ELECTRIC MACHINERY

fourth edition

ELECTRIC MACHINERY

A. E. Fitzgerald

Late Vice President for Academic Affairs
and Dean of the Faculty
Northeastern University

Charles Kingsley, Jr.

Associate Professor of Electrical Engineering, Emeritus
Massachusetts Institute of Technology

Stephen D. Umans

Principal Research Engineer
Department of Electrical Engineering and Computer Science
Electric Power System Engineering Laboratory
Massachusetts Institute of Technology

McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

ELECTRIC MACHINERY

1 2 3 4 5 6 7 8 9 0 DOCDOC 8 9 8 7 6 5 4 3 2

ISBN 0-07-021145-0

This book was set in Century Schoolbook by York Graphic Services, Inc.
The editors were T. Michael Slaughter and David A. Damstra;
the designer was Charles A. Carson;
the production supervisor was Leroy A. Young.
New drawings were done by J & R Services, Inc.
R. R. Donnelley & Sons Company was printer and binder.

Library of Congress Cataloging in Publication Data

Fitzgerald, A. E. (Arthur Eugene), date
Electric machinery.

(McGraw-Hill series in electrical engineering. Power and energy)
Includes index.
1. Electric machinery. I. Kingsley, Charles, date . II. Umans, Stephen D. III. Title.
IV. Series.
TK2181.F5 1983 621.31'042 82-7763
ISBN 0-07-021145-0 AACR2

PREFACE

Like the three preceding editions of this book, the fourth edition presents a unified treatment of the basic theory of electromechanical devices with especial emphasis on the applications, control, and limitations of rotating electric machinery. Much of the text is taken from the best parts of the second and third editions, rearranged in a more logical sequence, and rewritten.

The topic of power electronics has received considerable attention in the past decade. A number of textbooks have been written on the subject, and it is available as a separate course in many electrical engineering curricula. It certainly requires more than a single chapter to do it justice. As a result, the chapter on solid-state motor control has been eliminated from the fourth edition. However, the concepts of solid-state control as applied to electric machinery have been retained in the form of an article discussing dc motor drives in Chapter 6 and an article discussing variable-frequency ac motor drives in Chapter 10.

This bit of reorganization gave us room to add some new material and to reinstate some material which had been included in the second edition. For example, the technique of the transformation to dq0 variables, so useful in the transient analysis of ac machinery, is again included. In addition, the book has been rearranged with the hope that it will be easier to use.

The first four chapters provide a basic introduction to the topic of energy conversion and to rotating electric machinery. The material of the second chapter on electromechanical energy conversion is often felt to be too mathematical for an introductory undergraduate course on electric machinery. In this edition, it has been written in such a fashion that the first two sections can serve as an overview of the topic and the rest of the chapter can be omitted if desired without a loss of understanding in the remainder of the book.

The chapters on dc, synchronous, and induction machinery have been grouped in pairs. In each of these groupings, the first chapter serves as an introduction to the machine type, develops the basic models, and applies them to the steady-state performance of the machine. The second chapter then deals with the issues associated with the control and transient behavior of the machine as well as some of the more subtle issues not required for the development of the basic models.

This book does not require a level of mathematical sophistication beyond that given in undergraduate courses in basic physics and circuit theory. The emphasis is on physical understanding as the basis for mathematical models.

Obviously more material is presented than can be treated in the time available in the typical curriculum. Moreover, the detailed content and order of subject matter are naturally governed by local circumstances and the desires and enthusiasms of individual instructors. For these reasons, particular attention is given to flexibility of use without loss of continuity. Browsing in the book for an hour or two and reading the introductory paragraphs of each chapter should enable an instructor to outline a variety of courses with differing content and sequence. Furthermore the book should serve the student as a reference text during his subsequent professional career.

The untimely death of Dr. A. E. Fitzgerald in July 1978 was a great loss of a coauthor and personal friend of many years. His sound intuitive judgment and lucid writing style are contributions that are hard to replace. However, much of his writing taken from the earlier editions has been retained in this fourth edition.

During the preliminary planning for the fourth edition, Dr. Alexander Kusko, a coauthor of the third edition, decided that he could no longer spare the time from his active consulting business to take part in the revision.

With the loss of these two coauthors I feel very fortunate to have been able to interest Dr. Stephen D. Umans in joining me in this new edition. Without his energy, enthusiasm, and conscientious adherence to a time schedule, the production of the fourth edition would have been difficult. From my point of view I have found working with him to be a very satisfying experience.

Charles Kingsley, Jr.

