

Controlled Thermonuclear Reactions

L. A. ARTSIMOVICH

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CONTROLLED THERMONUCLEAR REACTIONS

L. A. ARTSIMOVICH

Edited by

A. C. KOLB and R. S. PEASE

Translated by

P. KELLY and A. PEIPERL

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FOREWORD TO THE RUSSIAN EDITION

Amongst the various purposes which the foreword to a book is intended to serve, the main one is commonly that of giving the author an opportunity to mitigate, by means of a properly devised defence, the forthcoming assault of the critics, the elements of this defence being on the one hand a frank admission of the book's defects and, on the other, an irrefutable demonstration that such defects are absolutely unavoidable. Such a form of literary self-defence is particularly necessary when the subject being expounded is a rapidly expanding field of science or technology in the initial phase of its development. This is precisely the present state of research into controlled thermonuclear reactions. The author must therefore begin this book with self-criticism.

At the very outset, the reader must be warned that the title of the book is not a wholly accurate reflection of its contents. The title represents the goal towards which we are striving, but for the present this goal only glimmers in the distance, and a long and difficult road has to be traversed before we get there. What the book describes is the small fraction of the road which has been travelled so far, together with an outline of the sort of obstacles which have yet to be overcome in order to solve the problem before us. The reader should not look here for the numerical and technological details of industrial thermonuclear reactors because at this time there are no such reactors. We find ourselves now at an earlier stage when our primary task is to determine the scientific principles which will underlie the thermonuclear technology of the future.

Thermonuclear reactions occur in a form of matter which is comparatively novel in the laboratory—high-temperature plasma. The studies which have been carried out on the properties of high-temperature plasma constitute the main subject of this book. An account is given of the principal theoretical concepts of the various processes which take place in plasma; methods of heating and of thermally insulating plasma are examined; devices built for carrying out experiments with high-temperature plasma are described, and the results of these experiments are evaluated. Plasma physics is the subject of the first four chapters; the remaining four are devoted to the main trends of experimental research on the problem of producing controlled thermonuclear reactions.

The choice and arrangement of the material have been governed by the belief that a clear understanding of the problems can be obtained without draping the meagre skeleton of experimental facts in a voluminous garment of mathematical elaboration; a rational balance must be maintained between theoretical and experimental information. For this reason the book in general includes only that minimum of theoretical knowledge

needed to guide the reader through the physics of high-temperature plasma. Lengthy derivations of formulae have been omitted; and in all cases where either clarity or pedantry had to be chosen, preference is given to the first.

With rare exceptions the book does not take account of research carried out after the end of 1960. The Gaussian c.g.s. system of units is used unless otherwise stated.

The author expresses deep gratitude to the large staff of the Plasma Research Department at the 'I.V. Kurchatov' Atomic Energy Institute of the U.S.S.R. Academy of Sciences, for the very valuable and friendly help they gave in preparing the book by making available experimental data, calculations, graphs and photographs, and by providing much valuable advice and comment.

Special mention must be made of the help given by R. Z. Sagdeev: § 4.7 of the book is basically his work, and also the greater part of § 7.7. The author is indebted to V. M. Glagolev for the analysis of the main methods of high-frequency confinement of plasma (§ 8.17 and § 8.18). Without the help of E. V. Artyushkov the publication of this book would probably have been delayed for a long time. It was he who did the very laborious work of preparing the manuscript for printing, and did it with great conscientiousness and skill. In addition he made numerous difficult and detailed calculations, and prepared a large number of the graphs used in the book.

L. A. ARTSIMOVICH

June 1961

FOREWORD TO THE ENGLISH EDITION

The problem of controlled thermonuclear fusion belongs to the border region between science and technology which can be called 'scientific invention'. When we survey the work on nuclear fusion as a whole we find that new ideas born of the creative imagination of inventors are intermingled with the results of theoretical and experimental research on plasma physics, and that usually these ideas have flowed directly from the research. The tremendous importance of the ultimate goal and the enormous difficulties that lie in the way combine to stamp the problem of thermonuclear fusion with a quite distinctive character. For the time being we find ourselves as it were on the dividing line between dream and reality. It is this that brings an element of emotionalism into the study of this problem—the succession of hope and despair, the searching doubts followed by assurance of success.

