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Contemporary Calculus

through applications

The North Carolina School of Science and Mathematics

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The Department of Mathematics and Computer Science
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Dedication

Those of us who teach at the North Carolina School of Science and Mathematics have two families, our own and our students. We dedicate this book first to our own families who have supported us throughout the long process of writing and revising this text. We also dedicate this book to our students, whose enthusiastic support of this course from its inception and throughout its many revisions has inspired us.

Preface

In 1985 the Department of Mathematics and Computer Science at the North Carolina School of Science and Mathematics began its precalculus curriculum reform project. For the next five years, the entire Department worked to develop a precalculus course that was applications-oriented and that took full advantage of technology. The textbook, *Contemporary Precalculus through Applications*, was published in 1991.

Even before the precalculus text was published, we saw the need to revise the course that follows precalculus. We wanted to continue with the major themes that had guided us through the development of our precalculus course and incorporate these themes into a calculus course. In some ways this new task was much more difficult than it had been with precalculus. Unlike precalculus, which is very loosely defined, calculus (even called "The Calculus" by some), seems to be much more rigorously defined. Nearly all calculus books contain exactly the same topics, and the Advanced Placement Syllabus defines what will be tested, and therefore, what will be included in most calculus textbooks.

Technology, more than anything else, dictates that calculus as we know it must change. We took this as a serious challenge and attempted to do more than layer technology on top of the existing curriculum. As a result, the use of technology is not an optional part of this text. Instructors cannot demonstrate ideas and students cannot do homework or complete labs or take tests without the use of technology.

The Tulane Conference held in January of 1986 sounded the warning that college calculus was not meeting the needs of many students. The follow-up conference, sponsored by the National Academy of Sciences and the National Academy of Engineering in the Fall of 1987, began to offer some suggestions for reform as noted in the Mathematical Association of America's publication *Calculus for a New Century*. The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* called for significant changes to both the mathematics curriculum and existing pedagogy. This textbook takes seriously the recommendations of the NCTM Standards and the MAA Notes. In particular, the text responds to the following three frequently cited concerns.

1. Students should be able to apply mathematics to problems that are stated in new and different contexts. In other words, students should understand mathematics, in contrast with merely substituting numbers into formulas.
2. Mathematics, and calculus in particular, is applied to virtually every field of human endeavor. Students should appreciate and understand a broad range of these applications, and they should be able to use mathematics as a tool for modeling a variety of real-world phenomena.
3. The numeric, symbolic, and graphic capabilities of computers and calculators have given technology a crucial role in the mathematics classroom.

This text defines an applications-oriented, investigative calculus course in which students are provided tools for understanding the world in which they live. The textbook involves students in both the development of problem statements and in their solution. Students learn to use the concepts of calculus to solve problems in a variety of contexts, many of which are discussed over extended periods of time. Concepts are presented in the context of real-world applications; calculators and computers are used to develop concepts and to solve problems; and the interpretation of problem solutions is given strong emphasis.

The fabric of the course is woven with seven themes that are an indispensable part of virtually every segment of the course. These themes address important student needs and are described here.

1. *Developing Understanding*: Calculus offers students a repertoire of new techniques for describing the world around them and should be accessible to as wide an audience as possible. For that reason, this text encourages intuitive understanding of many topics. Although mathematical rigor may be appropriate, it should not be an obstacle to the success of students. Every student may not be able to prove the theorems of calculus, however each should obtain a solid understanding of the concepts of calculus.

   To help students develop their conceptual understanding, graphical, numerical, and algebraic interpretations are used together whenever possible in each section of the text. Students are led to discover concepts for themselves as often as is feasible, primarily through mathematics laboratory experiences.

2. *Data, Applications, and Mathematical Models*: Much of our information about the world comes to us in the form of data. Problems presented in this textbook describe realistic situations that can be modeled using calculus. As much as possible, major concepts are introduced through data, applications, and mathematical modeling. These applications come from a wide variety of areas in which calculus is used as a tool to solve problems. While the physical sciences provide a wealth of applications of calculus, examples are drawn from many other areas of study. As students see a wide variety of applications they will begin to appreciate the power of calculus.

3. *Computer and Calculator as Tools*: Much of traditional calculus involves extensive paper-and-pencil manipulation. While students must be able to do some manipulations, technology makes obsolete the skills required for solving difficult problems with paper and pencil. Computer technology empowers us to solve problems with "messy" data and large numbers of computations, which is exactly how problems appear in the real world.

   Technology also is used to experiment with different values for parameters in problems, to try different strategies for solution, to test out various conjectures, and to ask "What if...?" questions that otherwise would not be feasible to investigate. Students must, however, know how to interpret results, when a result is reasonable, and when technology is not going to provide the best solution to a particular problem. Each of these requirements further emphasizes the need for students to develop a conceptual understanding of calculus.

