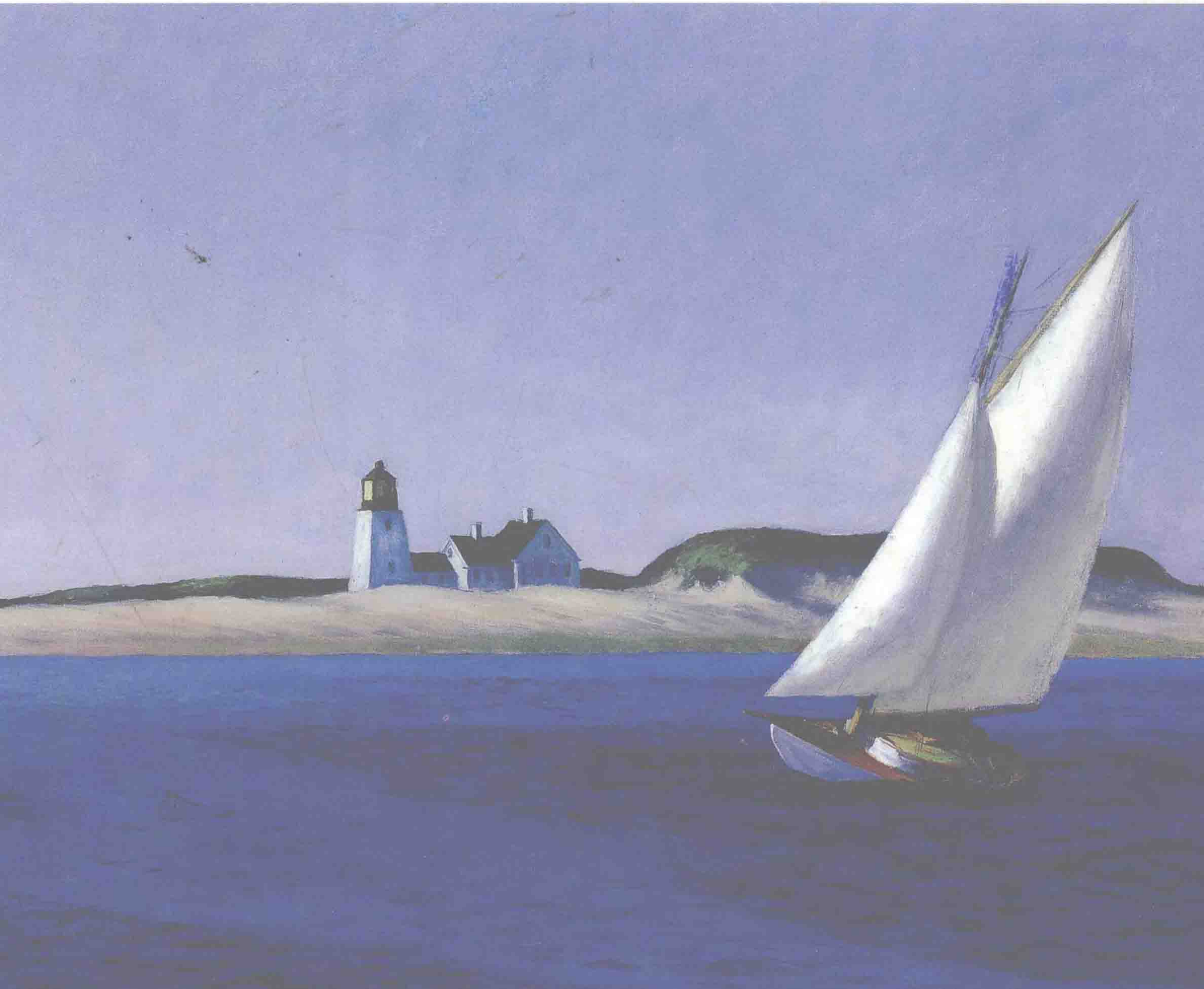
THIRD EDITION

COLLEGE ALGEBRA

David Cohen



COLLEGE ALGEBRA

Third Edition

DAVID COHEN
Department of Mathematics
University of California
Los Angeles

WEST PUBLISHING COMPANY
St. Paul New York Los Angeles San Francisco

FOR MY DAUGHTER JENNIFER

Copyediting: Susan Gerstein, Pamela McMurry
Text Design: Janet Bollow
Technical Illustrations: TECHarts, Boulder, Colorado
Composition: The Clarinda Company
Cover Design: Diane Beasley
Cover Image: Edward Hopper, *The Long Leg.* Virginia Steele Scott Collection, Huntington Library and Art Gallery.
Production, Prepress, Printing, and Binding by West Publishing Company.

610 Opperman Drive
P.O. Box 64526
St. Paul, MN 55164–0526

Printed in the United States of America
99 98 97 96 95 94 93 92 8 7 6 5 4 3 2

LIBRARY OF CONGRESS CATALOGING-IN-PUBLICATION DATA

Cohen, David, 1942 Sept. 28-
College algebra / David Cohen. — 3rd ed.
p. cm.
Includes index.
ISBN 0-314-93362-X (hard)
1. Algebra. I. Title.
QA154.2.C596 1992
512.9—dc20 91-37985
CIP

PREFACE

This text develops the elements of college algebra in a straightforward manner. As in the earlier editions, my goal has been to create a book that is *accessible* to the student. The presentation is student-oriented in three specific ways. First, I've tried to talk to, rather than lecture at, the student. Second, examples are consistently used to introduce, to explain, and to motivate concepts. And third, all of the initial exercises for each section are carefully correlated with the worked examples in that section.

AUDIENCE

In writing *College Algebra,* I have assumed that the students have been exposed to intermediate algebra, but that they have not necessarily mastered that subject. Also, for many college algebra students, there may be a gap of several years between their last mathematics course and the present one. For these reasons, the review material in Chapter 1 and in parts of Chapter 2 is unusually thorough.

FEATURES

