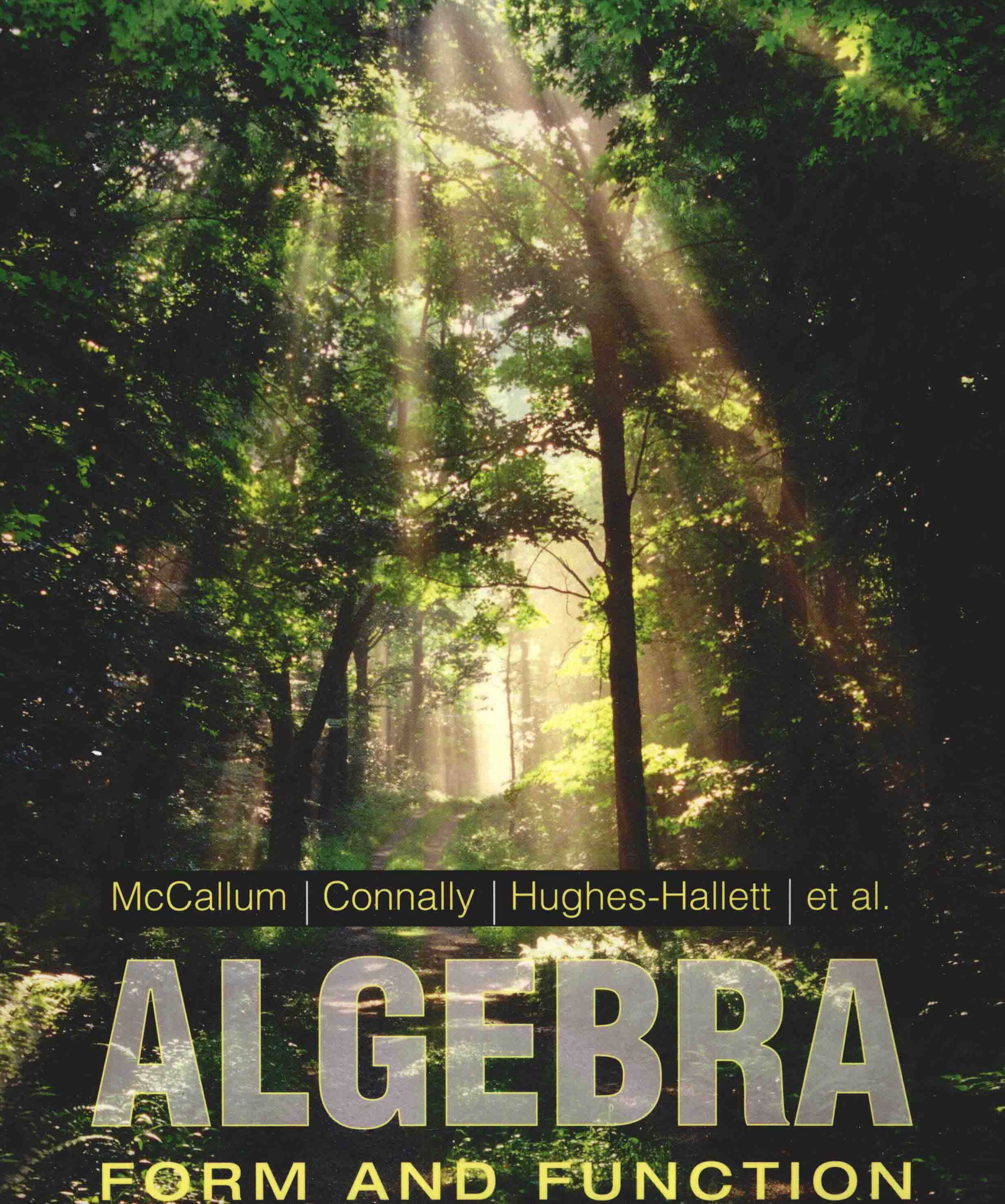

McCallum | Connally | Hughes-Hallett | et al.

ALGEBRA

FORM AND FUNCTION

ALGEBRA: FORM AND FUNCTION

Produced by the Calculus Consortium and initially funded by a National Science Foundation Grant.

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PREFACE

Algebra is fundamental to science, engineering, and business. Its efficient use of symbols to represent complex ideas has enabled extraordinary advances throughout the natural and social sciences. To be successful in any quantitative field, students need to master both symbolic manipulation and algebraic reasoning.