CONTENTS

3/ Rotating Machines: Basic Concepts 117

4/ Engineering Aspects of Practical Electric-Machine Performance 175

5/ DC Machines: Steady State 201

6/ DC Machine Dynamics and Control 247

1

Magnetic Circuits and Transformers

The object of this book is to study the devices used in the interconversion of electric and mechanical energy. Emphasis is placed on electromagnetic rotating machinery, by means of which the bulk of this energy conversion takes place. Attention is also paid to the transformer, which, although not an electromechanical energy-conversion device, is an important component in the overall problem of energy conversion. Moreover, in many respects its analysis uses techniques closely related to those required for rotating machinery.

Practically all transformers and electric machinery use magnetic material for shaping and directing the magnetic fields which act as the medium for transferring and converting energy. Thus the ability to analyze and describe magnetic field quantities is an essential tool for understanding these devices. Magnetic materials play a large role in determining the properties of a piece of electromagnetic equipment and affect its size and efficiency.

This chapter will develop some basic tools for the analysis of magnetic field systems and will provide a brief introduction to the properties of practical magnetic materials. These results will then be applied to the analysis of transformers. In later chapters they will be used in the analysis of rotating machinery.

In this book it is assumed that the reader has basic knowledge of magnetic and electric field theory such as given in a basic physics course for engineering students. Other readers may have had a course on electromagnetic field theory based on Maxwell's equations, but an understanding of Maxwell's equations is not a prerequisite for study of this book. The pertinent basic equations will be introduced in simplified form when required.

1-1 INTRODUCTION TO MAGNETIC CIRCUITS

The complete, detailed solution for magnetic fields in most situations of practical engineering interest involves the solution of Maxwell's equations along with various constitutive relationships which describe material properties. Although in practice exact solutions are often unattainable, various simplifying assumptions permit the attainment of useful engineering solutions.

The first assumption is that for the types of electric machines and transformers treated in this book the frequencies and sizes involved are such that the displacement-current term in Maxwell's equations can be neglected. This term accounts for magnetic fields being produced in space by time-varying electric fields and is associated with electromagnetic radiation. Neglecting this term results in the *magneto-quasi-static* form of Maxwell's equations. By this we mean that the magnetic field quantities are determined solely by the instantaneous values of the source currents and that the time variation of the magnetic fields follow directly from the time variations of the sources.

A second simplifying assumption involves the concept of the *magnetic circuit.* The general solution for the *magnetic field intensity* **H** and the *magnetic flux density* **B** in a structure of complex geometry is extremely difficult. However, a three-dimensional field problem can often be reduced to what is essentially a one-dimensional circuit equivalent, yielding solutions of acceptable engineering accuracy.

A magnetic circuit consists of a structure composed for the most part of high-permeability magnetic material. The presence of high-permeability material causes the magnetic flux to be confined to the paths defined by the structure, much as currents are confined to the conductors of an electric circuit. Use of this concept of the magnetic circuit is illustrated in this article and will be seen to apply quite well to many situations in this book.†

A simple example of a magnetic circuit is shown in Fig. 1-1. The core is

†For a more extensive treatment of magnetic circuits see A. E. Fitzgerald, D. E. Higgenbotham, and A. Grabel, "Basic Electrical Engineering." 5th ed., McGraw-Hill, New York, 1981, chap. 13; also E. E. Staff, MIT, "Magnetic Circuits and Transformers," MIT Press, Cambridge, Mass., 1965, chaps. 1 to 3.

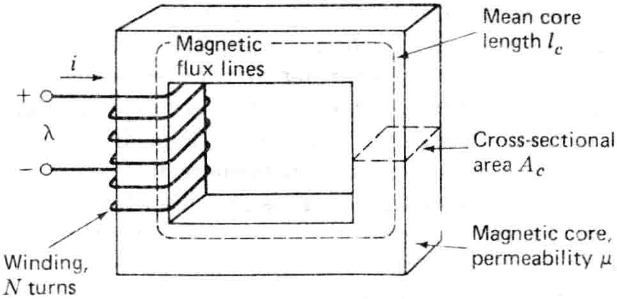

Fig. 1-1. Simple magnetic circuit.

assumed to be composed of magnetic material whose permeability is much greater than that of the surrounding air. The core is of uniform cross section and is excited by a winding of N turns carrying current i amperes (A). This winding produces a magnetic field in the core, as shown in the figure. The magnetic field can be visualized in terms of flux lines, which form closed loops interlinked with the winding. The basic relation between current i and the magnetic field intensity $\mathbf{H}$ states that the line integral of $\mathbf{H}$ around a closed path is equal to the net current enclosed by that path.