1. *Word problems and applications.* Beginning with the review material in Chapter 1, there is a constant emphasis on learning to use the *language* of algebra as a tool in problem-solving. Word problems and strategies for solving them are explained and developed throughout the book. For examples of this, see any of the following sections.

Sec. 2.1 (setting up equations)

Sec. 2.4 (word problems leading to linear and quadratic equations)

Sec. 5.3 (more on setting up equations)

Sec. 5.4 (maximum-minimum problems relating to quadratic functions)

Sec. 6.5 (applications of the exponential function)

2. *Emphasis on graphing.* Graphs and techniques for graphing are developed throughout the text, and graphs are used to explain and reinforce algebraic concepts. (See, for example, Sections 4.3 and 6.3.)

3. *Calculator exercises.* There are two broad categories of calculator exercises in this text:

(i) OPTIONAL GRAPHING CALCULATOR EXERCISES

Over the past several years, all of us in the mathematics teaching community have become increasingly aware of the graphing calculator and its potential for making a positive impact in our teaching. While it is clear that vast reforms lie ahead, many of the specific details are still evolving in workshops and in classrooms across the country. And indeed, at present, even within a given school, some instructors are teaching the course with the graphing calculator, while others are not. Thus, for this 1992 edition of *College Algebra*, I have labeled the graphing calculator exercises "optional," and I have placed them at the ends of the chapters.

There are graphing calculator exercises for most of the major topics in Chapters 3 through 9. (These exercises have been class-tested.) Although the exercises can be adapted for use with any graphing calculator, they were written specifically for use with the new Texas Instruments TI-81 Graphics Calculator. Since many of these exercises contain carefully detailed instructions for using this calculator, a minimum of class time is required for discussing its operation. Additionally, Section A.2 of the appendix (written by Professor Mickey Settle) contains a complete introduction to the basic features of the calculator.