Balance: Manipulation and Interpretation

The fact that algebra can be encapsulated in rules sometimes encourages students to try and learn the subject merely as a set of rules. However, both manipulative skill and understanding are required for fluency. Inadequate practice in manipulation leads to frustration; inadequate attention to understanding leads to misconceptions, which easily become firmly rooted. Therefore we include both drill and conceptual exercises to develop skill and understanding together.

By balancing practice in manipulation and opportunities to see the big picture, we offer a way for teachers to help students achieve fluency. Our approach is designed to give students confidence with manipulations as well as a solid understanding of algebraic principles, which help them remember the many different manipulations they need to master.

Laying the Foundation: Expressions and Equations

We start the book by revisiting the two fundamental ideas that underpin algebra: expressions and equations. The distinction between the two is fundamental to understanding algebra—and to choosing the appropriate manipulation. After introducing each type of function, we study the types of expressions and the types of equations it generates. We pay attention to the meaning and purpose of expressions and equations in various contexts. On these foundations we proceed to study how each type of function is used in mathematical modeling.

Achieving Algebraic Power: Strategic Competence

We help students use algebra effectively by giving them practice in identifying the manipulation needed for a particular purpose. For each type of function, we give problems about recognizing algebraic forms and understanding the purpose of each form.

Functions and Modeling

Students who have a grasp of both the basic skills and the basic ideas of algebra, and a strategic sense of how to deploy them, discover a new confidence in applying their knowledge to the natural and social sciences. Mastery of algebra enables students to attack the multi-step modeling problems that we supply for each type of function.

Technology and Pedagogy

The classes who used the preliminary edition of this book were taught in a variety of pedagogical styles and a mix of lecture and discussion. The students in these classes used a range of technologies, from none to computer algebra systems. The emphasis on understanding enables this book to be used successfully with all these groups.

Student Background

We expect students who use this book to have completed high school algebra. Familiarity with basic manipulations and functional forms enables students to build on their knowledge and achieve fluency.

After completing this course, students will be well-prepared for precalculus, calculus, and other subsequent courses in mathematics and other disciplines. The focus on interpreting algebraic form, supported by graphical and numerical representations, enables students to obtain a deeper understanding of the material. Our goal is to help bring students' understanding of mathematics to a higher level. Whether students go on to use "reform" texts, or more traditional ones, this knowledge will form a solid foundation for their future studies.

Content

This content represents our vision of how algebra can be taught.

Chapter 1: The Key Concepts of Algebra

In this chapter we look at the the basic ideas of expression and equation, and at the difference between them. We discuss the underlying principles for transforming expressions, and we construct, read, and analyze examples of expressions and equations.

Chapter 2: Rules for Expressions and the Reasons for Them

This chapter reviews the rules for manipulating expressions that flow from the basic rules of arithmetic, particularly the distributive law, which provides the underlying rationale for expanding, factoring, combining like terms, and many of the manipulations of algebraic fractions.

Chapter 3: Rules for Equations and the Reasons for Them

This chapter reviews the rules for transforming equations and inequalities, laying the foundation for the more complicated methods of solving equations that are covered in later chapters. It also covers equations involving absolute values.

Chapter 4: Functions, Expressions, and Equations

In this chapter we consider functions defined by algebraic expressions and how equations arise from functions. We consider other ways of describing functions—graphs, tables, and verbal descriptions—that are useful in analyzing functions. We look at the average rate of change and conclude the chapter with a discussion of proportionality as an example of modeling with functions.

Chapter 5: Linear Functions, Expressions, and Equations

In this chapter we introduce functions that represent change at a constant rate. We consider different forms for linear expressions and what each form reveals about the function it expresses. We see how linear equations in one and two variable arise in the context of linear functions. We use linear functions to model data and applications. We conclude the chapter with a discussion of systems of linear equations.

Chapter 6: Rules for Exponents and the Reasons for Them

This chapter reviews the rules for exponents, including fractional exponents, and rules for manipulating expressions involving radicals.

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Chapter 7: Power Functions, Expressions, and Equations

Power functions express relationships in which one quantity is proportional to a power of another. We relate the basic graphical properties of a power function to the properties of the exponent and use the laws of exponents to put functions in a form where the exponent can be clearly recognized. We consider equations involving power functions and conclude with applications that can be modeled by power functions.