4. *Numerical Algorithms*: Approximations for the derivative, the definite integral, and solutions to differential equations can be found using numerical algorithms. Since the techniques used in these algorithms reinforce the broad concepts of calculus, the algorithms provide more than just the solution to a problem. Numerical algorithms do have limitations, and the text will help students develop an understanding of the accuracy and the appropriateness of various numerical methods.

5. *Discrete Phenomena*: Calculus involves the study of phenomena that are represented by continuous functions. Because computations can be carried out very rapidly, it is possible to study these phenomena using discrete techniques. Recursive techniques, difference quotients, and other discrete concepts help students move from the discrete to the continuous domain. Links between discrete and continuous phenomena are emphasized frequently in the textbook.

6. *Computer and Calculator Laboratory Experiences*: Computer and calculator lab work is the centerpiece of the entire course. Students use technology as a tool for investigation and discovery. Labs activities allow students to discover concepts, to develop their intuition related to calculus, and to investigate extended problems in groups and individually. These labs are designed to culminate in written reports that summarize and analyze a particular investigation.

7. *Writing about Mathematics*: This textbook requires that students write about mathematics, including both concepts and the interpretations of these concepts. The students will use the language of mathematics and communicate their ideas to other individuals familiar with the subject. Writing about mathematics completes a process that begins with the translation of a problem statement into mathematical notation, then uses mathematics to investigate the problem, and ends with translating the results back into a verbal explanation and summary.

The NCSSM Contemporary Calculus project began in 1988 as a joint curriculum project, Project CALC, with Drs. David Smith and Lawrence Moore at Duke University, and funded by the National Science Foundation. The format for this text and much of the content areas grew out of early work with Project CALC. In 1992 the authors received funding, also from the National Science Foundation, to complete the work on a separate reformed calculus course and text. This text, *Contemporary Calculus through Applications,* is the direct result of that grant.

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This text was developed under two grants from the National Science Foundation. The first grant was to Duke University; we worked as a subcontractor to write modules and to field-test and modify materials. The second grant to the Department of Mathematics and Computer Science at the North Carolina School of Science and Mathematics (NCSSM) gave us the opportunity to take what we had learned from our work with Duke and write our own course.

Our thanks to Professors David Smith and Lawrence Moore of Duke University for including us in Project CALC. Drs. Smith and Moore were among the first university mathematicians to recognize the need for reforming calculus instruction. They included us in their first planning grant and again in their first major grant. Our initial involvement with Duke University included writing some sections, field-testing their materials in our classes and modifying their materials for our students. After we received funding to continue our project, Drs. Smith and Moore continued to work with us as members of our Advisory Board.

In addition to David Smith and Lawrence Moore, our Advisory Board included Henry Pollak, retired from Bell Labs, Joan Countryman of The Lincoln School, and Tom Tucker of Colgate University. Their careful reading of our materials reassured us that we were making good progress and resulted in changes which have greatly improved our efforts.

All of the members of the Department of Mathematics and Computer Science at the North Carolina School of Science and Mathematics, both current and former, have made significant contributions to the development of this text. John Goebel, Kevin Bartkovich, and Lawrence Gould worked with Duke during the first two years of the first grant. Lawrence attended planning meetings with Duke, observed classes at Duke, wrote some of the early units, and tested materials in our course classes. His untimely death in 1990 deprived us of his insight. Mary Malinauskus, Tracey Harting, and Julie Allen wrote solutions for many of the exercises and labs. Maria Hernandez, Peggy Craft, and Marilyn Schiermeier all taught the course during the writing of this text, made suggestions, and helped with the many tasks associated with preparing the text, the *Solutions Manual*, and the *Instructor's Guide*. In addition to members of the Mathematics Department, we would like to thank Dr. John Kolena of the NCSSM Physics Department and Dr. Myra Halpin of the Chemistry Department for their help with the physics and chemistry applications in the text.

Our thanks to our field testers who taught all or part of the course and sent us valuable input. These field testers were: Sharon Adam, Pensacola High School, Pensacola, FL; David Bannard, Collegiate Schools, Richmond, VA; Michael Lutz, Elkhart Central High School, Elkhart, IN; Nannette Dyas, Montgomery Blair High School, Silver Spring, MD; Gale Farmer, Wichita Collegiate School, Wichita, KS; Landy Godbold, Westminster School, Atlanta, GA; Ted Gott, Southern High School, Galesville, MD; Peter Gufstafson, The Pennington School, Pennington, NJ; Cheryl Haley, Norview High School, Norfolk, VA; Roger Jardine, Caribou High School, Caribou, ME; Beverly Johnson, Webb School of Knoxville, Knoxville, TN; John King, Dwight-Englewood School, Englewood, NJ; Beth Krantz, Vallivue Sr. High, Caldwell, ID; Brad Morris, Friends Central School, Wynnewood, PA; Donavon Nagel, John Marshall High School, Rochester, MN; Mel Noble, Olympic High School, Bremerton, WA; Steve Unruhe, Riverside High School, Durham, NC.

