

Günter Fandel

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# Theory of Production and Cost



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by

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## Preface

Production theory and the theory of cost both belong to the central areas of business administration, for all considerations concerning the economic organization of industrial manufacturing processes start from these. Two developments in the past 30 years have had a considerable influence on the structure and the concentration on points of emphasis in this book. I am referring to findings from KOOPMANS' activity analysis and to the formulation by GUTENBERG of a production function concept that focuses on industrial production processes. Activity analysis has made it possible to develop, from a uniform approach, different types of production functions which describe the concrete principles of production in the productive sector of a business enterprise; this has created a common basis for all production concepts in business administration. The Gutenberg Production Function with its different kinds of adjustment to a changing output has opened up a flexibility to theoretical and practical considerations that gave rise to a large number of additional studies in this area. Considerations in cost theory were in particular need of considerable extensions in the direction of cost minimal combined adjustment processes. By means of the organization of its contents, this book will take both approaches into due account. In that way, it is vastly different from other books dealing with the same subject.

As a matter of course, traditional analytical methods and ways of thinking also constitute a large part of the book. They have been supplemented to a considerable degree by stochastic, dynamic and empirical approaches in which production processes are looked at from an economic point of view. In addition to that, we will revert to concepts describing the technical basis of manufacturing processes, at least as far as they contribute to the understanding of the normally goods-related input-output-analyses of production theory.

Large parts of the book were developed from correspondence lessons that I had written for my teaching at the Fernuniversität. As a result, I was able to include in this book version many positive suggestions coming from critical students. For this I am grateful and hope that the students who take this textbook into their hands may profit from this.

I would also like to thank all those who work in my department and who assisted in the publication of this book. I am particularly thankful to my assistants Dr. DYCKHOFF and Dr. REESE for their constant readiness to discuss with me the contents and the organization as regards the subject matter of this book. Their criticism and suggestions were a valuable help for me. I would also like to thank Miss RIPKE and Miss DEVENTER for their help in translating the book into English. Finally, I would like to thank C. DÜCHTING, T. GIERSEPEN, J. HEAPHY, M. KWAST, L. LANWER, F. REH, P. SCHADE, G. SCHIMRICH, A. SCHMIDT, R. SCHMIEDGEN, and D. SUPPLE for their patience in preparing the manuscript for printing, Mr. P. FRANÇOIS, MBA, for initiating and also Mr. K.-M. GUBITZ, MBA, for coordinating this project.

GÜNTER FANDEL

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# Table of Contents

Preface .....	VII
Chapter 1:   Introductory Survey of the Production Field .....	1
I.       Integration of Production into Business Administration .....	1
1.   The Notion of Production .....	1
2.   Institutional Division of Business Administration .....	2
3.   Functional Division of Business Administration .....	4
II.      Basic Conditions and Tasks of Entrepreneurial Activity .....	7
III.     Delimitation of the Research Subject .....	11
IV.     Object of the Theory of Production and Cost .....	13
V.      Different Fields of Production Planning .....	18
1.   Planning the Production Program .....	18
2.   Choice of the Production Procedure .....	19
3.   Organizing the Productive Potential .....	20
4.   Planning the Production Process .....	21
VI.     Production and Management Planning .....	21
VII.    Summarizing Survey .....	23

## Part One

### Production Theory

Chapter 2:   Fundamentals of Production Theory .....	29
I.       Survey of Developments in Production Theory .....	29
1.   Description of Productive Principles by Technologies .....	29
2.   Types of Production Functions .....	30

	3. Empirical Validity of Production Functions .....	34
	4. Stochastic and Dynamic Extensions .....	34
II.	Basic Elements of Production .....	37
	1. Products, Production Factors, Goods .....	37
	2. Activity and Technology .....	41
III.	Technologies .....	45
	1. General Assumptions .....	45
	2. Special Forms of Technologies .....	47
	3. Production Matrix and Goods Restrictions for Linear Technologies .....	51
IV.	The Efficiency Criterion .....	57
V.	Production Functions .....	61
	1. Deduction of the Production Function from Technology .....	61
	2. Relations between Factors and between Products .....	64
	3. Basic Notions of Production Theory for Characterizing Production Functions .....	69
Chapter 3:	Substitutional Production Functions .....	76
I.	Marginal Rate of Substitution, Complementarity, Substitutional Elasticity .....	76
II.	The Classical Production Function (Classical Law of Returns) ..	84
III.	Neoclassic Production Functions .....	91
	1. The Cobb–Douglas Production Function .....	92
	2. The CES Production Function .....	100
	3. Extensions of the CES Production Function .....	106
Chapter 4:	Limitational Production Functions .....	111
I.	Leontief Production Function .....	111
	1. Considerations Based on a Single Production Process .....	111
	2. Investigation of Cases with more than One Production Process .....	118