Chapter 8: More on Functions

In this chapter we use what we have learned about analyzing algebraic expressions to study functions in more depth. We consider the possible inputs and outputs of functions (domain and range), see how functions can be built up from, and decomposed into, simpler functions, and consider how to construct inverse functions by reversing the operations from which they are made up.

Chapter 9: Quadratic Functions, Expressions, and Equations

We start this chapter by looking at quadratic functions and their graphs. We then consider the different forms of quadratic expressions—standard, factored, and vertex form—and show how each form reveals a different property of the function it defines. We consider two important techniques for solving quadratic equations: completing the square and factoring. The first technique leads to the quadratic formula.

Chapter 10: Exponential Functions, Expressions, and Equations

In this chapter we consider exponential functions such as 2^x and 3^x, in which the base is a constant and the variable is in the exponent. We show how to interpret different forms of exponential functions. For example, we see how to interpret the base to give the growth rate and how to interpret exponents in terms of growth over different time periods. We then look at exponential equations. Although they cannot be solved using the basic operations introduced so far, we show how to find qualitative information about solutions and how to estimate solutions to exponential equations graphically and numerically. We conclude with a section on modeling with exponential functions.

Chapter 11: Logarithms

In this chapter we develop the properties of logarithms from the properties of exponents and use them to solve exponential equations. We explain that logarithms do for exponential equations what taking roots does for equations involving powers: They provide a way of isolating the variable so that the equation can be solved. We consider applications of logarithms to modeling, and conclude with a section on natural logarithms and logarithms to other bases.

Chapter 12: Polynomials

In this chapter, as in the chapter on quadratics, we consider the form of polynomial expressions, including the factored form, and study what form reveals about different properties of polynomial functions. We conclude with a section on the long-run behavior of polynomials. If desired, Chapters 12 and 13 could be taught immediately after Chapter 9.

Chapter 13: Rational Functions

In this chapter we look at the graphical and numerical behavior of rational functions on both large and small scales. We examine the factored form and the quotient form of a rational function, and consider horizontal, vertical, and slant asymptotes.

Chapter 14: Summation Notation

This brief chapter of one section introduces subscripted variables and summation notation in preparation for the following three chapters.

Chapter 15: Sequences and Series

In this chapter we consider arithmetic and geometric sequences and series, and their applications. We also look briefly at recursively defined sequences.

Chapter 16: Matrices and Vectors

This chapter introduces matrices and vectors from an algebraic point of view, using concrete examples to motivate matrix multiplication and multiplication of vectors by matrices. We cover the use of matrices in describing linear equations and discuss the purpose of echelon form, concluding with an introduction to row reduction.

Chapter 17: Probability and Statistics

In this chapter we discuss the mean and standard deviation as ways of describing data sets, and provide a brief introduction to concepts of probability, including a discussion of conditional probability and independence.

Changes since the Preliminary Edition

In response to reviewer comments and suggestions, we have made the following changes since the preliminary edition.

- Added four new chapters on summation notation, sequences and series, matrices, and probability and statistics.
- Split the initial chapter reviewing basic skills into three shorter chapters on rules and the reasons for them.
- Added Solving Drill sections at the end of the chapters on linear, power, and quadratic functions. These sections provide practice solving linear, power, and quadratic equations.
- Added material on radical expressions to Chapter 6, the chapter on the exponent rules.
- Added material on solving inequalities, as well as absolute value equations and inequalities, to Chapter 3, the chapter on rules for equations.
- Added more exercises and problems throughout.

Supplementary Materials

The following supplementary materials are available for the First Edition:

- **Instructor's Manual** (ISBN 978-0470-57088-3) contains information on planning and creating lessons and organizing in-class activities. There are focus points as well as suggested exercises, problems and enrichment problems to be assigned to students. This can serve as a guide and check list for teachers who are using the text for the first time.
- **Instructor's Solution Manual** (ISBN 978-0470-57258-0) with complete solutions to all problems.
- **Student's Solution Manual** (ISBN 978-0471-71336-4) with complete solutions to half the odd-numbered problems.
- **Student's Study Guide** (ISBN 978-0471-71334-0) with key ideas, additional worked examples with corresponding exercises, and study skills.

