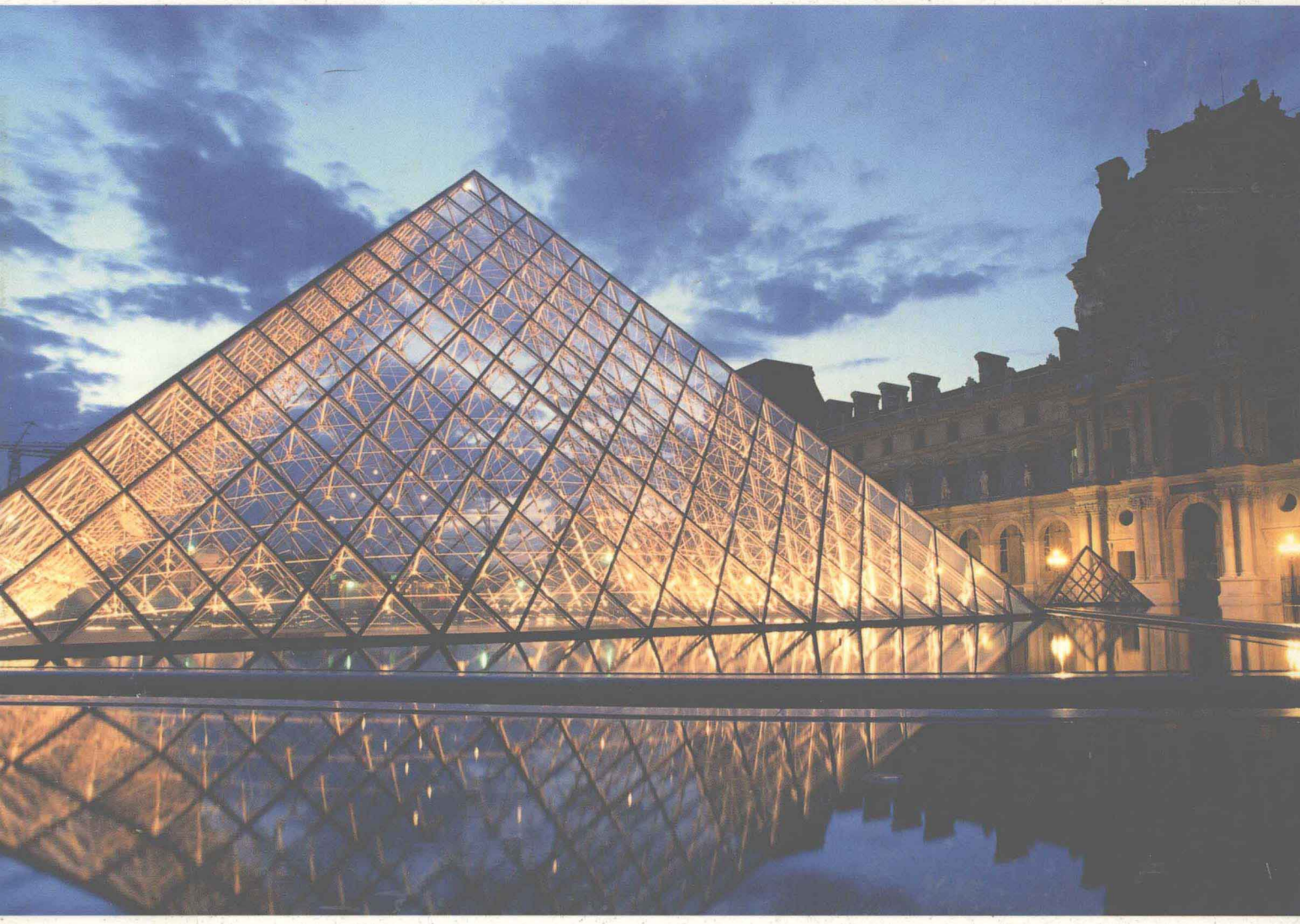
Single Variable Calculus

SECOND EDITION



Gerald L. Bradley ▲ Karl J. Smith

Single Variable CALCULUS

SECOND EDITION

GERALD L. BRADLEY

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PRENTICE HALL

Upper Saddle River, New Jersey 07458

Library of Congress has cataloged another edition as follows:

Bradley, Gerald L. (date)
Calculus/Gerald L. Bradley, Karl J. Smith. — 2nd ed.
p. cm.
Includes index.
ISBN 0-13-660135-9
1. Calculus. I. Smith, Karl J. II. Title.
QA303.B88218 1999 98-18962
515--dc21 CIP