It is natural that everything should be in motion in such a living field of study and therefore any book on controlled thermonuclear reactions can at best represent only a snapshot photograph which shows the situation as it is for a very short period of time. The present work was published in the U.S.S.R. in the summer of 1961 and the author has therefore introduced a number of amendments and additions to the original Russian text. Major alterations have been made to passages bearing on systems of the theta-pinch type; the sections dealing with toroidal discharges have been supplemented with fresh data; and in respect of plasma stability in magnetic traps, a series of new results is given which has been derived from recent theoretical and experimental studies. These additions do not, of course, cover all the recent developments in this new scientific field, but if this book in its English edition proves useful to young scientists and, in particular, if it can show them that the problem of thermonuclear fusion is not as easy to solve as it sometimes seems at first glance, then the author will consider his task accomplished.

I regard it as my pleasant obligation to thank P. Kelly and A. Peiperl for translating this book, and Drs. R. S. Pease and A. C. Kolb for editing the translation and making a number of corrections which have significantly improved the presentation. I also express thanks to my English and American publishers.

L. A. ARTSIMOVICH

20th September 1962

EDITORS' NOTE

We believe that this book is of value on a number of grounds to those studying controlled thermonuclear reactions and plasma physics: it constitutes a major review of the subject by one of the most distinguished pioneers in the field; it succeeds in presenting difficult ideas in lucid and simple fashion without losing the essential physical content; the extensive research programme in the Soviet Union is authoritatively discussed, including the underlying scientific motivations; and finally it contains independent assessment of much of the research programmes carried out in western countries.

The text used for the translation is basically that of the first Russian edition. Corrections and some extensive additions sent to us by Academician Artsimovich have been added and also, in a few cases, emendations and clarifications suggested by us and discussed with him. The text is now close to, but not identical with, that of the second Russian edition which we received while the present text was in page proof.

The subject does not, in all aspects, possess a widely accepted standard terminology. In editing this translation we have had to make some arbitrary decisions in order to provide as far as possible a self-consistent notation related to that used in already published English-language books. While much of this is self-explanatory, the following points may assist the reader. In discussing devices possessing or treated as possessing cylindrical symmetry, such as the linear (z -) pinch or the θ -pinch, the cylindrical polar coordinates r , θ , z are denoted where necessary radial, azimuthal and axial respectively. Devices with toroidal or topologically toroidal geometry, such as the stabilised toroidal pinch, the Tokomak systems and the Stellarator, may possess two axes to which the cylindrical polar coordinates can be related. If an initially straight tube of circular cross-section is bent into a closed toroid, then the axis about which it is bent (marked z in Figs. 28 and 29) we have called the major axis, and quantities referring to this axis are prefixed major. For example, R of Fig. 29 is the major radius. The circular cross-section—shown in Fig. 30—of the initially straight tube we refer to as the minor cross-section, and quantities pertaining to it are prefixed minor. The locus of the centre of the minor cross-sections forms the centre line of the tube bore. Vector quantities parallel to this centre line we refer to as longitudinal; thus the current J of Fig. 29 is a longitudinal current. In some discussions of topologically toroidal devices, the cylindrical polar coordinate system is used with the z -axis coincident with the centre line of the tube bore (assumed straight). The term 'magnetic axis' is reserved for the line defined at the beginning of Chapter VIII. We have reserved the term

'longitudinal' for the above-mentioned use alone, with the exception that the well-established term 'longitudinal wave' is retained for waves in which the particle motion is parallel to the wave vector.