Special thanks to Steve Unruhe who has taught the entire course at Riverside High School in Durham, NC, for several years, has given us a wealth of feedback and input, and has helped us with the *Instructor's Guide*. Special thanks also to Ted Gott, who has also taught the at Southern High School in Galesville, MD, for several years and has demonstrated that the course can be taught effectively using graphing calculators.

Many instructors participated in the Calculus workshops we taught during the summers from 1991 through 1995. We thank these instructors for their reactions to preliminary drafts of the text that helped to shape the final outcome.

We appreciate the efforts of our publisher Barbara Janson and our editor Eric Karnowski in helping us create a book from a manuscript that has been undergoing continuous change.

We would like to thank Sally Berenson and Glenda Carter of the Center for Research in Mathematics and Science Education at North Carolina State University who were the formal evaluators for our project.

We would also like to thank the NCSSM administration for helping us pursue funding for this project, the personnel and business offices for cheerfully accepting the extra work that the project required of them, and Terry Brown, the department secretary, who helped us complete all of our school responsibilities in the midst of our work on this book.

Our students would have been without a photocopied text, our field testers would have been stranded, and the project would have ground to a halt if not for the support provided daily by Pam Smith, the secretary for this project. We thank her for her commitment to the success of our development effort.

Features of this Text

The seven major themes detailed in the Preface permeate this text but the following features of the text deserve special mention.

Laboratory Experiences

Throughout this text students will be expected to complete lab assignments which will either aid them in the discovery of important concepts or give them the opportunity to use the calculus they have learned to solve real-world applications. We strongly encourage our students to work in pairs on the labs. These labs are an integral part of this text and are not optional. We have left the labs somewhat open-ended so that students will have the freedom to explore and use different techniques to discover the concepts or solve the problems in the labs. This open-endedness may be unfamiliar to some students, but one of the major goals of this text is to empower students with the skills and confidence they will need to solve problems on their own. For this reason, suggested solutions to the labs are not included in the *Solutions Manual*, but are available in the *Instructor's Guide*. This will allow instructors to provide hints to students as needed. We recommend that these hints be given very sparingly.

Use of Technology

Just as the labs are essential to the successful completion of this text, so is the use of technology. In the earliest stages of development of this text we depended almost entirely on Mathcad®, an electronic mathematics scratchpad, to do all of the labs and much of the classwork and homework. The course has been successfully taught both with a graphing calculator that handles data well and also with a spreadsheet that has good graphing capabilities.

End of Chapter Exercises

We have provided summary exercises at the end of each of Chapters 2 through 6. These exercises, though not exhaustive, summarize the important concepts in the chapter and give the students an opportunity to review these concepts out of the context of the section in which they were taught. In some cases, the ideas of the chapter are extended in these exercises.

Investigations

The end of Chapter 3 and the entirety of Chapter 8 consist of investigations. These "super labs", like the regular labs, come from a wide variety of fields, but are intended to be more open-ended than the other labs and are not specifically based on material covered in any one section of the text. The intent is for students, working in groups, to have an opportunity to think about all of the calculus they have learned up to the point in the text where the investigations appear and to use the appropriate calculus to develop a reasonable solution to the problem. Many of the investigations can be solved by more than one method, and we encourage multiple solutions. We also encourage students to try the extensions that are often provided, or to look for their own extensions to the problems. In these investigations, students have the opportunity to pull together the calculus they have studied, to demonstrate their problem-solving skills, and to communicate their ideas both orally and in writing.

Contents

CHAPTER 1

How Things Change

1.1 Introduction

Why is mathematics, and calculus in particular, important? One of the main reasons we study mathematics is because it helps us to understand the phenomena we experience in the world around us. This understanding often allows us to predict what will happen in the future or explain what happened in the past. Prediction and explanation are two ways that mathematics helps us to solve problems.

Calculus is especially important for solving problems involving how phenomena change because *calculus is a mathematical language of change*. How crucial is change in understanding phenomena in the world around us? Consider the following observation:

At the beginning of 1987, the world population passed 5 billion.