Acquisitions Editor: George Lobell
Editor-in-Chief: Jerome Grant
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Interior Designer: Geri Davis, The Davis Group, Inc.
Cover Designer: Joseph Sengotta
Cover Photo: Louvre Museum and Pyramid. Tom Craig/FPG International
Photo Researcher: Beth Boyd
Photo Research Administrator: Melinda Reo
Art Studio: Monotype Composition
Historical Quest Portraits: Steven S. Nau
Copy Editor: Susan Reiland

Simon & Schuster/A Viacom Company
Upper Saddle River, New Jersey 07458

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-639279-2

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
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Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

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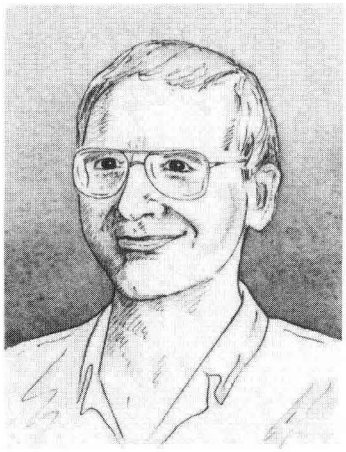

Gerald L. Bradley

Karl J. Smith

Gerald L. Bradley received his B.S. degree in mathematics from Harvey Mudd College in 1962, as a member of the second full graduating class, and his Ph.D. from the California Institute of Technology in 1966. A former NSF and Woodrow Wilson Fellow, Professor Bradley has taught at Claremont McKenna College since 1966, and twice has served as chairman of the mathematics department. His primary field of interest and research is matrix theory, and he is the author of *A Primer of Linear Algebra*, also with Prentice Hall. He has a strong interest in the development of undergraduate mathematics, and has co-authored a top-selling business calculus text as well as a basic text in finite mathematics. His personal interests include history, archaeology, and bridge, but he spends most of his spare time with his passion: writing science fiction novels.

Karl J. Smith received his B.A. and M.A. (in 1967) degrees in mathematics from UCLA. Then he moved in 1968 to northern California to teach at Santa Rosa Junior College, where he has been ever since. Along the way, he served as department chair, and he received a Ph.D. in 1979 in mathematics education at Southeastern University. A past president of the American Mathematical Association of Two-Year Colleges, Professor Smith is very active nationally in mathematics education. He was founding editor of *Western AMATYC News*, a chairperson of the committee on Mathematics Excellence, and a NSF grant reviewer. He was a recipient in 1979 of an Outstanding Young Men of America Award, in 1980 of an Outstanding Educator Award, and in 1989 of an Outstanding Teacher Award. Professor Smith is the author of several successful textbooks. In fact, over one million students have learned mathematics from his textbooks.

Foreword

Over the years I have seen and used a number of calculus texts. I have not always viewed "improvements" in these as real improvements, at least for my students. However, when I was asked to take a look at the manuscript for the 2nd edition of the Bradley/Smith text, I was struck by how readable it is and how basically friendly. It is by no means a "once-over-lightly" treatment of calculus, but at the same time it does not wear out the reader with excessive detail and mind-numbing rigor. This is, after all, a text intended for a course at a level that does not require all that much rigor. The explanations are careful without being pedantic and fussy, well illustrated with examples without exhausting the reader with an overabundance, and uncluttered with alternate ways of doing things.

In particular, I like the historical-biographical essays with associated projects and the calculator/computer projects, all placed where the instructor can use them or not, depending on taste or what the length of the term will allow. Some are provocative and open-ended, allowing for additional time to be spent by students who are ready for more of a challenge. In fact, this expandability would even allow the use of the text in an honors section while the text is also used for regular sections, thus allowing students to transfer in and out of honors sections during the calculus sequence.

The applications are interesting and wide-ranging. A number of them were new to me. Yet I was convinced that they would be understandable to students. In my experience with a number of recent calculus texts, some applications are so far from the experience of most of my students that they end up being mainly frustrating, rather than beneficial. Such texts also use language students don't know and ideas from other fields in which students have no background. This is not the case with Bradley/Smith.