II.	Gutenberg Production Function .....	124
1.	Basic Assumptions and Fundamental Considerations .....	124
2.	Different Kinds of Adjustment as Parameters of Action .....	128
3.	Production Relations between End Product Quantity and Use of Potential Factors for Different Kinds of Adjustments .....	134
4.	The Concept of the Consumption Function for Consumption Factors .....	137
5.	Production Relations between End Product Quantity and Consumption Factor Input for Different Adjustments .....	140
Chapter 5:	Some Further Approaches in the Field of Static-Deterministic Production Functions .....	147
I.	Introductory Remarks .....	147
II.	Heinen Production Function .....	148
III.	Engineering Production Functions .....	158
1.	Development and General Formal Description of Engineering Production Functions .....	158
2.	Engineering Production Functions for Single Aggregates .....	164
3.	Engineering Production Functions for Branches of Industry .....	167
4.	Engineering Production Function for a High-Voltage Line ..	170
5.	Engineering Production Function in Aircraft Construction ..	176
IV.	PICHLER's Concept of Throughput Functions .....	177
V.	KLOCK's Input-Output-Analysis Approach .....	180
Chapter 6:	Dynamic and Stochastic Extensions in the Field of Production Functions .....	185
I.	Preliminary Remarks .....	185
II.	The Dynamizing of Production Functions .....	186
1.	Points of Contact between the Static and the Dynamic Approach .....	186
2.	Reasons for a Dynamic Approach .....	190
3.	Dynamizing Forms .....	193



III.	Time-Dependent Technologies Resulting from Innovations – Autonomous Technical Progress .....	197
1.	Description of Time-Related Input-Output Relations for Leontief Processes .....	197
2.	Technical Progress in the Gutenberg Production Function ..	202
3.	KRELLE's Dynamic Production Function .....	204
IV.	Inclusion of Learning Processes in a Theory of Production – Induced Technical Progress .....	206
1.	A Theoretical Concept of Learning Processes in Manufacturing .....	206
2.	Inclusion of Learning Processes in Different Types of Production Functions .....	210
V.	KÜPPER's Dynamic Production Function .....	217
VI.	Considering Uncertainties of Production by Stochastization of the Production Function .....	222
VII.	Model of a Stochastic Production Function on the Basis of the Classical Law of Returns .....	226
Chapter 7: Empirical Validity of Production Functions .....		231
I.	Different Approaches .....	231
II.	A Formal Framework for Analysing the Empirical Validity of Production Functions .....	232
III.	Evaluating the Empirical Validity of Specific Production Functions .....	234
1.	The Classical Law of Returns .....	234
2.	The Leontief Production Function .....	238
3.	The Gutenberg Production Function .....	240
4.	The Heinen Production Function .....	243
5.	The Kloock Production Function .....	245
6.	Engineering Production Functions .....	246
7.	Summary .....	246
IV.	The Empirical Significance of the Gutenberg Production Function in the Light of Practical Research .....	247
1.	Chances and Limitations of the Gutenberg Production Function .....	247

2. Lines of Empirical Determination .....	249
3. Results of Empirical Studies .....	250

## Part Two

### Cost Theory

Chapter 8:	Fundamentals of a Cost Theory and	
	Minimal Cost Combination .....	265
I.	Transition from Production Theory to Cost Theory .....	265
II.	Costs and Cost-Influencing Factors .....	267
	1. The General Notion of Cost .....	267
	2. Cost-Influencing Factors .....	270
III.	Special Notions of Cost .....	277
IV.	The Selection Problem in Cost Theory: The Minimal Cost	
	Combination .....	285
	1. Notion and Content of the Minimal Cost Combination .....	285
	2. Minimal Cost Combination for Substitutional Production ...	288
	3. Minimal Cost Combination for Linear-Limitational	
	Production with a Single Production Process .....	295
	4. Minimal Cost Combination for Linear-Limitational	
	Production with Several Production Processes .....	298
V.	Minimal Cost Combination for Dynamic Considerations of	
	Production and Cost .....	301
VI.	Historical Contributions to Cost Theory .....	306
Chapter 9:	Cost Functions on the Basis of Special Production	
	Functions .....	315
I.	Deriving a Cost Function from a Production Function .....	315
II.	Cost Functions on the Basis of the Law-of-Return	
	Production Functions .....	318