In the discussion of wave motion in plasmas, we have retained the translated Russian terminology. The English terminology is at present very confused, and the reader can place reliance only on the mathematical description of the mode when comparing the discussion in different texts. The reader is particularly warned that the Russian usage of the terms 'ordinary' and 'extraordinary' is at variance with that of some English-language authors. We have eliminated some, but not all cases of synonymous terms. Thus the term Larmor frequency is not used and the frequency eH/mc is referred to throughout as the cyclotron frequency; on the other hand the heating produced by the collisional resistance to the passage of current is referred to variously as Joule and ohmic heating. We have added a detailed table of contents, a subject index, a number of additional references and a list of the principal symbols.

We wish to thank Academician Artsimovich and a number of our colleagues for discussions which have clarified our understanding of the text; and we are particularly grateful to J. Fortna and J. D. Jukes for reading the entire proofs of the translation. We are very grateful to P. Kelly and A. Peiperl, and to Messrs. Oliver & Boyd and Messrs. Gordon and Breach for their cooperation in producing this translation.

A. C. KOLB

R. S. PEASE

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INTRODUCTION

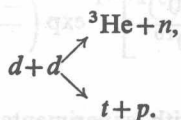
§ 1.1 Thermonuclear reactions arise from collisions between the fast atomic nuclei in matter heated to a very high temperature. If the colliding nuclei have sufficiently large relative speed, they can overcome the potential barrier of mutual electrostatic repulsion, approach very closely to one another and react. When the temperature T is not too high, so that the quantity kT , representing the energy of the random thermal motion of the particles, has a value small compared to the barrier potential, only an insignificant proportion of the thermal collisions will result in nuclear reactions. As T is raised, the thermonuclear reaction rate increases rapidly.

Thermonuclear reactions are, apparently, the main source of stellar energy and must therefore play an important part in astrophysical processes. Inside stars, the temperature and the density are very high. Therefore in stellar matter an intensive process of nuclear synthesis must go on, during which the basic component of matter—hydrogen—is converted into helium by a succession of nuclear fusion reactions, and a vast amount of energy is released.

It is natural that the idea of producing similar reactions here on earth, in order to make use of the power from them, should have been current for many years. The problem of how to do this has now come to the forefront of nuclear energy research. Its solution will provide access to the well-nigh inexhaustible energy resources of the light elements—energy which can be released by thermonuclear fusion processes at very high temperatures, but cannot be obtained in surplus by any other known method.

From the point of view of both scientific research and practical applications, the greatest interest attaches to producing nuclear fusion in deuterium or in a mixture of deuterium and tritium. Here only the minimum temperature, relatively speaking, is required in order to obtain intense thermonuclear reactions.

The reactions in deuterium can take place in two ways, as follows:



The two branches of the reaction have almost equal probability. The energy released is 3.3 MeV for the reaction giving a neutron and 4.0 MeV for that giving a proton.

Figure 1 shows the total cross-section for the $d-d$ reaction as a function of the deuteron energy W_d (in the co-ordinate system where one deuteron is at rest). When $W_d \leq 1.5 \times 10^5$ eV the variation of this cross-section can be expressed with sufficient accuracy by the formula

$$\sigma = (2.4 \times 10^{-19}/W_d) \exp(-1.4 \times 10^3/W_d^{1/2}); \text{ cm}^2, W_d \text{ in eV.} \quad \text{.....(1.1)}$$