The historical and biographical sketches, along with associated projects involving further reading and exploration, lend a humane touch to a subject this is often viewed by the unconverted as cold, mechanical, and not the work of real human beings. The authors here have done a nice job of making it clear that the calculus as we know it has been developed over many years by a variety of investigators. It is the work of some of the best scientific minds of history. And if the subject does not come easily and without help, it is not surprising. The subject is clear to us now but it is the result of centuries of human effort and ingenuity. Calculus remains one of the greatest of scientific achievements, not only a useful subject but one that forms a profound and cohesive body of knowledge. It's worth the effort it takes to master it.

Gerald L. Alexanderson
Santa Clara University

Preface

Calculus teaching is undergoing great changes, most of which will be of lasting benefit to students. We applaud teaching by using numerical, graphical, and analytical approaches to important concepts. The goal is clear student understanding of concepts, not simply the ability to "manipulate symbols." You will find this book rich with tables, graphs, and algebraic characterizations of each main concept. Our goal in writing this text was to blend the best aspects of calculus reform with the reasonable goals and methodology of traditional calculus. In incorporating so much of calculus reform, we made a deliberate effort not "to throw the baby out with the bath water." Calculus should not be a terminal course, but rather one that prepares students in engineering, science, math, and other related areas to move on to more advanced and necessary career or professional courses. Unlike some reform books, this text addresses topics such as continuity, the mean value theorem, l'Hopital's rule, parametric equations, polar coordinates, sequences, and series. In short, this text is an attempt at Reform with Reason.

The acceptance and response from our first edition has been most gratifying. In spite of the fact that many professors are reluctant to adopt a first edition textbook, we found widespread acceptance for our book and appreciate the many suggestions we received. With this second edition, we checked and rechecked the accuracy of the text material, and have taken extraordinary effort to ensure the accuracy of the answers. We incorporated many of your suggested improvements:

- This edition correctly reflects the precalculus mathematics being taught at most colleges and universities. We assume knowledge of the trigonometric functions, as well as e^x and $\ln x$. These functions are reviewed in Chapter 1. We also assume a knowledge of the conic sections and their graphs.
- It is possible to begin the course with Chapter 1 or Chapter 2 (where the calculus topics begin).
- Modeling was added as a major theme in this edition. Modeling is introduced in Section 2.1, and then appears in almost every section of the book. These applications are designed MODELING PROBLEMS.
- We have taken the introduction of differential equations seriously. Students in many allied disciplines need to use differential equations early in their studies, and consequently cannot wait for a post calculus

course. In this edition, we introduce differential equations in a natural and reasonable way. Slope fields are introduced as a geometric view of antidifferentiation in Section 5.1, and then are used to introduce a graphical solution to differential equations in Section 5.6. We consider separable differential equations in Chapter 5 and first-order linear equations in Chapter 7, and demonstrate the use of both modeling a variety of applied situations. Exact and homogeneous differential equations appear in Chapter 15, along with an introduction to second-order linear equations. The "early and often" approach to differential equations is intended to illustrate their value in continuous modeling and to provide a solid foundation for further study.

- This edition offers an early presentation of transcendental functions: logarithmic, exponential, and trigonometric functions are heavily integrated into all chapters of this book (especially Chapters 1–5).
- We continue to exploit the *humanness* of mathematics, but instead of simply including *Historical Notes*, as we did in the first edition, we have transformed these notes into *problems* that lead the reader from the development of a concept to actually participating in the discovery process. The problems are designated as Historical Quest problems. The problems are not designed to be "add-on or challenge problems," but rather are intended to become an integral part of the usual assignment. The level of difficulty of Quest problems ranges from easy to difficult. An extensive selection of biographies of noted mathematicians can be found on the Internet site accompanying this text.
- We are aware that most calculus books grow "bigger and bigger" as they progress from one edition to the next. As they grow, they add "more features" and become more and more expensive for the student. We are determined that this does not happen with this book. In an effort to keep down costs, we made the decision that a full color edition, while visually appealing, does not add to *mathematical* understanding enough to justify the added cost. Consequently, this edition uses color functionally as a pointer and not as a decoration. We ask for your feedback regarding our decision.

The major issue driving calculus reform is the poor performance of students trying to master the concepts of calculus. Much of this failure can be attributed to the way most students learn mathematics in high school, which often involves stressing rote memorization over insight and understanding. On the other hand, some reform texts are perceived as spending so much time with the development of insight and understanding that students are not given enough exposure to important computational and problem-solving skills to perform well in more advanced courses. In our view, it is equally wrong to foster a situation in which the student understands too little about a lot or a lot about too little. This text aims at a middle ground by providing sound development, stimulating problems, and well-developed pedagogy within a framework of a traditional topic structure. "Think then do" is a fair summary of our approach.