(ii) (ORDINARY) CALCULATOR EXERCISES
As in the previous edition, there are many calculator exercises integrated throughout the regular exercise sets in the text. Some of these exercises reinforce or supplement the core material; some of these exercises contain surprising results that motivate subsequent theoretical questions; and a few of these exercises demonstrate that the use of a calculator cannot replace thinking or the need for mathematical proofs.

4. *End of chapter material.* Each chapter concludes with a detailed chapter summary, a *Writing Mathematics* section, an extensive chapter-review exercise set, and a chapter test. Also, as described above, for Chapters 3 through 9 there are graphing calculator exercises.

The *Writing Mathematics* questions are new to this edition. In these exercises, the student is asked to organize his or her thoughts and respond in complete sentences. Some of the questions are simply *true or false* questions that can be explained in just a sentence or two. In other cases, a more elaborate response is required. For example, in Chapter 3 (page 166), the Writing Mathematics section describes a geometric technique for solving quadratic equations, and the student is asked to explain why the method is valid.

CHANGES IN THIS EDITION

Comments and suggestions from students, instructors, and reviewers have helped me to revise this text in a number of ways that I believe will make the book more useful to the instructor and more accessible to the student.

In many sections I've included boxed summaries entitled "Errors to Avoid." These summaries warn the student about some of the "popular" errors in college algebra. For instance, the three messages

$$\frac{1}{x}+\frac{1}{y}\neq\frac{1}{x+y} \qquad \sqrt{a+b}\neq\sqrt{a}+\sqrt{b} \qquad f(a+b)\neq f(a)+f(b)$$

are delivered (with examples and comments) on pages 10, 34, and 177, respectively. To help the student internalize these messages, there are often follow-up exercises that ask for examples showing that a purported rule is, in general, false.

Section 1.9 on factoring has been revised. There are more examples and exercises, and many of these emphasize *patterns*. (See, for instance, Example 3 on page 51.)

At the beginning of Chapter 2, there is now a preparatory section on setting up equations. I have used George Polya's simple two-column format for translating English into algebraic notation. The pace and the subject matter in this warmup section are deliberately limited, and all of the exercises are tied directly to the worked examples. Class testing has convinced me that this is a very effective way to begin the chapter.

In Chapter 3, the material on slopes and lines, formerly two sections, is now in one section (3.3). (The applications of slope, such as marginal cost and velocity, appear in Chapter 5 in the context of linear functions.)

In Chapter 4, Section 4.2, *The Graph of a Function,* has been expanded slightly to include the geometric ideas of increasing and decreasing functions, turning points, and maximum and minimum values. The material on techniques in graphing in Section 4.3 has been streamlined and the presentation is now, I believe, easier to teach and easier to learn. (Some of the material on translation and coordinates is presented again later, but from a different viewpoint, in Chapter 9.)

The treatment of linear functions at the beginning of Chapter 5 has been expanded in this edition to a full section. The idea of slope as a rate of change is emphasized through examples involving marginal cost and velocity. The concepts of a scatter diagram and the least-squares line are introduced in the text rather than the exercises, and the treatment is expanded.

In Section 5.5 on graphs of polynomial functions, the behavior of $f(x)$ when $|x|$ is very large is given greater emphasis both in the text and in the exercises. (See, for example, Figure 4 and Example 1 on pages 276–277.) The material on graphing polynomials in factored form has been revised. We now make use of the earlier work in Chapter 2 on solving inequalities. Also, more details are given about how to analyze a graph in the vicinity of an x-intercept.

Chapter 6 introduces exponential functions from a more practical or contemporary viewpoint. We indicate how exponential functions are used to model real-life situations. (See Figures 1 and 2 on page 306 and Example 9 on page 349.)

In Chapter 9, conic sections are discussed in greater detail than is found in most other college algebra texts today. In this edition, there is a new section presenting the focus-directrix properties of these curves.

