MATHEMATICAL APPLICATIONS

for the Management, Life, and Social Sciences



HARSHBARGER / REYNOLDS

Fifth Edition

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for the Management, Life, and Social Sciences

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Design: Kenneth Hollman

Art Editing: Diane Grossman/Carol Reid

Production Coordination: Lisa Merrill/Charles Dutton

Permissions: Margaret Roll

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Printed in the United States of America.

International Standard Book Number: 0-669-39840-3

Library of Congress Catalog Number: 95-68036

10 9 8 7 6 5 4 3 2

PREFACE

To paraphrase English mathematician, philosopher, and educator Alfred North Whitehead, the purpose of education is not to fill a vessel but to kindle a fire. In particular, Whitehead encouraged students to be creative and imaginative in their learning and to continually form ideas into new and more exciting combinations. This desirable goal is not always an easy one to realize in mathematics with students whose primary interests are in areas other than mathematics. The purpose of this text, then, is to present mathematical skills and concepts and to apply them to ideas that are important to students in the management, life, and social sciences. We hope that this look at the relevance of mathematical ideas to a broad range of fields will help inspire the imaginative thinking and excitement for learning that Whitehead spoke of. The applications included allow students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years of high school algebra or its equivalent, this text begins with a brief review of algebra which, if covered, will aid in preparing students for the work ahead.

Pedagogical Features

Important pedagogical features that have been retained in this new edition are the following.

Intuitive Viewpoint. The book is written from an intuitive viewpoint, with emphasis on concepts and problem solving rather than on mathematical theory. Each topic is carefully explained, and examples illustrate the techniques involved. Many exercises stress computation and drill, but there are enough challenging problems to stimulate students in thoughtful investigations.

Flexibility. At different colleges and universities, the coverage and sequencing of topics may vary according to the purpose of this course and the nature of the student audience. To accommodate alternative approaches, the text has a great deal of flexibility in the order in which topics may be presented and the degree to which they may be emphasized.

Chapter Warm-ups. A Warm-up appears at the beginning of each chapter and invites students to test themselves on the skills needed for that chapter. The Warm-ups present several prerequisite problem types that are keyed to the appropriate sections in the upcoming chapter where those skills are needed. Students who have difficulty with any particular skill are directed to specific sections of the text for review. Instructors may also find the Warm-ups useful in creating a course syllabus that includes an appropriate scope and sequence of topics.

Applications. We have found that integrating applied topics into the discussions and exercises helps to provide motivation within the sections and demonstrate the relevance of each topic. In addition, we have found that offering separate lessons on applied topics such as cost, revenue, and profit functions brings the preceding mathematical discussions into clear and concise focus and provides a thread of continuity as mathematical sophistication increases. There are eleven such sections in this text. Overall, of the more than 5300 exercises in the book, more than 1900 are applied problems.

Objective Lists. Every section begins with a brief list of objectives that outlines the goals of that section for the student.

Procedure/Example and Property/Example Tables. Appearing throughout the text, these tables aid student understanding by giving step-by-step descriptions of important procedures and properties with illustrative examples worked out beside them.

Boxed Information. All important information is boxed for easy reference, and key terms are highlighted in boldface.

Review Exercises. At the end of each chapter, a set of Review Exercises offers the student extra practice on the topics in that chapter. These exercises are annotated with section numbers so that any student who has difficulty can turn to the appropriate section for review.

Changes in the Fifth Edition

The major goals of this revision were to include numerous real data problems, enhance the graphical and numeric approaches to problem solving, add other pedagogical features, lay a better groundwork for the presentation of the mathematics, and provide opportunities for the use of graphing and computer technologies for those instructors inclined to use them. These goals were achieved in the following ways.

- 1. We have made an extensive addition of applications involving real data and mathematical modeling in exercises and examples throughout the text. Over 220 problems among the exercise applications—about one of every eight applications—are developed around real data and contain source notes. Many sections include problems with functions that are modeled from real data, and some problems ask students to model functions from the data.
- 2. The pedagogy includes an enhanced graphical and numerical viewpoint or approach, which can be used effectively during instruction, whether or not the instructor chooses to require student use of graphics calculators or other graphing utilities. We have added a large number of graphs to our discussions, examples, and exercises that allow students to see relationships and that are designed to improve their ability to read and interpret graphs. In addition, the inclusion of real data problems provides increased opportunities to develop a numerical perspective in understanding and solving mathematical applications.

- 3. Special highlighted subsections that discuss the use of graphing utilities are added throughout the text. After the introduction to graphing with technology in new Section 1.3, Graphs and Graphing Utilities, discussion of the use of technology is placed in subsections and in examples within sections. These presentations are highlighted with a technology icon which includes graphics are exercises that benefit from the use of technology—which includes graphics calculators, computer programs, and computer spreadsheets—are also highlighted with the technology icon. Of course, many additional problem investigations can also profit from the use of technology, at the option of the instructor. Technology is used to graph functions and to discuss generalizations, applications, and implications of problems being studied. It is also used for modeling functions from real data. The power that these tools bring to our investigations now permits the inclusion of problems that are larger in scope and that contain more data points.
- 4. Each section begins with a new Application Preview that establishes the context and direction for the concepts that will be presented. The preview motivates the mathematical discussion in the upcoming section and is then revisited in a worked-out example appearing later in the lesson.
- 5. New Checkpoint problems are introduced in each section. These in-lesson exercises ask questions and pose problems that allow students to check their understanding of the skills and concepts under discussion as they progress through the section. Solutions to Checkpoint problems appear before lesson exercises.
- 6. The Consultant's Corner feature has been expanded to appear at the end of every chapter. These case studies can be investigated with or without the use of graphing utilities and are excellent means of providing cooperative learning opportunities for students.
- 7. Exercise sets include labeled applications in which the subject matter of each problem is identified. Instructors and students can use these labels to easily select applications that are of special interest or are particularly suitable for their class.
- A new end-of-chapter Key Terms and Formulas index precedes the Review Exercises in each chapter.