Conceptual Understanding Through Verbalization

Besides developing some minimal skills in algebraic manipulation and problem solving, today's calculus text should require students to cul tivate verbal skills in a mathematical setting. This is not just because real mathematics wields its words precisely and compactly, but because verbalization should help students think conceptually.

COOPERATIVE LEARNING (GROUP RESEARCH PROJECTS) In July 1991, the National Science Foundation funded a group of instructors who met on the campus of New Mexico State University to discuss the topic, "Discovering Calculus through Student Projects." The participants concluded that encouraging students to work on significant projects in small groups acts as a counterbalance to traditional lecture methods and can serve to foster both conceptual understanding and the development of technical skills. However, many instructors still believe that mathematics can be learned only through independent work. We feel that independent work is of primary importance, but that students must also learn to work with others in group projects. After all, an individual's work in the "real world" is often done as part of a group and almost always involves solving problems for which there is no answer "in the back of the book." In response to this need, we have included challenging exercises (the Journal and Putnam problems) and a number of group research projects, each of which appears at the end of a chapter and involves intriguing questions whose mathematical content is tied loosely to the chapter just concluded. These projects have been developed and class-tested by their individual authors, to whom we are greatly indebted. Note that the complexity of these projects increases as we progress through the book, and the mathematical maturity of the student is developed.

MATHEMATICAL COMMUNICATION We have included several opportunities for mathematical communication in terms that can be understood by nonprofessionals. The guest essays provide alternate viewpoints. The questions that follow are called MATHEMATICAL ESSAYS and are included to encourage individual writing assignments and mathematical exposition. We believe that students will benefit from individual writing and research in mathematics. Shorter problems encouraging written communication are included in the problem sets and are designed by the logo WHAT DOES THIS SAY? Another pedagogical feature is the "**What this says:**" box in which we rephrase mathematical ideas in everyday language. In the problem sets we encourage students to summarize procedures, processes, or to describe a mathematical result in everyday terms. Concept problems are found throughout the book as well as at the end of each chapter, and these problems, as well as the **mathematical essay** problems, are included to prove that mathematics is more than "working problems and getting answers." Mathematics education *must* include the communication of mathematical ideas.

Integration of Technology

COMPUTATIONAL WINDOWS Reform is driven partly by the need to embrace the benefits technology brings to the learning of mathematics. Simply adding a lab course to the traditional calculus is possible, but this may lead to unacceptable work loads for all involved. We choose to include technology as an aid to the understanding of calculus, rather than to write a calculus course developed around the technology. While we have included over 130 windows devoted to the use of technology, we strive to keep such references "platform neutral" because specific calculators and computer programs frequently change and are better considered in separate technology manuals. The technology in the book is organized under the title "TECHNOLOGY WINDOWS" to give insight into how technological advances can be used to help understand calculus. TECHNOLOGY WINDOWS also appear in the exercises and involve problems requiring a graphing calculator or software and computer. On the other

sight into the type of problems that are asked in mathematical competitions. The annual national competition is given under the auspices of the Mathematical Association of America. The problems are designed to recognize mathematically talented college and university students.

THINK TANK PROBLEMS It has been said that mathematical discovery is directed toward two major goals—the formulation of proofs and the construction of counterexamples. Most calculus books focus only on the first goal (the body of proofs and true statements), but we feel that some attention should be paid to the formulation of counterexamples for false statements. Throughout this book we ask the student to formulate an example satisfying certain conditions. We have designated this type of problem as a *think tank* problem.

Topics

STUDENT MATHEMATICS HANDBOOK The content of this text adapts itself to either semester or quarter systems, and both differentiation and integration can be introduced in the first course. We begin calculus with a minimum of review. The prerequisite material which is often included in a calculus textbook has been bound separately in a companion book, *Student Mathematics Handbook*. Our handbook is offered *free of charge* with every *new* copy of the textbook. The Handbook not only includes the necessary review material and formulas, but also contains a catalog of curves and a complete integral table. We feel this is an important supplement to the textbook because we have found that the majority of errors our students make in a calculus class are not errors in calculus, but errors in basic algebra and trigonometry. A unified and complete treatment of this prerequisite material, easily referenced and keyed to the textbook, has been a valuable tool for our students taking calculus. Those portions of the text that benefit from an appropriate precalculus review are marked by the symbol SMH

SEQUENCE OF TOPICS We resisted the temptation to label certain sections as optional, because that is a prerogative of individual instructors and schools. However, the following sections could be skipped without any difficulty: 4.7, 5.9, 6.5 (delay until Sec. 13.6), 7.8, 12.8, and 13.8. To assist instructors with the pacing of the course, we have written the material so each section reasonably can be covered in one classroom day, but to do so requires that the students read the text in order to tie together the ideas which might be discussed in a classroom setting.