SUPPLEMENTARY MATERIALS

1. The *Student's Solutions Manual,* by Ross Rueger, contains complete solutions for the odd-numbered exercises and for all of the test questions at the end of each chapter.
2. The *Instructor's Solutions Manual,* by Ross Rueger, contains answers or solutions for every exercise in *College Algebra.*
3. The *computer-generated testing program* Brownstone's DIPLOMA IV© is available to schools adopting *College Algebra.* (There are versions for both the Macintosh and the IBM PC or compatibles.)
4. The *Test Bank,* by Charles Heuer, is a package that contains two versions of multiple-choice tests, as well as two open-ended tests, for each chapter in *College Algebra.*
5. *GraphToolz,* by Tom Saxton, is a software program for graphing and evaluating

functions. This ingenious easy-to-use software, along with *Drill and Enrichment Exercises* by David Cohen, can make many of the topics in *College Algebra* really seem to come alive. *GraphToolz* is free to qualified adopters. (Available for the Macintosh family of computers.)

6. *Transparency masters* for many of the key figures or tables appearing in the text are available to schools adopting *College Algebra*.

7. *College Algebra: In Simplest Terms* (a series of videotapes produced by Annenberg/CPB Collection) is available to qualified adopters. This series covers the major topics in college algebra, and it includes some excellent applications.

ACKNOWLEDGMENTS Many students and teachers from both colleges and high schools have made useful constructive suggestions about the text and exercises, and I thank them for that. Special thanks go to Professor Charles Heuer for his careful work in checking the text and the exercise solutions for accuracy. I am grateful to Professor Mickey Settle for allowing us to reproduce his copyrighted work, *Using the TI-81 Graphics Calculator* in Section A.2 of the appendix. In preparing and revising the manuscript, I received valuable suggestions and comments from the following reviewers.

Lynn H. Brown,
Illinois State University

Lawrence O. Cannon,
Utah State University

Paula C. Gnepp,
Cleveland State University

William L. Grimes,
Central Missouri State University

Thomas G. Lucas,
University of North Carolina at Charlotte

George Miller,
Central Connecticut State University

William A. Miller,
Central Michigan University

Richard Nadel,
Florida International University

Sandra J. Olson,
Winona State University

Nathaniel Silver,
The City College, The City University of New York

Janine Smolin,
University of Wisconsin at Madison

William B. Wallace,
Phoenix College

Peter L. Waterman,
Northern Illinois University

Paul M. Weichsel,
University of Illinois at Urbana

James A. Wood,
Virginia Commonwealth University

I would like to thank Professor Ross Rueger, who wrote the supplementary manuals and created the answer section for the text; and Susan Gerstein and Pam McMurry, who copyedited the manuscript. Finally, to Mark Jacobsen and to Peter Marshall at West Publishing Company, thank you for all your work and help in bringing this manuscript into print.

David Cohen
Lunada Bay, California, 1992

COLLEGE ALGEBRA

Third Edition

CONTENTS

FOUR FUNCTIONS 173

FIVE POLYNOMIAL AND RATIONAL FUNCTIONS. APPLICATIONS TO OPTIMIZATION 239

SIX EXPONENTIAL AND LOGARITHMIC FUNCTIONS 305

SEVEN SYSTEMS OF EQUATIONS 366

EIGHT ROOTS OF POLYNOMIAL EQUATIONS 447

CHAPTER ONE

FUNDAMENTAL CONCEPTS

INTRODUCTION

This first chapter sets the foundations for our work in algebra. The central topics here are the real numbers, polynomials, and operations with polynomials. In the last section of the chapter we describe the complex number system. The notation and the results in this chapter will be used repeatedly throughout the rest of the book.

1.1 NOTATION AND LANGUAGE

Perhaps Pythagoras was a kind of magician to his followers because he taught them that nature is commanded by numbers. There is a harmony in nature, he said, a unity in her variety, and it has a language: numbers are the language of nature.

Jacob Bronowski

Ever since the time of Descartes, the terms "real" and "imaginary" numbers have been used in mathematics, despite their misleading associations.

