaying the system model are less well developed. Indeed, few such diagrams are very explicit about just what effects are to be included in the model. The basic structure of the model may not be evident from the diagram. A bond graph is more abstract than the type of diagrams shown in Figure 1.1, but it is explicit and has the great advantage that all the models shown in Figure 1.1 would be represented using exactly the same set of symbols. For mixed systems such as Figure 1.1d, a universal language such as bond graphs provide is required in order to display the essential structure of the system model.

1.2 SYSTEMS, SUBSYSTEMS, AND COMPONENTS

In order to model a system it is usually necessary to first break up the system into smaller parts that can be modeled and perhaps studied experimentally and then to assemble the system model from the parts. Often, the breaking up of the system is conveniently accomplished in several stages. In this book major parts of a system will be called *subsystems* and primitive parts of subsystems will be called *components*. Of course, the hierarchy of components, subsystems, and systems can never be absolute, since even the most primitive part of a system could be modeled in such detail that it would be a complex subsystem. On the other hand, in many engineering applications, the subsystem and component categories are fairly obvious.

Basically, a subsystem is a part of a system that will be modeled as a system itself; that is, the subsystem will be broken into interacting component parts. A component, on the other hand, is modeled as an entity and is not thought of as composed of simpler parts. One needs to know how the component interacts with other components and one must have a characterization of the component, but otherwise a component is treated as a “black box” without any need to know what caused it to act as it does.

To illustrate these ideas, consider the vibration test system shown in

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Figure 1.2. The system is intended to subject a test structure to a vibration environment specified by a signal generator. For example, if the signal generator delivers a random noise signal, it may be desired that the acceleration of the shaker table be a faithful reproduction of the electrical noise signal waveform. In a system that is assembled from physically separate pieces, it is natural to consider the parts that are assembled by connecting wires, hydraulic lines, or by mechanical fasteners as subsystems. Certainly, the electronic boxes labeled signal generator, controller and electrical amplifier are subsystems, as are the electrohydraulic valve, the hydraulic shaker, and the test structure. It may be possible to treat some of these subsystems as components if their interactions with the rest of the system can be specified without knowledge of the internal construction of the subsystem. The electrical amplifier is obviously composed of many components, such as resistors, capacitors, transistors, and the like, but if the amplifier is sized correctly so that it is not overloaded, then it may be possible to treat the amplifier as a component specified by the manufacturers input-output data. Other subsystems may require a subsystem analysis in order to achieve a dynamic description suitable for the overall system study.

Consider, for example, the electrohydraulic valve. A typical servo valve is shown in Figure 1.3. Clearly, the valve is composed of a variety of electrical, mechanical, and hydraulic parts that work together to produce the dynamic response of the valve. For this subsystem the components might be the torque motor, the hydraulic amplifier, mechanical springs, hydraulic passages, and the spool valve. A subsystem dynamic analysis can reveal weaknesses in the subsystem design that may necessitate the substitution of another subsystem or a reconfiguration of the overall system. On the other hand, such an analysis may indicate that, from the

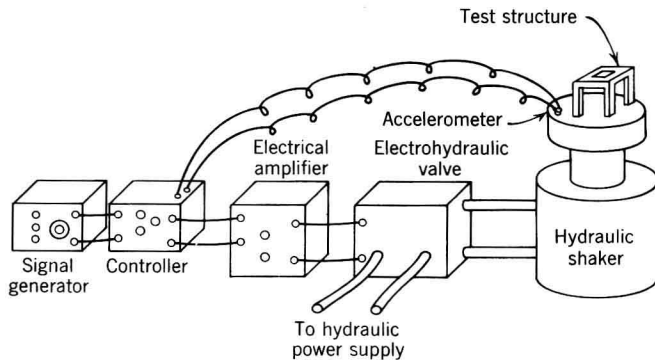


Figure 1.2. Vibration test system.