PROOFS One of the trends in the move to reform calculus is to minimize the role of a mathematical theorem. We do not agree with this aspect of reform. Precise reasoning has been, and we believe will continue to be, the backbone of good mathematics. While never sacrificing good pedagogy and student understanding, we present important results as *theorems*. We do not pretend to prove every theorem in this book; in fact we often only outline the steps one would take in proving a theorem. We refer the reader to Appendix B for certain longer proofs, or sometimes to an advanced calculus text. Why then do we include the heading "PROOF" after each theorem? It is because we want the student to know for a result to be a theorem there must be a proof. We use the heading not necessarily to give a complete proof in the text, but to give some direction to where a proof can be found, or an indication of how it can be constructed.

Supplementary Materials

SSM *Student Mathematics Handbook and Integration Table for CALCULUS* by Karl J. Smith offers a review of prerequisite material, a catalog of curves, and a complete integral table. This handbook is presented free of charge along with the purchase of a new book.

SMH A *Student Survival and Solutions Manual* by Karl J. Smith offers a running commentary of hints and suggestions to help ensure the students' success in calculus. Since this manual is written by one of the authors of this text, the solutions given in the manual complement all of the procedures and development of the textbook. The problem numbers in the book shown in color indicate the solutions included in the *Survival Manual.*

Technology Manuals by John Gresser offers computer applications keyed to the sections in the book. These manuals are identical except for the specific keystrokes on TI calculators, *Mathematica*, and *Maple*. There are similar manuals for the HP and MATLAB by Frank Hagin and Jack Cohen.

A Complete Solutions Manual by Karl J. Smith contains a brief solution for every problem in the book.

An Answer Book by Karl J. Smith contains only the answers to most problems in the book.

An Instructor's Guide offers sample tests and reviews for each chapter in the book. This guide also includes sample transparencies.

Computerized Computer Testing Program is available in both IBM and Macintosh formats.

Resources for Calculus, Volumes 1-5, A. Wayne Roberts (Project Director) available from the Mathematical Association of America, 1993:

Dudley, Underwood (ed), *Readings for Calculus*, MAA Notes, No. 28

Fraga, Robert (ed), *Calculus Problems for a New Century*, MAA Notes, No. 28

Jackson, Michael B., and Ramsay, John (eds), *Problems for Student Investigation*, MAA Notes, No. 30

Snow, Anita E. (ed), *Learning for Discovery: A Lab Manual for Calculus*, MAA Notes, No. 27

Straffin, Philip (ed), *Applications of Calculus*, MAA Notes, No. 29

This is a valuable collection of resource materials for calculus instructors. All material may be reproduced for classroom use.

Acknowledgments

The writing and publishing of a calculus book is a tremendous undertaking. We take this responsibility very seriously because a calculus book is instrumental in transmitting knowledge from one generation to the next. We would like to thank the many people who helped us in the preparation of this book. First, we thank our editor,

George Lobell, who led us masterfully through the development and publication of this book. We sincerely appreciate Donald Gecewicz, who read and critiqued each word of the manuscript. We also appreciate the work of Barbara Mack in college production, who kept us all on track, and we especially thank Susan Reiland for her meticulous attention to detail.

Of primary concern is the accuracy of the book. We had the assistance of many: Jerry Alexanderson and Mike Ecker, who read the entire manuscript and offered us many valuable suggestions; the accuracy checkers of the first edition, Ken Sydel, Diana Gerardi, Kurt Norlin, Terri Bittner, and Mary Toscano; and of the second edition, Nancy and Mary Toscano. We also would like to thank the following readers of the text for their many suggestions for improvement:

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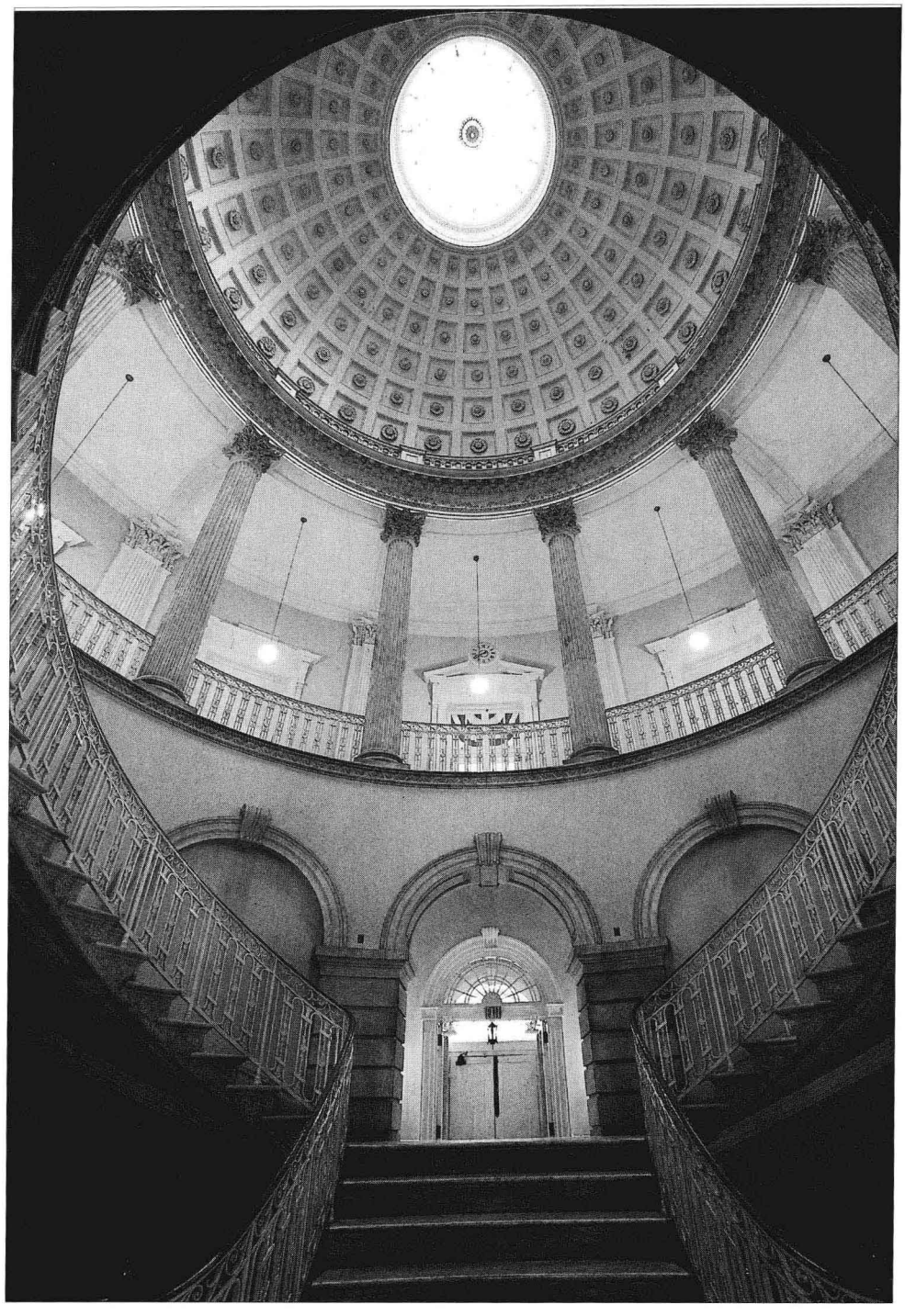
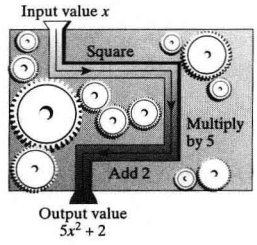


The rotunda of City Hall, in Manhattan, New York City.
Designed by John McComb, Jr., reputedly the first full-time, American-born architect, and built in the early nineteenth century in the neoclassical style. (photo credit: Behrenholtz)