Alfred Hooper

Here, as in your previous mathematics courses, most of the numbers we deal with are *real numbers*. These are the numbers used in everyday life, in the sciences, in industry, and in business. Perhaps the simplest way to define a real number is this: A **real number** is any number that can be expressed in decimal form. Some examples of real numbers are

$$7\ (=7.000\cdots) \qquad -\tfrac{2}{3}\ (=-0.\overline{6}) \qquad \sqrt{2}\ (=1.4142\cdots)$$

In algebra we often use letters to stand for numbers. This simple idea is what gives algebra much of its power and applicability in problem solving. By using letters, we can often see patterns and reach conclusions that might not be so apparent otherwise.

Arithmetic, and therefore algebra too, begins with the *operations* of addition, subtraction, multiplication, and division. Table 1 will serve to remind you

TABLE 1

Operation	Arithmetic Example	Algebraic Example
Addition (+)	$5 + 7.01$	$p + q$
Subtraction (−)	$13 - \sqrt{5}$	$r - s$
Multiplication (×)	3×4	$a \times b$ or $a \cdot b$ or ab
Division (÷)	$5 \div 6$	$x \div y$ or $\frac{x}{y}$ or x/y

that, for the most part, the notation for these operations in both arithmetic and algebra is the same. One exception: as the table indicates, the product $a \times b$ is often abbreviated simply by ab.

There are several other notational conventions that will seem familiar to you from previous courses. A sum such as $a + a + a$ is denoted by $3a$, while a product such as $a \cdot a \cdot a$ is written a^3. In general, if n is a **positive integer** (i.e., one of the numbers 1, 2, 3, 4, ...), then

$$na \quad \text{means} \quad \underbrace{a + a + a + \cdots + a}_{n \text{ terms}}$$

and

$$a^n \quad \text{means} \quad \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

In the expression na, the number n is called the **coefficient** of a. For example, in the expression $3a$, the coefficient is 3. When we write a^n, we refer to a as the **base** and n as the **exponent** or **power** to which the base is raised. The process of raising a base to a power is called **exponentiation.** The notation a^2 is read "a squared," because the area of a square with sides each of length a is $a \times a$ or a^2. Similarly, a^3 is read "a cubed," because the volume of a cube with sides each of length a is $a \times a \times a$ or a^3.

There are several important conventions regarding the order in which the operations of arithmetic and algebra are to be performed. Again, these will seem familiar to you from previous courses, and we summarize them in the box on the next page.

Examples 1 through 3, which follow, serve to summarize the ideas discussed so far. In Example 4, we translate some expressions from words into algebraic notation. This is an important skill that will be developed in more detail in Chapters 2 and 5.

EXAMPLE 1 Rewrite each expression using exponential notation.

(a) $x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
(b) $(a + 3)(a + 3)(b - 1)(b - 1)(b - 1)$

Solution **(a)** $x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y = x^3y^4$
(b) $(a + 3)(a + 3)(b - 1)(b - 1)(b - 1) = (a + 3)^2 (b - 1)^3$

PROPERTY SUMMARY THE ORDER OF OPERATIONS

CONVENTION	EXAMPLE
1. If no grouping symbols (such as parentheses) are present:	
(a) First do the exponentiations, working from right to left;	$3 + 4^2 = 3 + 16 = 19$; $5 \times 2^3 = 5 \times 8 = 40$; $2^{3^2} = 2^9 = 512$
(b) Next do the multiplications and divisions, working from left to right;	$9 - 2 \times 3 = 9 - 6 = 3$; $12 + 8 \div 2 = 12 + 4 = 16$; $4 + 2 \times 3^2 = 4 + 2 \times 9 = 4 + 18 = 22$
(c) Then do the additions and subtractions, working from left to right.	$1 + 2 \times 3 - 4 = 1 + 6 - 4 = 7 - 4 = 3$ $15 + 6^2 \div 4 - 2 \times 3^2 = 15 + 36 \div 4 - 2 \times 9 = 15 + 9 - 18 = 24 - 18 = 6$
2. If grouping symbols are present:	
(a) First do the operations within the grouping symbols;	$(1 + 2) \times 5 = 3 \times 5 = 15$; $(3 + 4)^2 = 7^2 = 49$
(b) If grouping symbols appear within grouping symbols, the operations within the innermost set are to be done first.	$3[8 - 2(6 - 4)] = 3(8 - 2 \times 2) = 3(8 - 4) = 3 \times 4 = 12$