ConcepTests

ConcepTests (ISBN 978-0470-59253-3), modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Our evaluation data show students taught with ConcepTests outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems. A new supplement to *Algebra: Form and Function*, containing ConcepTests by section, is available from your Wiley representative.

About the Calculus Consortium

The Calculus Consortium was formed in 1988 in response to the call for change at the "Lean and Lively Calculus" and "Calculus for a New Century" conferences. These conferences urged mathematicians to redesign the content and pedagogy used in calculus. The Consortium brought together mathematics faculty from Harvard, Stanford, the University of Arizona, Southern Mississippi, Colgate, Haverford, Suffolk Community College and Chelmsford High School to address the issue. Finding surprising agreement among their diverse institutions, the Consortium was awarded funding from the National Science Foundation to design a new calculus course. A subsequent NSF grant supported the development of a precalculus and multivariable calculus curriculum.

The Consortium's work has produced innovative course materials. Five books have been published. The first edition Calculus was the most widely used of any first edition calculus text ever; the precalculus book is currently the most widely used college text in the field. Books by the Consortium have been translated into Spanish, Portuguese, French, Chinese, Japanese, and Korean. They have been used in Australia, South Africa, Turkey, and Germany.

During the 1990s, the Consortium gave more than 100 workshops for college and high school faculty, in addition to numerous talks. These workshops drew a large number of mathematicians into the discussion on the teaching of mathematics. Rare before the 1990s, such discussions are now part of the everyday discourse of almost every university mathematics department. By playing a major role in shaping the national debate, the Consortium's philosophy has had widespread influence on the teaching of mathematics throughout the US and around the world.

During the 1990s, about 15 additional mathematics faculty joined the Consortium. The proceeds from royalties earned under NSF funding were put into a non-profit foundation, which supported efforts to improve the teaching of mathematics.

Since its inception, the Calculus Consortium has consisted of members of high school, college, and university faculty, all working together toward a common goal. The collegiality of such a disparate group of instructors is one of the strengths of the Consortium.

Acknowledgments

We would like to thank the many people who made this book possible. First, we would like to thank the National Science Foundation for their trust and their support; we are particularly grateful to Jim Lightbourne and Spud Bradley.

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To Students: How to Learn from this Book

- This book may be different from other mathematics textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the *meaning* (in algebraic, practical, graphical or numerical terms) of the symbols you are using. There is more emphasis on the interpretation of expressions and equations than you may expect. You will often be asked to explain your ideas in words.

  Why does the book have this emphasis? Because *understanding* is the key to being able to remember and use your knowledge in other courses and other fields. Much of the book is designed to help you gain such an understanding.
- The book contains the main ideas of algebra written in plain English. It was meant to be read by students like yourself. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.
- There are few examples in the text that are exactly like the homework problems, so homework problems can't be done by searching for similar-looking "worked out" examples. Success with the homework will come by grappling with the ideas of algebra.
- Many of the problems in the book are open-ended. This means that there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common sense ideas that are not stated in the problem explicitly but which you know from everyday life.
- The following quote from a student may help you understand how some students feel. "I find this course more interesting, yet more difficult. Some math books are like cookbooks, with recipes on how to do the problems. This math requires more thinking, and I do get frustrated at times. It requires you to figure out problems on your own. But, then again, life doesn't come with a cookbook."
- This book attempts to give equal weight to three skills you need to use algebra successfully: interpreting form and structure, choosing the right form for a given application, and transforming an expression or equation into the right form. There are many situations where it is useful to look at the symbols and develop a strategy before going ahead and "doing the math."
- Students using this book have found discussing these problems in small groups helpful. There are a great many problems which are not cut-and-dried; it can help to attack them with the other perspectives your classmates can provide. Sometimes your teacher may organize the class into groups to work together on solving some of the problems. It might also be helpful to work with other students when doing your homework or preparing for exams.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of algebra as well as a real sense of how mathematics is used in the age of technology.

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  - Algorithmic graphing problems in WileyPLUS allow students to draw lines and curves on graphs to answer graphing problems, simulating paper-and-pencil homework. This provides students with extensive graphing practice to improve their course grade.