III.	Cost Functions of the Neoclassic Production Functions .....	326
IV.	Cost Functions for the Leontief Production Functions .....	333
V.	Cost Functions on the Basis of the Gutenberg Production Function .....	342
Chapter 10:	Combined Processes of Adjustment for Several Functionally Identical Aggregates .....	355
I.	Approaches to Combined Adjustment in the Literature .....	355
II.	Adjustments of Time, Intensity, and Quantity of Aggregates that Differ in Costs .....	357
	1. Formulation of the Problem and Assumptions .....	357
	2. The Solution Approach of Pre-Optimized Marginal Costs Functions .....	361
	3. The Solution Method of Dynamic Programming .....	366
III.	Combined Adjustment Processes without Adjustments of Time .	370
	1. Preliminary Remarks .....	370
	2. Adjustment Processes in Cases of Constant Output Intensities .....	372
	3. Adjustment Processes in Cases of Intensity Splitting .....	380
References	.....	387
Name Index	.....	395
Subject Index	.....	397

## Important Symbols

$k = 1, \dots, K$	Type of good
$j = 1, \dots, J$	Output type
$s = 1, \dots, S$	Type of intermediate product
$i = 1, \dots, I$	Input type
$m = 1, \dots, M$	Type of aggregate (type of potential factor)
$n_m = 1, \dots, N_m$	Aggregate of type $m$
$v_k$	Good quantity
$x_j$	Output quantity
$y_s$	Quantity of intermediate product
$r_i$	Input quantity
$b_{n_m}$	Power output of the aggregate
$v = (v_1, \dots, v_K)$	Activity
$T$	Technology
$\pi = 1, \dots, \Pi$	Type of process
$a, d$	Production coefficients
$\lambda$	Output intensity
$a(\lambda), \rho(\lambda)$	Consumption function
$t, T$	Time, time interval
$q_i$	Factor price
$K$	Total costs
$k$	Average costs

## **Chapter 1**

# **Introductory Survey of the Production Field**

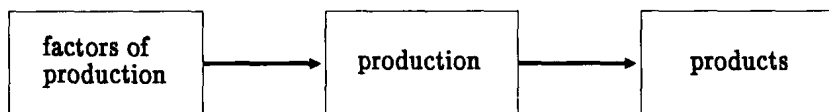
## **I. Integration of Production into Business Administration**

### **1. The Notion of Production**

Productive processes are characterized by the fact that by the combination resp. transformation of goods new goods are produced. This combined process of production that takes place in enterprises – the terms enterprise and firm will be used synonymously hereafter – is set going and realized by people intentionally and actively according to plan. The activity of enterprises that becomes apparent hereby and orientates itself by certain superior criteria and terms of reference is called managing. It serves to provide other economic subjects such as households, enterprises, and the state with demanded goods. Thus, productive processes of a firm do not constitute an end in themselves. On the contrary, according to their purpose they are integrated into the general economic process of production and exchange of goods.

Goods that are put into the productive process are called input goods, resources, factors of production or in a word, input. Goods which result from the process of combination and transformation are called output goods or products; they are also called output for short. The process of transformation that serves as the basis for a productive process proceeds according to technical principles. This process is outlined in Fig. 1.1. The technical principles may belong to the physical, chemical, biological or other fields. Material goods and services can be considered as input and output goods. Material goods are of a material kind. Durable goods (real estates, buildings, machines, fittings) as well as sales goods in the form of resources, materials, auxiliary materials, wares, and finished products belong to this group. Services, on the other hand, are goods of an immaterial kind. They appear in the form of transport, insurance, bank services, arrangements, and human labor or in similar forms. In business administration, human labor, machines, and materials, which are used as input goods in

production, are usually called elementary factors. Human labour is only included in so far as it refers to production-related activities in the firm. That part of human labor that goes into the planning and control of internal procedures, however, is called the dispositive factor.



**Fig. 1.1.** Production as a process of combination

The notion production thus refers to all processes in which – assisted by material goods and services – different material goods and services are produced. In this broad sense, production consists of any kind of productive performance; it may be executed in processing plants, finishing plants, plants for the production of raw materials or in service industries. But this broad definition of the notion production has not always been in common use in business administration; it has only developed gradually in recent times and has to be seen in close correlation to the shift of perspective that has taken place in the meantime and by which business administration suggests today the investigation of productive processes as a subsection of this branch of science.

## **2. Institutional Division of Business Administration**

In the past, the structure of the different sections of business economics was orientated to the different economic sectors. From this, special theories of business administration were developed as so-called institutional theories, which deal with single economic problems of enterprises in particular sections of the economic practice. This procedure was guided by the fundamental idea that enterprises of the same economic sector have to perform activities of the same kind and consequently show a more or less strong uniformity in their structures and their operational procedures. Consequently, the micro economic analysis of industrial processes under homogeneous criteria is facilitated. According to those institutional theories, business administration has been divided into the following sections (KERN 1976, pp. 756):

- Industrial management (HEIDEBROEK 1923; KALVERAM 1960)
- Handicraft management (RÖSSLE 1952)
- Commercial management (SCHAR 1921; SEYFERT 1972)
- Banking management (HASENACK 1925; KALVERAM 1950)
- Insurance management (HILBERT 1914; PATZIG 1925)
- Transportation management (PIRATH 1949; LECHNER 1963)
- Agricultural and forestry management (AEREBOE 1917; DIETERICH 1941, 1942, 1948).