This statement tells us the size of the world population at a particular time, but says nothing about what the population was before 1987 or will be after 1987. The fact that the world population was 5 billion in 1987 may be less significant than the fact that the population doubled between 1950 and 1987. Assessing the significance of the current population level is difficult without knowing the rate at which the population is changing. The mathematics of populations addresses a number of questions of special relevance to our generation. Are we in the midst of a global population explosion? If so, what approaches might be effective in dealing with this problem? Suppose there is no impending population explosion. Will the world population eventually level off? If so, at what level? Will the size of the limiting population be so large that we will have a host of other problems in the future? If the population does not level off, will the population oscillate between different levels? If the population does oscillate, will the oscillations include wild swings between very high and very low levels, or will the swings be relatively small? All of these questions involve the concept of the rate of change of population and can be investigated mathematically with calculus.

Numerical information presented by the media often has the characteristic that the rate of change is more significant than the current value. For instance, scientific reports about the ozone layer usually describe the rate at which ozone is being destroyed. Rarely do reports mention the actual amount of ozone that currently exists. In fact, such a figure would probably be meaningless to the average reader. What is significant is the rate at which ozone is being depleted. A decrease in the rate of ozone depletion is considered good news by most people. Unfortunately, such a decrease does not mean that the ozone layer is being replenished, but only that the ozone layer is being destroyed less quickly than in the past.

Calculus encompasses much mathematics used for answering the questions mentioned above, as well as related questions concerning economics, medicine, and the analysis of data. Furthermore, calculus is essential for understanding the motion of atomic particles, automobiles, satellites, rockets, and galaxies. Calculus is basic to the study of flow in rivers, currents in the oceans, and movement of air over airplane wings. Businesses use calculus in cost and profit analysis. This list can go on and on, yet the characteristic common to all of these phenomena is the central importance of how quantities change.

On a mathematical level, these examples share another common theme. They all involve related variables (such as population and time), but the information given by the rate of change of one variable with respect to the other is more important than the values of the variables. In precalculus courses, the study of functions often emphasizes graphs and ordered pairs. For instance, if $f(x) = x^2 - 3x$, we know that $f(2) = -2$ and that the point $(2, -2)$ is an ordered pair of the function. The calculus view of functions is more dynamic. In calculus courses we want to know more than, "If $x = 2$, then $y = -2$." We will ask, "Around $x = 2$, how are the y values changing? Are they increasing or decreasing? At what rate are the y values changing?" One of the primary purposes of calculus is to quantify rates of change. As the above examples illustrate, rates of change are important because knowing how a phenomenon changes gives valuable information about the phenomenon.

1.2 Phenomena Modeled by Discrete Change Expressions

Figure 1.1 shows a graph of 1995 first class postage costs versus weight. Notice that the value for the weight of a letter can be any positive number. On the other hand, the set of values for the postage costs is discrete, since the possible values for postage costs belong to the set {32, 55, 78, 101,...}. The discrete nature of the set of postage costs gives the graph a stair-step appearance. How does the U.S. Postal Service treat mail that weighs exactly some integer amount? A letter that weighs less than or equal to 1 ounce costs 32 cents to mail first class; however, a letter that weighs over 1 ounce but less than or equal to 2 ounces is charged the next highest rate, which is 55 cents. Graphically this is indicated by the use of open endpoints on the left end of each step and closed endpoints on the right.

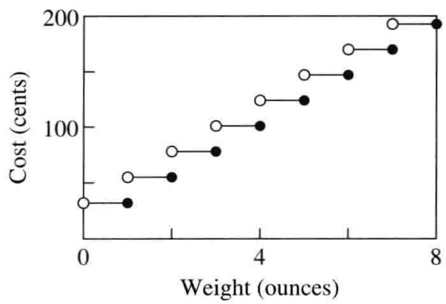

Figure 1.1 Postage cost versus weight

Suppose you have a checking account that earns no interest. The balance in the account changes only when you deposit or withdraw money. Assume that on January 1 the balance is \$200. At the end of each month you make a deposit of \$1000, but then \$850 is automatically withdrawn to pay your bills; therefore, the account has a net increase of \$150 each month. On February 1 you will have \$350, on March 1 you will have \$500, on April 1 you will have \$650, and so on. In general, we find the balance in any month by adding the change in the balance to the previous month's balance. Since the change occurs at the constant rate of \$150 per month, each month's balance is simply the previous month's balance plus \$150. Figure 1.2 shows the balance in the account during the seven months after January 1. As in the postage stamp example, one variable, the number of months, can have any non-negative value; the other variable, balance, has only certain discrete values. The step graph in Figure 1.2 shows that the balance in your account is the same from the beginning of any month to the end of that month.