EXAMPLE 2 (a) Evaluate $x - 4y^2$ when $x = 19$ and $y = 2$.

(b) Evaluate $(x - 4)y^2$ using the values for x and y given in part (a).

Solution (a)
$$x - 4y^2 = 19 - 4 \times 2^2 = 19 - 4 \times 4 = 19 - 16 = 3$$

(b) $(x - 4)y^2 = (19 - 4) \times 2^2 = 15 \times 4 = 60$ ■■■

EXAMPLE 3 (a) Evaluate each of the following expressions using $x = 5$ and $y = 2$.

$$(x + y)^2 \qquad x^2 + y^2 \qquad x^2 + 2xy + y^2$$

(b) Compare the results in part (a). What conclusions can be drawn?

Solution (a) Using $x = 5$ and $y = 2$ in each of the expressions, we have

$$(x + y)^2 = (5 + 2)^2 = 7^2 = 49$$
$$x^2 + y^2 = 5^2 + 2^2 = 25 + 4 = 29$$
$$x^2 + 2xy + y^2 = 5^2 + 2(5)(2) + 2^2 = 49$$

(b) On the one hand, using $x = 5$ and $y = 2$, we found that the values of $(x + y)^2$ and $x^2 + y^2$ are not the same. This shows that, in general $(x + y)^2 \neq x^2 + y^2$. (It takes only one counterexample to show that something is not, in general, true.) On the other hand, again using $x = 5$ and $y = 2$, we did find that the values of $(x + y)^2$ and $x^2 + 2xy + y^2$ are the same. But this does not *prove*, in general, that $(x + y)^2 = x^2 + 2xy + y^2$. It only shows that the equation is valid in one particular case. [Actually, as you may recall from a previous course (and as you'll see in Section 1.8), this last equation is, in fact, true in general. But, the point is, what we just did does not constitute a proof of this fact.] ■■■

EXAMPLE 4 Translate each of the following into algebraic notation:

(a) five more than twice the number x;
(b) three times the sum of x and y^3;
(c) the sum of three times x and y^3.

Solution **(a)** $\underbrace{2x}_{\text{twice } x} + \underbrace{5}_{\text{five more}}$

(b) $\underbrace{3\times}_{\text{three times}} \underbrace{(x+y^3)}_{\text{the sum of } x \text{ and } y^3} = 3(x+y^3)$

(c) $\underbrace{3x+y^3}_{\text{the sum of } 3x \text{ and } y^3}$

We conclude this section with some comments about exponents and calculators. Suppose, for example, that we wish to evaluate the expression

$$3^{2^4}$$

According to Property 1(a) in the box on page 3, we have

$$3^{2^4} = 3^{16} \qquad \text{because} \qquad 2^4 = 16$$

Now we need to evaluate 3^{16}. Although this can certainly be computed "by hand," a more sensible approach here is to use a calculator. All scientific calculators have keys for computations involving exponents. The details vary, however, from brand to brand, and even between different models of the same brand. The examples that follow indicate how 3^{16} is calculated on four common types of calculators. If you have questions about how your calculator operates in this respect, you should consult the owner's manual.