From the point of view of the institutional structure of business administration, industrial and handicraft management most of all required the study of production-orientated relations. Both types of enterprises are characterized by combining productive equipment in order to produce material goods. In contrast to the production of services which is the main task of the other institutions, the production of material goods makes possible a relatively simple measurement of the output. A precondition has thereby been established that makes possible the investigation of the law between the quantity of products and the input factors of production. These considerations have their effect on the procurement of production factors as well as on the sales of products and, finally, on the other sectors of a firm.

With increasing mechanization and automation in the production area of industrial enterprises the craftman's business, however, has lost its significance for the investigation of productive connections. In craftsmen's businesses, most of the production is still done manually. In contrast to this, the frequently repeated and regular productive processes in the industrial area allow an easier access to the empirical deduction of the rules which the production is subject to. Therefore, it is not astonishing that most of the production-orientated model descriptions are based on product structures of industrial enterprises. As a result of emphasizing the most essential criterion of activity in these industrial enterprises - as far as the institutional division of business administration is concerned - it turned out to be necessary to deal with productive processes more intensively.

Different fields of activity and highly differing quantities of sales and numbers of employees in industrial enterprises suggest that the industrial firm does not exist as such, just as little as production-orientated conditions and connections in these enterprises allow the formulation of general statements at all times. For special purposes of investigation, it therefore seemed to be useful

to summarize identical phenomena according to objective aspects. This was done with the help of the standardization of industrial enterprises (SCHÄFER 1969, 1971). Commonly used classifying criteria for the characterization of industrial enterprises are:

- The kind of activity of a firm with division into branches, groups of industry or economic groups.
- The size of firms according to sales, number of employees and balance-sheet total.
- The legal form of firms, i.e. whether they are registered as joint-stock companies, limited liability companies or cooperatives.
- The intended usage of the products: This leads to a differentiation between enterprises of the capital-goods industry and the consumer-goods industry.
- The dominant input goods: Firms of a large intensity with regard to materials, labor or investment.
- The organization of production: Firms with job-shop, flow-shop or building-site production.
- The position in the economic goods cycle: Firms for winning, processing, finishing, end-processing and recovering.

Further classifications, like a division according to the marketing structure (commission- or sales-orientated), the size of the production program or the number of production stages (firms producing one or several products, single-stage or multi-stage production), and the batch-size etc. are possible. Most of the classifications show overlappings, doubts about the right classification, and difficulties as to their delimitation. On the other hand, they can in some ways be very helpful. If one is interested in the consequences of wage increases on a firm's cost structure, one will choose the classification as to the predominant factor of production. In this way, classifications of industrial enterprises can be applied to the structuring of investigating production-orientated problems. Which form of classification will actually be chosen depends on the sort of problem.

### **3. Functional Division of Business Administration**

Because of different developments in single problem areas and because of the possibility to make use of solutions for similar cases which have been made



available from other scientific areas, business administration in recent times has turned away more and more from the classification scheme given by institutional theories. For the purpose of general business administration, it has turned to the question of which tasks are common to all enterprises, independent of the economic branch they belong to. This has led to a division of the problem areas according to the different functions of a firm (GUTENBERG 1958).

The question of such functions can be answered most easily from the catalog of activities in enterprises. For the purpose of satisfying human needs, enterprises procure the following means of production: labor, machines, and material. They are combined with each other in a process of transformation, and goods and services they produced are then marketed. The managerial functions of procurement, investment, production, and marketing can be concluded from these. For the procurement of the means of production, adequate funds have to be made available. The solution to this problem falls within the scope of the duties of financing. Usually, not all of the goods procured are put into production immediately, and not all of the goods produced can be marketed directly. The tasks of materials management and stockkeeping, which are closely connected with the enterprise's productive sector, result from that.

The mastering of these managerial partial tasks, which exist in small enterprises of food manufacturing as well as in large-scale enterprises of the steel-producing industry, and which grow more and more complex with increasing size, requires the use of the executive instruments of planning, organization, and control. Their handling falls within the scope of the management's duty as a dispositive factor. Whereas planning deals with the question of which tasks have to be performed in what order with regard to the enterprise's object, organization serves to realize planning. Control checks whether the realization of the planning has led to the expected success.

This brief survey of entrepreneurial activities and fields of duties makes it possible to recognize the division of business administration into functional sections as is expressed in the following functional systems:

- Management (planning, organization, control),
- Procurement (procuring of raw materials and supplies),
- Personnel (manpower requirements, personnel recruiting, planning of personnel deployment),
- Investment and financing (procuring of operational equipment, licenses, capital, planning and analysis of financing),