

**THIRD
EDITION**

Applied Linear Regression Models

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To
Dorothy, Ron, David,
Nancy, Michelle, Allison,
Maureen, Abigael, Andrew,
Cathy, Christopher, Timothy, Randall, Erin, Fiona

Preface

Linear regression models are widely used today in business administration, economics, engineering, and the social, health, and biological sciences. Successful applications of these models require a sound understanding of both the underlying theory and the practical problems that are encountered in using the models in real-life situations. While *Applied Linear Regression Models*, Third Edition, is basically an applied book, it seeks to blend theory and applications effectively, avoiding the extremes of presenting theory in isolation and of giving elements of applications without the needed understanding of the theoretical foundations.

The third edition differs from the second in a number of important respects.

1. We have reorganized the chapters in order to get to multiple linear regression analysis more quickly. Old Chapter 1 on basic results in probability and statistics has been placed in Appendix A, old Chapter 9 on polynomial regression has been interwoven in the discussion of multiple linear regression, and old Chapter 10 on qualitative predictor variables now comes after a full discussion of multiple regression model building and diagnostics.

2. We have expanded substantially the discussion of diagnostics and remedial measures. In Chapters 3 and 10, we have added robust tests for constancy of the error variance, smoothing techniques to explore the shape of the regression function, robust regression and nonparametric regression techniques, bootstrapping methods for evaluating the precision of sample estimates for complex situations, and estimation of the variance and standard deviation functions to obtain weights for weighted least squares.

3. The chapter on logistic regression (Chapter 14) has been extensively revised and expanded to include an introduction to polytomous logistic regression, Poisson regression, and generalized linear models, as well as greater coverage of diagnostic procedures using the model deviance, deviance residuals, and simulated envelopes.

4. We now include several case studies at strategic places to assist with the integration of the statistical methods discussed.

5. We have given greater emphasis to maximum likelihood methods and have substantially expanded the use of graphic presentation, including the use of scatter

plot matrices, three-dimensional rotating plots, three-dimensional response surface and contour plots, and conditional effects plots.

6. Throughout the text, we have made extensive revisions in the exposition on the basis of classroom experience to improve the clarity of the presentation.

We have included in this book not only the more conventional topics in regression, but also topics that are frequently slighted, though important in practice. We devote three chapters (Chapter 8–10) to the model-building process for regression, including computer-assisted selection procedures for identifying “good” subsets of predictor variables and validation of the chosen regression model. Two chapters (Chapters 11 and 14) are devoted to indicator variables, covering both response and predictor indicator variables. The use of residual analysis and other diagnostics for examining the appropriateness of a regression model is a recurring theme throughout this book. So is the use of remedial measures that may be helpful when the model is not appropriate. In the analysis of the results of a study, we give greater emphasis to the use of estimation procedures than to significance tests, because estimation is often more meaningful in practice. Also, since practical problems seldom are concerned with a single inference, we stress the use of simultaneous inference procedures.

Theoretical ideas are presented to the degree needed for good understanding in making sound applications. Proofs are given in those instances where we feel they serve to demonstrate an important method of approach. Emphasis is placed on a thorough understanding of the regression models, particularly the meaning of the model parameters, since such understanding is basic to proper applications. A wide variety of examples and cases is presented to illustrate the use of the theoretical principles, to show the great diversity of applications of regression models, and to demonstrate how analyses are carried out for different problems.

We use “Notes” and “Comments” sections in each chapter to present additional discussion and matters related to the mainstream of development. In this way, the basic ideas in a chapter are presented concisely and without distraction.

Applications of regression models frequently require extensive computations. We take the position that a computer or programmable calculator is available in most applied work and that almost every computer user has access to program packages for regression analysis. Hence, we explain the basic mathematical steps in fitting a regression model but do not dwell on computational details. This approach permits us to avoid many complex formulas and enables us to focus on basic principles. We make extensive use in this text of computer capabilities for performing computations and preparing graphic plots, and we illustrate a variety of computer printouts and plots and explain how they are used for analysis.