CALCULATOR	SEQUENCE OF KEYSTROKES	OUTPUT
Sharp EL-506H	3 (y^x) 16 (=)	43,046,721
Casio fx-7000G	3 (x^y) 16 (EXE)	43,046,721
Hewlett Packard 11C	3 (ENTER) 16 (y^x)	43,046,721
Texas Instruments TI-81	3 (∧) 16 (ENTER)	43,046,721

EXERCISE SET 1.1

A

In Exercises 1–10, rewrite the given expressions using coefficients and exponents.

1. $x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y$
2. $x\cdot x\cdot y\cdot y\cdot z$
3. $(x+1)(x+1)(x+1)$
4. $(a^2+b^2)(a^2+b^2)(a^2+b^2)$
5. (a) $x+x+x+x$
   (b) $(x^2+1)+(x^2+1)+(x^2+1)$
6. (a) a^5+a^5
   (b) $(a^4+1)+(a^4+1)$
7. (a) $a+a+a+b+b$
   (b) $a^2+a^2+a^2+b^2+b^2$
8. (a) $t^4+t^4+t^4+z^2+z^2$
   (b) $(t^4+1)+(t^4+1)+(t^4+1)$
9. (a) $(2a+1)(2a+1)(2a+1)(2b+1)(2b+1)$
   (b) $(1+2a)(2a+1)(1+2a)(2b+1)(1+2b)$

10. (a) $(x + y^2)(x + y^2)(x^2 + y)(x^2 + y)$
(b) $(x + y^2)(y^2 + x)(x^2 + y)(y + x^2)$

For Exercises 11–26, evaluate the expressions using the given values of the variables.

11. $x + 3y;\ x = 4,\ y = 6$
12. $a - 4b^2;\ a = 20,\ b = 2$
13. $a^2 + b^2;\ a = 3,\ b = 4$
14. $(a + b)^2;\ a = 3,\ b = 4$
15. $x^2 - 4y^2;\ x = 10,\ y = 4$
16. $x^2 - xy + y^2;\ x = 1,\ y = 1$
17. $x^2 - x + 1;\ x = 5$
18. $(x + 1)(x^2 - x + 1);\ x = 5$
19. $2x^3 - 3y^2;\ x = 3,\ y = 2$
20. $2(x^3 - 3y^2);\ x = 3,\ y = 2$
21. $1 \div a^2 + b^2;\ a = 1,\ b = 1$
22. $1 \div (a^2 + b^2);\ a = 1,\ b = 1$
23. $\dfrac{1}{a^2} + \dfrac{1}{b^2} - \dfrac{1}{a^2 + b^2};\ a = \dfrac{1}{2},\ b = \dfrac{1}{2}$
24. $\dfrac{1}{1 - 1/x};\ x = 2$
25. A^{B^C} for (a) $A = B = 2,\ C = 3$; (b) $A = C = 2,\ B = 3$; (c) $B = C = 2,\ A = 3$
26. A^{B^C} for (a) $A = 2, B = 1, C = 3$; (b) $A = 2, B = 3, C = 1$; (c) $A = 1,\ B = 2,\ C = 3$
27. Which is larger: $(2^3)^4$ or 2^{3^4}?
28. Compute $2^{3^2} - (2^3)^2$.

In Exercises 29 and 30, use a calculator to evaluate A^{B^C}, using the given values.

29. (a) $A = 5,\ B = 2,\ C = 3$
(b) $A = 5,\ B = 3,\ C = 2$
30. (a) $A = 4,\ B = 3,\ C = 2$
(b) $A = 3,\ B = 4,\ C = 2$
31. (a) Without using a calculator, decide which of the quantities, 2^{2^5} or 5^{2^2}, is larger. (You should be able to do this without completely evaluating both expressions.)
(b) Use a calculator to evaluate the expressions and check your answer in part (a).
32. Follow Exercise 31 using the expressions 2^{3^3} and 3^{3^2}.

In Exercises 33–46, translate each phrase into algebraic notation.