Contents

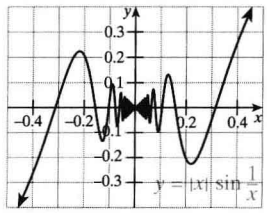
Foreword xiii
Preface xv

CHAPTER 1 ***Functions and Graphs*** ***1***



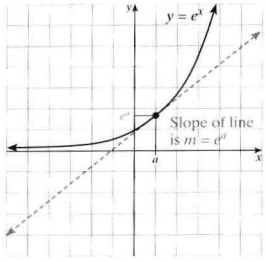
1.1 Preliminaries 2
1.2 Lines in the Plane 15
1.3 Functions 24
1.4 Graphs of Functions 34
1.5 Inverse Functions; Inverse Trigonometric Functions 44
1.6 Exponential and Logarithmic Functions 54
Chapter 1 Review 68

CHAPTER 2 ***Limits and Continuity*** ***73***



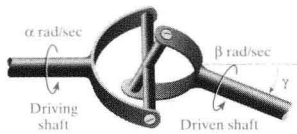
2.1 What Is Calculus? 74
2.2 The Limit of a Function 85
2.3 Properties of Limits 97
2.4 Continuity 107
Chapter 2 Review 117
GUEST ESSAY: "Calculus Was Inevitable," John Troutman 122

CHAPTER 3 Differentiation 127



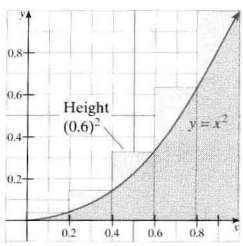
3.1 An Introduction to the Derivative: Tangents 128
3.2 Techniques of Differentiation 143
3.3 Derivatives of Trigonometric, Exponential, and Logarithmic Functions 152
3.4 Rates of Change: Rectilinear Motion 159
3.5 The Chain Rule 169
3.6 Implicit Differentiation 177
3.7 Related Rates and Applications 189
3.8 Linear Approximation and Differentials 197
Chapter 3 Review 209
GROUP RESEARCH PROJECT: Chaos 213

CHAPTER 4 Additional Applications of the Derivative 215



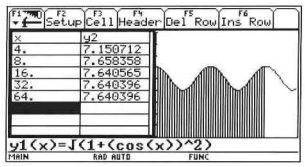
4.1 Extreme Values of a Continuous Function 216
4.2 The Mean Value Theorem 228
4.3 First-Derivative Test 235
4.4 Concavity and the Second-Derivative Test 246
4.5 Curve Sketching: Limits Involving Infinity and Asymptotes 259
4.6 Optimization in the Physical Sciences and Engineering 272
4.7 Optimization in Business, Economics, and the Life Sciences 286
4.8 l'Hôpital's Rule 300
Chapter 4 Review 309
GROUP RESEARCH PROJECT: Wine Barrel Capacity 315

CHAPTER 5 Integration 317



5.1 Antidifferentiation 318
5.2 Area as the Limit of a Sum 329
5.3 Riemann Sums and the Definite Integral 337
5.4 The Fundamental Theorems of Calculus 350
5.5 Integration by Substitution 357
5.6 Introduction to Differential Equations 364
5.7 The Mean Value Theorem for Integrals; Average Value 377
5.8 Numerical Integration: The Trapezoidal Rule and Simpson's Rule 383
5.9 An Alternative Approach: The Logarithm as an Integral 391
Chapter 5 Review 396
GUEST ESSAY: "Kinematics of Jogging," Ralph Boas 401

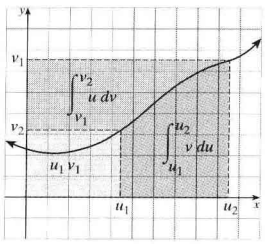
CHAPTER 6



Additional Applications of the Integral 405

6.1 Area Between Two Curves 406
6.2 Volume by Disks and Washers 415
6.3 Volume by Shells 426
6.4 Arc Length and Surface Area 433
6.5 Physical Applications: Work, Liquid Force, and Centroids 441
Chapter 6 Review 454
GROUP RESEARCH PROJECT: Houdini's Escape 460
Cumulative Review Problems for Chapters 1–6 461

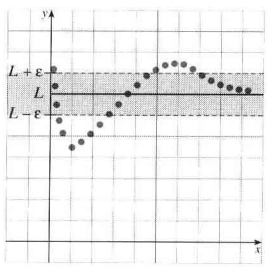
CHAPTER 7



Methods of Integration 465

7.1 Review of Substitution and Integration by Table 466
7.2 Integration by Parts 475
7.3 Trigonometric Methods 481
7.4 The Method of Partial Fractions 488
7.5 Summary of Integration Techniques 499
7.6 First-Order Differential Equations 503
7.7 Improper Integrals 515
7.8 The Hyperbolic and Inverse Hyperbolic Functions 525
Chapter 7 Review 533
GROUP RESEARCH PROJECT: Buoy Design 538

