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Mark Zegarelli

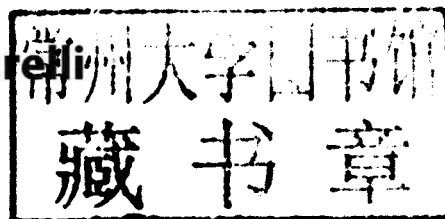
Math tutor and writer



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FOR
DUMMIES®
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by Mark Zegarelli



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Mark Zegarelli is the author of *Logic For Dummies* (Wiley), *Basic Math & Pre-Algebra For Dummies* (Wiley), and numerous books of puzzles. He holds degrees in both English and math from Rutgers University, and he lives in Long Branch, New Jersey, and San Francisco, California.

Dedication

For my brilliant and beautiful sister, Tami. You are an inspiration.

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Introduction

Calculus is the great Mount Everest of math. Most of the world is content to just gaze upward at it in awe. But only a few brave souls attempt the ascent.

Or maybe not.

In recent years, calculus has become a required course not only for math, engineering, and physics majors, but also for students of biology, economics, psychology, nursing, and business. Law schools and MBA programs welcome students who've taken calculus because it requires discipline and clarity of mind. Even more and more high schools are encouraging students to study calculus in preparation for the Advanced Placement (AP) exam.

So perhaps calculus is more like a well-traveled Vermont mountain, with lots of trails and camping spots, plus a big ski lodge on top. You may need some stamina to conquer it, but with the right guide (this book, for example!), you're not likely to find yourself swallowed up by a snowstorm half a mile from the summit.

About This Book

You *can* learn calculus. That's what this book is all about. In fact, as you read these words, you may well already be a winner, having passed a course in Calculus I. If so, then congratulations and a nice pat on the back are in order.

Having said that, I want to discuss a few rumors you may have heard about Calculus II:

- ✓ Calculus II is harder than Calculus I.
- ✓ Calculus II is harder, even, than either Calculus III or Differential Equations.
- ✓ Calculus II is more frightening than having your home invaded by zombies in the middle of the night and will result in emotional trauma requiring years of costly psychotherapy to heal.

Now, I admit that Calculus II is harder than Calculus I. Also, I may as well tell you that many — but not all — math students find it to be harder than the two semesters of math that follow. (Speaking personally, I found Calc II to be easier than Differential Equations.) But I'm holding my ground that the long-term psychological effects of a zombie attack far outweigh those awaiting you in any one-semester math course.

The two main topics of Calculus II are integration and infinite series. *Integration* is the inverse of differentiation, which you study in Calculus I. (For practical purposes, integration is a method for finding the area of unusual geometric shapes.) An *infinite series* is a sum of numbers that goes on forever, like $1 + 2 + 3 + \dots$ or $+++ \dots$. Roughly speaking, most teachers focus on integration for the first two-thirds of the semester and infinite series for the last third.

This book gives you a solid introduction to what's covered in a college course in Calculus II. You can use it either for self-study or while enrolled in a Calculus II course.

So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section in case you want to refresh yourself on the basics.

Here are two pieces of advice for math students (remember them as you read the book):

✔ **Study a little every day.** I know that students face a great temptation to let a book sit on the shelf until the night before an assignment is due. This is a particularly poor approach for Calc II. Math, like water, tends to seep in slowly and swamp the unwary!

So, when you receive a homework assignment, read over every problem as soon as you can and try to solve the easy ones. Go back to the harder problems every day, even if it's just to reread and think about them. You'll probably find that over time, even the most opaque problem starts to make sense.

✔ **Use practice problems for practice.** After you read through an example and think you understand it, copy the problem down on paper, close the book, and try to work it through. If you can get through it from beginning to end, you're ready to move on. If not, go ahead and peek, but then try solving the problem later without peeking. (Remember, on exams, no peeking is allowed!)

Conventions Used in This Book

Throughout the book, I use the following conventions:

- ✓ *Italicized* text highlights new words and defined terms.
- ✓ **Boldfaced** text indicates keywords in bulleted lists and the action parts of numbered steps.
- ✓ Monofont text highlights web addresses.
- ✓ Angles are measured in radians rather than degrees, unless I specifically state otherwise. (See Chapter 2 for a discussion about the advantages of using radians for measuring angles.)

What You're Not to Read

All authors believe that each word they write is pure gold, but you don't have to read every word in this book unless you really want to. You can skip over sidebars (those gray shaded boxes) where I go off on a tangent, unless you find that tangent interesting. Also feel free to pass by paragraphs labeled with the Technical Stuff icon.

If you're not taking a class where you'll be tested and graded, you can skip paragraphs labeled with the Tip icon and jump over extended step-by-step examples. However, if you're taking a class, read this material carefully and practice working through examples on your own.

Foolish Assumptions

Not surprisingly, a lot of Calculus II builds on topics introduced in Calculus I and Pre-Calculus. So here are the foolish assumptions I make about you as you begin to read this book:

- ✓ If you're a student in a Calculus II course, I assume that you passed Calculus I. (Even if you got a D-minus, your Calc I professor and I agree that you're good to go!)
- ✓ If you're studying on your own, I assume that you're at least passably familiar with some of the basics of Calculus I.