33. Two more than x
34. Two less than x
35. Four times the sum of a and b^2
36. The sum of four times a and b^2
37. The sum of x^2 and y^2
38. The square of the sum of x and y
39. The sum of x and twice the square of x
40. Twice the sum of x and the square of x
41. The average of x, y, and z
42. The square of the average of x, y, and z
43. The average of the squares of x, y, and z
44. The square of the average of the squares of x, y, and z
45. One less than twice the product of x and y
46. The cube of three more than the product of x^2 and y^3

B

For Exercises 47 and 48, evaluate the expressions using the given values of the variables.

47. (a) $\dfrac{1000}{19 - \dfrac{12}{1 - (1/x)}};\ x = 3$

(b) $\dfrac{1}{1 + \dfrac{1}{1 + \dfrac{1}{1 + (1/x)}}};\ x = 2$

48. (a) $[p(p^2 - 2^p)]^{p+1};\ p = 3$

(b) $\dfrac{\dfrac{a+b}{a-b} + \dfrac{a-b}{a+b}}{\dfrac{a-b}{a+b} - \dfrac{a+b}{a-b}} \cdot \dfrac{ab^3 - a^3b}{a^2 + b^2};\ a = \dfrac{5}{4},\ b = \dfrac{1}{4}$

49. (a) In each case, use a calculator to determine which of the two quantities is larger.
(i) 6^7; 7^6 (ii) 7^8; 8^7 (iii) 9^8; 8^9
(b) Find a positive whole number a such that $(a + 1)^a > a^{a+1}$. (There are only two such numbers; some experimenting is called for.)

1.2 PROPERTIES OF THE REAL NUMBERS

Today's familiar plus and minus signs were first used in 15th-century Germany as warehouse marks. They indicated when a container held something that weighed over or under a certain standard weight.

Martin Gardner in "Mathematical Games" (*Scientific American*, June 1977)

I do not like × as a symbol for multiplication, as it is easily confounded with x; . . . often I simply relate two quantities by an interposed dot . . .

G. W. Leibniz in a letter dated July 29, 1698

In this section we first will list the basic properties for the real number system. After that we'll summarize some of the more down-to-earth implications of those properties, such as the procedures for working with signed numbers and fractions. (Section A.3 of the appendix, at the back of this book, presents a more theoretical treatment of some of these ideas.)

The set of real numbers is **closed** with respect to the operations of addition and multiplication. This just means that when we add or multiply two real numbers, the result (that is, the **sum** or the **product**) is again a real number. Some of the other most basic properties and definitions for the real-number system are listed in the following box. In the box, the lowercase letters a, b, and c denote arbitrary real numbers.

PROPERTY SUMMARY SOME FUNDAMENTAL PROPERTIES OF THE REAL NUMBERS

Commutative properties	$a + b = b + a \qquad ab = ba$
Associative properties	$a + (b + c) = (a + b) + c \qquad a(bc) = (ab)c$
Identity properties	**1.** There is a unique real number 0 (called **zero** or the **additive identity**) such that $a + 0 = a$ and $0 + a = a$ **2.** There is a unique real number 1 (called **one** or the **multiplicative identity**) such that $a \cdot 1 = a$ and $1 \cdot a = a$
Inverse properties	**1.** For each real number a there is a real number $-a$ (called the **additive inverse** of a or the **negative** of a) such that $a + (-a) = 0$ and $(-a) + a = 0$ **2.** For each real number $a \neq 0$, there is a real number denoted by $\frac{1}{a}$ (or $1/a$ or a^{-1}), and called the **multiplicative inverse** or **reciprocal** of a, such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$
Distributive properties	$a(b + c) = ab + ac \qquad (b + c)a = ba + ca$

On reading this list of properties for the first time, many students ask the natural question, "Why do we even bother to list such obvious properties?" One reason is that all the other laws of arithmetic and algebra (including the "not-so-obvious" ones) can be derived from our rather short list. For example, the rule $0 \cdot a = 0$ can be proved using the distributive property, as can the rule that the product of two negative numbers is a positive number. (See Section A.3