As applied to the magnetic circuit of Fig. 1-1, the source of the magnetic field in the core is the ampere-turn product Ni. In magnetic-circuit terminology Ni is the *magnetomotive force* (mmf) $\mathfrak{F}$. Although Fig. 1-1 shows only a single coil, transformers and most rotating machines have at least two windings and Ni is the algebraic sum of the ampere-turns of all the windings. Under the assumption of uniform magnetic flux density across the core cross section, the line integral of $\mathbf{H}$ becomes simply the scalar product $H_c l_c$, of the magnitude of $\mathbf{H}$ along the mean flux path whose length is l_c. Thus, the relationship between the mmf and the magnetic field intensity can be written in magnetic-circuit terminology as

$$\mathfrak{F} = Ni = H_c l_c \tag{1-1}$$

The direction of H_c in the core can be found from the *right-hand rule,* which can be stated in two equivalent ways. (1) Imagine a current-carrying conductor held in the right hand with the thumb pointing in the direction of current flow; the fingers then point in the direction of the magnetic field created by that current. (2) Equivalently, if the coil in Fig. 1-1 is grasped in the right hand (figuratively speaking) with the fingers pointing in the direction of the current, the thumb will point in the direction of the magnetic fields.

The relationship between the magnetic field intensity $\mathbf{H}$ and the magnetic flux density $\mathbf{B}$ is a property of the region in which the field exists; thus

$$\mathbf{B} = \mu\mathbf{H} \tag{1-2}$$

where μ is the *permeability*. In SI units **B** is in *webers per square meter,* known as *teslas* (T), and μ is in *webers per ampere-turn-meter,* or equivalently *henrys per meter*. In SI units the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$. The permeability of ferromagnetic material can be expressed in terms of μ_r, its value relative to that of free space, or $\mu = \mu_r\mu_0$. Typical values of μ_r range from 2000 to 80,000 for materials used in transformers and rotating machines. The characteristics of ferromagnetic materials are described in Arts. 1-3 and 1-4. For the present we shall assume that μ_r is a known constant, although it actually varies appreciably with magnetic flux density.

Because of the high permeability of the magnetic core, the magnetic flux is confined almost entirely to the core, the field lines follow that path defined by the core, and the flux density is essentially uniform over a cross section because the cross-sectional area is uniform.

The *magnetic flux* ϕ crossing an area is the surface integral of the normal component of **B**; thus

$$\phi = \int_S \mathbf{B} \cdot d\mathbf{a} \tag{1-3}$$

In SI units ϕ is in *webers*. In terms of field theory the continuity-of-flux equation

$$\oint_S \mathbf{B} \cdot d\mathbf{a} = 0 \tag{1-4}$$

states that the net magnetic flux crossing all surfaces of a three-dimensional closed surface (equal to the surface integral of B over that closed surface) is zero. This is equivalent to saying that all the flux which enters the surface enclosing a volume must leave that volume over some other portion of that surface because magnetic flux lines form closed loops. When flux outside the core is neglected, Eq. 1-3 reduces to the simple scalar equation

$$\phi_c = B_c A_c \tag{1-5}$$

where ϕ_c = the flux in core
B_c = flux density in the core
A_c = cross-sectional area of the core

The area is assumed to be constant throughout the length of the magnetic path. Because the field lines form closed loops, the flux is continuous throughout the length of the core.

Transformers are wound on closed cores like that of Fig. 1-1. Energy-conversion devices which incorporate a moving element must have air gaps in their magnetic circuits. A magnetic circuit with an air gap is shown in Fig. 1-2. When the air-gap length g is much smaller than the dimensions of the adja-

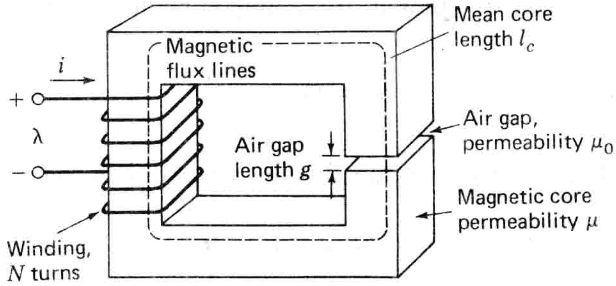

Fig. 1-2. Magnetic circuit with air gap.

cent core faces, the magnetic flux ϕ is constrained essentially to reside in the core and the air gap and is continuous throughout the magnetic circuit.

Thus, the configuration of Fig. 1-2 can be analyzed as a magnetic circuit with two series components, a magnetic core of permeability μ and mean length l_c, and an air gap of permeability μ_0 and length g. In the core the flux density is uniform, and the cross-sectional area is A_c; thus in the core

$$B_c = \frac{\phi}{A_c} \tag{1-6}$$

and in the air gap

$$B_g = \frac{\phi}{A_g} \tag{1-7}$$

The magnetic field lines bulge outward somewhat as they cross the air gap, as illustrated in Fig. 1-3. The effect of the fringing fields is to increase the effective cross-sectional area A_g of the air gap. Various empirical methods have been developed to account for this effect. A correction for such fringing fields in short air gaps can be made by adding the gap length to each of the two dimensions making up its cross-sectional area. In this book the effect of fring-