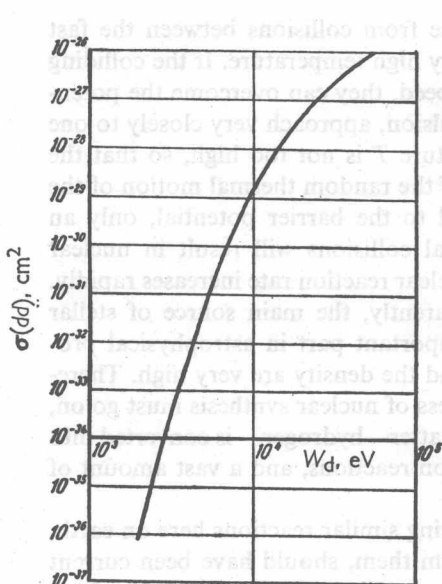


FIG. 1. Dependence of the total $d-d$ reaction cross-section on deuteron energy

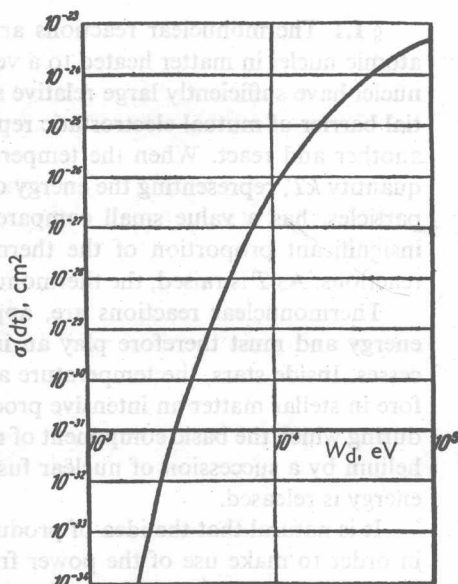


FIG. 2. Dependence of the $d-t$ reaction cross-section on deuteron energy

In a mixture of deuterium and tritium, the reaction



takes place, liberating 17.6 MeV of energy; 80% of this energy, i.e. about 14.1 MeV, is acquired by the neutron. The cross-section as a function of deuteron energy (the tritium nucleus is assumed to be at rest) is shown in Fig. 2. The cross-section for this reaction is given approximately by the formula

$$\sigma = (6 \times 10^{-17}/W_d) \left[1 + \frac{(W_d - 10^5)^2}{3 \times 10^{10}} \right]^{-1} \exp\left(\frac{-1.5 \times 10^3}{W_d^{1/2}}\right); \text{ cm}^2, W_d \text{ in eV.} \quad \text{.....(1.2)}$$

This is in good agreement with experimental data for energies below about 1 MeV. When $W_d \leq 10^5$ eV the $d-t$ reaction cross-section is about a hundred times greater than the total $d-d$ reaction cross-section. This is because the $d-t$ reaction involves a resonance process.

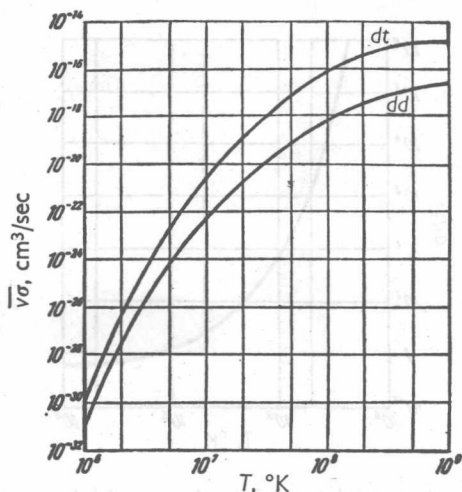
Given the cross-section as a function of particle energy, it is possible to calculate the thermonuclear reaction rate in matter. The number of nuclear reactions taking place in 1 cm^3 during 1 sec is given by the expression

$$g = n_1 n_2 \bar{v}\sigma. \quad \dots\dots(1.3)$$

Here n_1 and n_2 are the numbers of nuclei per cm^3 (number density) of the two reacting species, and $\bar{v}\sigma$ is the product of the relative velocity of the nuclei and the reaction cross-section averaged over the velocity distribution of the nuclei. In calculating the reaction rate in pure deuterium the product $n_1 n_2$ is replaced by $n^2/2$, where n is the number density of deuterium nuclei.