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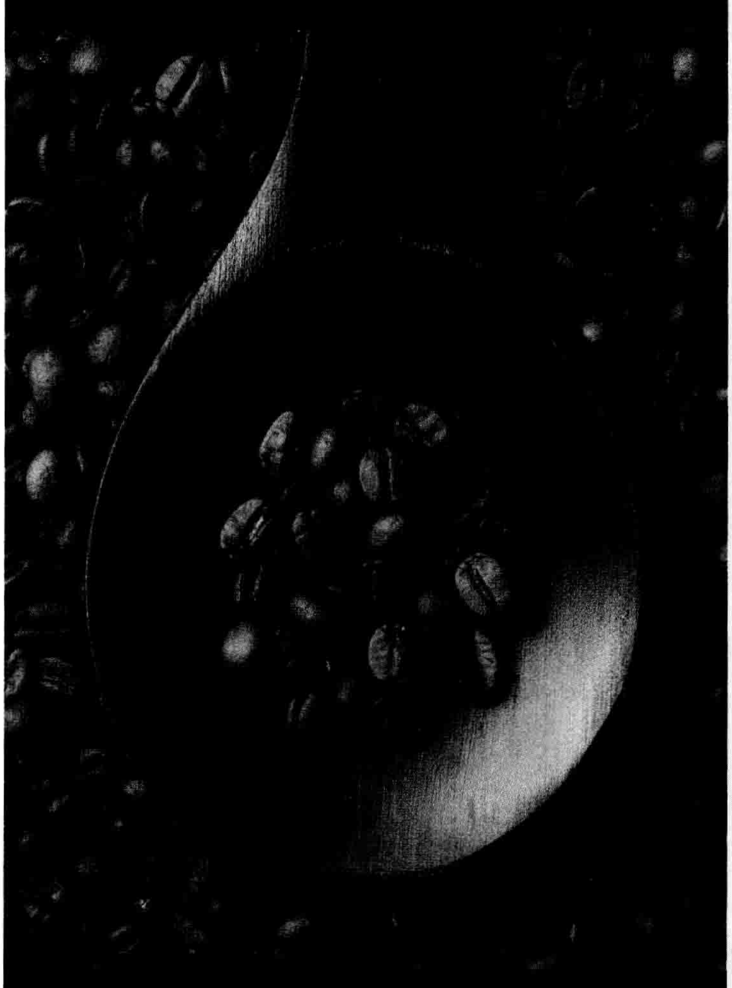
Chapter 1

The Key Concepts of Algebra



CONTENTS

1.1 EXPRESSIONS

An *algebraic expression* is a way of representing a calculation, using letters to stand for numbers. For example, the expression

$$\pi r^2 h,$$

which gives the volume of a cylinder of radius r and height h, describes the following calculation:

- square the radius
- multiply by the height
- multiply by π.

Notice that the verbal description is longer than the expression and that the expression enables us to see in compact form all the features of the calculation at once.

Example 1 (a) Describe a method for calculating a 20% tip on a restaurant bill and use it to calculate the tip on a bill of \$8.95 and a bill of \$23.70.
(b) Choosing the letter B to stand for the bill amount, represent your method in symbols.

Solution (a) Taking 20% of a number is the same as multiplying it by 0.2, so

$$\text{Tip on } \$8.95 = 0.2 \times 8.95 = 1.79 \text{ dollars}$$
$$\text{Tip on } \$23.70 = 0.2 \times 23.70 = 4.74 \text{ dollars.}$$

(b) The tip on a bill of B dollars is $0.2 \times B$ dollars. Usually in algebra we leave out the multiplication sign or represent it with a dot, so we write

$$0.2B \quad \text{or} \quad 0.2 \cdot B.$$

We call the letters r, h, and B *variables*, because they can stand for various different numbers.

Evaluating Algebraic Expressions

If we give particular values to the variables, then we can find the corresponding value of the expression. We call this *evaluating* the expression.

Example 2 Juan's total cost for 5 bags of chips at \$$c$ each and 10 bottles of soda at \$$s$ each is given by

$$\text{Total cost} = 5c + 10s.$$

If a bag of chips costs \$2.99 and a bottle of soda costs \$1.29, find the total cost.

Solution We have $c = 2.99$ and $s = 1.29$, so

$$\text{Total cost} = 5c + 10s = 5 \cdot 2.99 + 10 \cdot 1.29 = 27.85 \text{ dollars.}$$

Usually, changing the value of the variables changes the value of the expression.

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