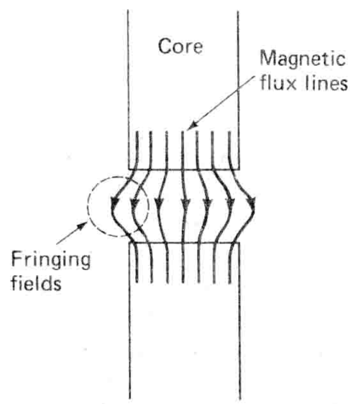

Fig. 1-3. Air-gap fringing fields.

ing fields will usually be ignored. If fringing is neglected, $A_g = A_c$ and

$$B_g = B_c = \frac{\phi}{A_c} \tag{1-8}$$

Application of Eqs. 1-1 and 1-2 to this magnetic circuit yields

$$\mathfrak{F} = Ni = H_c l_c + H_g g \tag{1-9}$$

$$\mathfrak{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \tag{1-10}$$

Here Ni is the total ampere-turns applied to the magnetic circuit. Thus we see that a portion of the mmf is required to excite the magnetic field in the core while the remainder excites the magnetic field in the air gap.

For practical magnetic materials (as discussed in Art. 1-3), B_c and H_c are not simply related by a known permeability μ. In fact, B_c is often a nonlinear, multivalued function of H_c. Thus, although Eq. 1-9 continues to hold, it does not lead directly to a simple expression relating the mmf and the flux densities, such as that of Eq. 1-10. Instead the specifics of the nonlinear B_c-H_c relation must be used, either graphically or analytically. However, in many cases, the concept of core permeability gives results of acceptable engineering accuracy and is frequently used.

From Eq. 1-8, Eq. 1-10 can be rewritten in terms of the total flux ϕ

$$\mathfrak{F} = \phi \frac{l_c}{\mu A_c} + \phi \frac{g}{\mu_0 A_c} \tag{1-11}$$

in which fringing at the air gap is neglected and the flux is assumed to go straight across the gap. The terms which multiply the flux in this equation are known as the *reluctance* $\mathfrak{R}$, of the core and air gap, respectively,

$$\mathfrak{R}_c = \frac{l_c}{\mu A_c} \tag{1-12}$$

$$\mathfrak{R}_g = \frac{g}{\mu_0 A_c} \tag{1-13}$$

and thus

$$\mathfrak{F} = \phi(\mathfrak{R}_c + \mathfrak{R}_g) \tag{1-14}$$

The fraction of the total mmf required for each portion of the magnetic circuit varies inversely as its reluctance. From Eq. 1–14 we see that the core reluctance becomes small as its permeability increases and can often be made much smaller than that of the air gap; i.e., for $\mu \gg \mu_0$, $\mathfrak{R}_c \ll \mathfrak{R}_g$. In this case, the flux and hence B can be found from Eq. 1-14 in terms of $\mathfrak{F}$ and the air-gap properties alone

$$\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F}\mu_0 A_c}{g} = Ni\frac{\mu_0 A_c}{g} \tag{1-15}$$

The term which multiplies the mmf is known as the *permeance* $\mathcal{P}$; thus the permeance of the air gap is

$$\mathcal{P}_g = \frac{1}{\mathcal{R}_g} = \frac{\mu_0 A_c}{g} \tag{1-16}$$

As will be seen in Art. 1-3, practical magnetic materials have permeabilities which are not constant but vary with flux level. From Eqs. 1-12 to 1-14 we see that as long as this permeability remains sufficiently large, its variation will not significantly affect the performance of the magnetic circuit.

We have now described the basic principles for reducing a magneto-quasi-static field with simple geometry to a magnetic-circuit model. Our limited purpose in this article is to introduce some of the concepts and terminology used by engineers in solving practical design problems. It should be emphasized that this type of thinking depends quite heavily on engineering judgment and intuition. For example, we have tacitly assumed that the permeability of the "iron" parts of the magnetic circuit is a constant known quantity, although this is not true in general (see Art. 1-3), and that the magnetic field is confined to the core and its air gaps. As we shall see later in this book, when two or more windings are placed on a magnetic circuit, as in a transformer or rotating machine, the fields outside the core, called *leakage fields,* are extremely important in determining the coupling between the windings.

EXAMPLE 1-1

The magnetic circuit as shown in Fig. 1-2 has dimensions $A_c = 9\ \text{cm}^2$; $A_g = 9\ \text{cm}^2$; $g = 0.050$ cm; $l_c = 30$ cm; $N = 500$ turns. Assume the value $\mu_r = 70{,}000$ for the iron. Find (*a*) current i for $B_c = 1$ T; (*b*) flux ϕ and flux linkage $\lambda = N\phi$.

Solution

(*a*) From Eq. 1-10 the ampere-turns for the circuit are

$$Ni = \frac{B_c l_c}{\mu_r \mu_0} + \frac{B_g g}{\mu_0}$$