FIG. 3

Mean $\bar{v}\sigma$ values ($\text{cm}^3 \text{ sec}^{-1}$) for $d-d$ and $d-t$ reactions when the distribution of particle energy in the plasma is Maxwellian



Where the value of T is fixed† (i.e. where there is a given value for the mean kinetic energy of the random motion of the particles) the quantity $\bar{v}\sigma$ is still not determined uniquely because it is very sensitive to the particle velocity distribution. Where temperatures are not too high ($T \leq 10^8$), the marked dependence of the cross-section on the relative velocity v results in the major part of the thermonuclear reactions being due to collisions between particles whose energy is several times greater than the mean value of the thermal energy $3/2kT$. Thus the reaction rate depends markedly on the detailed shape of that part of the energy spectrum made up by the small proportion of particles with an energy considerably in excess of the mean value. It may be assumed that where 'randomising' processes take place rapidly compared to all other processes, a Maxwellian velocity distribution will be established. Figure 3 shows values of $\bar{v}\sigma$ for the $d-d$ and $d-t$ reactions in the temperature range 10^6 to 10^9 °K, i.e. from 10^2 to 10^5 eV, for Maxwellian velocity distributions. When the temperatures do not exceed 10^8 °K (10^4 eV), use can also be made of the following formulae to determine the reaction rate:

† Henceforth T will be used to signify temperature measured in °K. Temperature in electron-volts will be designated by θ . As is familiar, $T = 11,610 \theta$.

$$g_{dd} = 7.5 \times 10^{-10} (n^2/T^{\frac{3}{2}}) \exp(-4.25 \times 10^3/T^{\frac{1}{2}}) \text{ reactions cm}^{-3} \text{ sec}^{-1}; \quad \dots\dots\dots(1.4a)$$

$$g_{dt} = 1.6 \times 10^{-7} (n_1 n_2/T^{\frac{3}{2}}) \exp(-4.52 \times 10^3/T^{\frac{1}{2}}) \text{ reactions cm}^{-3} \text{ sec}^{-1}. \quad \dots\dots\dots(1.4b)$$

However, the assumption of a Maxwellian velocity distribution is not by any means justified in all cases of interest to us. When a heating pulse of short duration takes place in a low density plasma, it may well be that there is insufficient time to establish a Maxwellian velocity distribution. This may mean that, although the random motion of the particles can be

defined by a specific temperature (meaning by this a quantity proportional to the mean particle energy), the proportion of par-

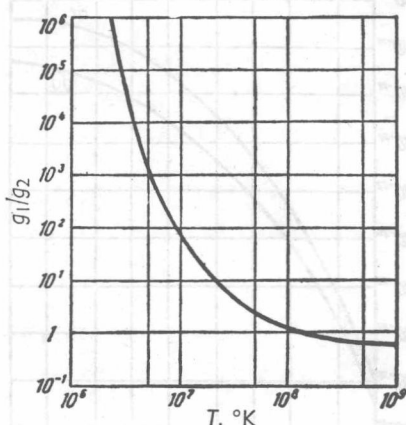


FIG. 4

Ratio of the d - d reaction rates for two different particle velocity distributions

ticles which populate the very high energy part of the energy spectrum is nevertheless negligible. In such a situation the reaction rate is reduced significantly. Figure 4 shows how the d - d reaction rate compares for two different velocity distributions, the temperatures being the same: g_1 corresponds to a Maxwellian distribution, g_2 to the case where all the particles have the same energy. Admittedly the second of these distributions, here selected for purposes of comparison, is not a realistic one; nevertheless the comparative values of g_1 and g_2 illustrate how greatly the shape of the energy spectrum influences the reaction rate. We see that for a temperature of about 10^6 °K a significant departure from a Maxwellian distribution can diminish the nuclear reaction rate by several orders of magnitude. As T increases, the ratio g_1/g_2 decreases until at $T = 10^8$ °K it approaches unity. This change in g_1/g_2 is qualitatively quite understandable: when the energy is high, σ is not nearly so strongly dependent on W_d .

From the above data on thermonuclear reaction rates, it follows that the experimentalist can hope to detect the first signs of nuclear reactions in heated matter only when the temperature reaches something like a million degrees. If thermonuclear processes are to be of any practical value for energy production, a considerably higher temperature—hundreds of millions of degrees—is required.