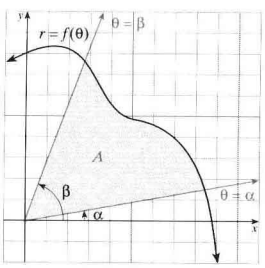
CHAPTER 8



Infinite Series 539

8.1 Sequences and Their Limits 540
8.2 Introduction to Infinite Series: Geometric Series 552
8.3 The Integral Test: p-Series 561
8.4 Comparison Tests 569
8.5 The Ratio Test and the Root Test 575
8.6 Alternating Series; Absolute and Conditional Convergence 582
8.7 Power Series 592
8.8 Taylor and Maclaurin Series 602
Chapter 8 Review 619
GROUP RESEARCH PROJECT: Elastic Tightrope Project 623

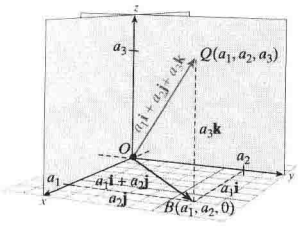
CHAPTER 9



Polar Coordinates and Parametric Forms 625

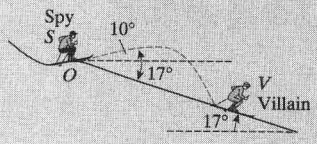
9.1 The Polar Coordinate System 626
9.2 Graphing in Polar Coordinates 631
9.3 Area and Tangent Lines in Polar Coordinates 641
9.4 Parametric Representation of Curves 651
Chapter 9 Review 661
GROUP RESEARCH PROJECT: Security Systems 664

CHAPTER 10 Vectors in the Plane and in Space 667



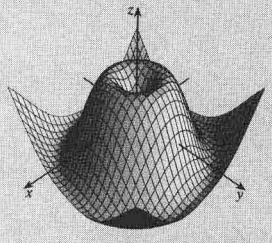
10.1 Vectors in the Plane 668
10.2 Quadric Surfaces and Graphing in Three Dimensions 676
10.3 The Dot Product 686
10.4 The Cross Product 696
10.5 Lines and Planes in Space 705
10.6 Vector Methods for Measuring Distance in $\mathbb{R}^3$ 716
Chapter 10 Review 721
GROUP RESEARCH PROJECT: Star Trek 726

* CHAPTER 11 Vector-Valued Functions 727



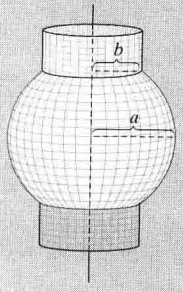
11.1 Introduction to Vector Functions 728
11.2 Differentiation and Integration of Vector Functions 736
11.3 Modeling Ballistics and Planetary Motion 747
11.4 Unit Tangent and Normal Vectors; Curvature 757
11.5 Tangential and Normal Components of Acceleration 770
Chapter 11 Review 777
GUEST ESSAY: "The Stimulation of Science," Howard Eves 781
Cumulative Review Problems for Chapters 7–11 786

CHAPTER 12 Partial Differentiation 789



12.1 Functions of Several Variables 790
12.2 Limits and Continuity 800
12.3 Partial Derivatives 807
12.4 Tangent Planes, Approximations, and Differentiability 817
12.5 Chain Rules 827
12.6 Directional Derivatives and the Gradient 835
12.7 Extrema of Functions of Two Variables 848
12.8 Lagrange Multipliers 861
Chapter 12 Review 871
GROUP RESEARCH PROJECT: Desertification 877

CHAPTER 13 Multiple Integration 879



13.1 Double Integration Over Rectangular Regions 880
13.2 Double Integration Over Non-Rectangular Regions 889
13.3 Double Integration in Polar Coordinates 897
13.4 Surface Area 906
13.5 Triple Integrals 914
13.6 Mass, Moments, and Probability Density Functions 924
13.7 Cylindrical and Spherical Coordinates 935
13.8 Jacobians: Change of Variables 944
Chapter 13 Review 950
GROUP RESEARCH PROJECT: Space-Capsule Design 956

*The shaded Chapters 11–15 appear only in *Multivariable Calculus*. Chapter 10 appears in both *Single Variable Calculus* and *Multivariable Calculus*.