A selection of problems is provided at the end of each chapter. Here readers can reinforce their understanding of the methodology and use the concepts learned to analyze data. We have been careful to supply data-analysis problems that typify genuine applications. In most problems the calculations are best handled on a programmable calculator or computer. To facilitate data entry, a diskette in ASCII format is provided with the text that includes the data sets for all examples, problems, exercises, and projects, as well as for the data sets in Appendix C. The README.TXT file on the diskette provides information about the identification of the data sets.

We assume that the reader of *Applied Linear Regression Models*, Third Edition, has had an introductory course in statistical inference, covering the material outlined in Appendix A. Should some gaps in the reader’s background exist, the relevant portions of an introductory text can be studied, or the instructor of the class may use supplemental materials for covering the missing segments. Appendix A is primarily

intended as a reference of basic statistical results for continuing use as the reader progresses through the book.

Calculus is not required for reading *Applied Linear Regression Models*, Third Edition. In a number of instances, we use calculus to demonstrate how some important results are obtained, but these demonstrations are confined to supplementary comments or notes and can be omitted without any loss of continuity. Readers who do know calculus will find these comments and notes in natural sequence so that the benefits of the mathematical developments are obtained in their immediate context. Some basic elements of matrix algebra are needed for multiple regression. Chapter 5 introduces these elements of matrix algebra in the context of simple regression for easy learning.

Applied Linear Regression Models, Third Edition, is intended for use in undergraduate and graduate courses in regression analysis and in second courses in applied statistics. The extent to which material presented in this text is used in a particular course depends upon the amount of time available and the objectives of the course. The basic elements of regression are covered in Chapters 1, 2, 3, 4 (Sections 4.1–4.3), 5, 6, 7, 8, 9, and 10. Chapters 11, 12, 13, 14, and 15 can be covered as time permits and interests dictate.

This book can also be used for self-study by persons engaged in the fields of business administration, economics, engineering, and the social, health, and biological sciences who desire to obtain competence in the application of regression models.

An *Instructor Solutions Manual*, containing detailed solutions to all numerical problems and analytical exercises, is available from the publisher, Irwin, for use by instructors. To facilitate learning by students, instructors may order from the publisher, Irwin, copies of the *Student Solutions Manual* for use by students. The *Student Solutions Manual* provides intermediate and final numerical results for easy self-checking of solutions for selected problems. We use a check mark (✓) in front of the problem number to designate the problems for which the solutions appear in the *Student Solutions Manual*.

A book such as this cannot be written without substantial assistance from numerous persons. We are indebted to the many contributors who have developed the theory and practice discussed in this book. We also would like to acknowledge appreciation to our students, who helped us in a variety of ways to fashion the method of presentation contained herein. We are grateful to the many users of *Applied Linear Statistical Models* and *Applied Linear Regression Models* who have provided us with comments and suggestions based on their teaching with these texts. We are also indebted to Professors James E. Holstein, University of Missouri, and David L. Sherry, University of West Florida, for their review of *Applied Linear Statistical Models*, First Edition; to Professors Samuel Kotz, University of Maryland at College Park, Ralph P. Russo, University of Iowa, and Peter F. Thall, The George Washington University, for their review of *Applied Linear Regression Models*, First Edition; to Professors John S. Y. Chiu, University of Washington, James A. Calvin, University of Iowa, and Michael F. Driscoll, Arizona State University, for their review of *Applied Linear Statistical Models*, Second Edition; to Professor Richard Anderson-Sprecher, University of Wyoming, for his review of *Applied Linear Regression Models*, Second Edition; and to Professors Alexander von Eye, The Pennsylvania State University, Samuel Kotz, University of Maryland at College Park, and John B. Willett, Harvard University, for their review of *Applied Linear Statistical Models*, Third Edition. These reviews provided many important suggestions, for which we are most grateful.

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John Neter
Michael H. Kutner
Christopher J. Nachtsheim
William Wasserman

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PART



Simple Linear Regression