Other significant content improvements include the following.

CHAPTER 1 Linear Equations and Functions

The power and limitations of graphing technology are discussed, paving the way for future discussions involving tabular, graphical, and analytic methods of solving problems. The discussion of operations with functions, in Section 1.4, now includes composition of functions.

CHAPTER 2 Matrices

Section 2.4, Inverse of a Square Matrix, now includes a brief discussion of finding determinants for 2x2 matrices and using technology for finding determinants of larger square matrices. The discussion emphasizes the usefulness of technology in performing operations with large matrices and finding matrix inverses.

CHAPTER 3 Inequalities and Linear Programming

Section 3.3 now includes the use of technology to find the corners of feasible regions in linear programming. Two Consultant's Corners are included, one of which discusses sensitivity analysis. There is also an increased emphasis in converting real data tables and figures to matrices.

CHAPTER 4 Quadratic Functions, and **CHAPTER 5** Exponential Functions

These chapters now exhibit a much stronger modeling emphasis with significant additions of real data problems.

CHAPTER 6 Mathematics of Finance

An increased emphasis on the use of graphical comparisons provides a more visual exploration of functional relationships and the idea that "time is money." Also, compound interest problems are now given a more student-friendly pace, focusing in early exercises on identification of interest rate per period and number of periods.

CHAPTER 7 Introduction to Probability

The chapter has been rewritten and reorganized to present both the "set" and "event" notation styles/interpretations of probability theory. One-trial and multiple-trial experiments are now treated in separate lessons.

CHAPTER 8 Further Topics in Probability; Statistics

Chapter 8 has been rewritten and reorganized to extend the text's presentation of probability topics and emphasize a variety of probability distributions. The discussion of expected value has been relocated from Chapter 7 to new Section 8.2, Discrete Probability Distributions; Decision Making. New Section 8.4 presents a consolidated discussion of descriptive statistics concepts.

CHAPTER 9 Derivatives

Section 9.1, Limits, has been reorganized to strengthen the pedagogical links among the graphic, numeric, and symbolic approaches to limits. Examples and exercises throughout the chapter now place much greater emphasis on graphical

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relationships between functions and their derivatives, as well as greater emphasis on interpretation problems, both for limits and derivatives. The discussion in Section 9.2 now includes limits at infinity.

CHAPTER 10 Applications of Derivatives

There is now increased emphasis in Sections 10.1 and 10.2 on interpretation of the graphs of functions and their first and second derivatives and, in particular, what can be learned from the relationships among them.

CHAPTER 14 Functions of Two or More Variables

Section 14.2, Partial Differentiation, now combines coverage of first-order and higher-order partial derivatives into a single section. Section 14.5 now includes a graphical approach to solving constrained max-min problems.

Supplements

Instructor's Guide. This supplement contains answers to all even-numbered exercises in the text; full solutions to all even-numbered exercises; solutions to all Consultant's Corner problems; four forms of chapter tests for each chapter; answer keys to all test forms; and transparency masters for important tables and figures from the text.

Study & Solutions Guide, by Gordon Shilling. In addition to the solutions to all odd-numbered exercises in the text, this guide contains supplementary exercises that reinforce the concepts and techniques presented in the text. Answers to these problems are also provided.

ipsTest Computerized Testing. This computerized testing package, available in MS-DOS and Macintosh formats, combines algorithmic and fixed-item testing in one convenient software package. The carefully constructed algorithms use random number generators to produce different, yet equivalent, versions of each problem. A nearly unlimited number of different tests and quizzes can be generated quickly and easily. Questions may be output as either multiple choice or free response items. The Macintosh version features an editing function for changing existing problems or creating additional problems. In custom-designing tests, instructors can also draw from among the 2000 fixed items in the test bank, add their own test items to the bank, or edit existing items. Tests may be prepared using any combination of both algorithmic and fixed items from the software package. User notes and sample problem printouts are included with the program.

Test Item File. This is a printed version of all of the test items appearing in the computerized testing program. Fixed items are correlated to appropriate sections of the textbook. The Test Item File also includes sample problem printouts for each algorithmic testing item, organized by chapter.

BestGrapher, Version 4.0, by George Best. Available in MS-DOS and Macintosh formats, this tool software and its accompanying workbook emphasize the concepts of calculus. The software can be used for graphing, evaluating functions, and symbolic differentiation among other tasks. Graphing features now also include zoom, trace, and simultaneous plot capabilities. The workbook provides activities that encourage experimentation and discovery about functions, derivatives, areas, and other topics.

Acknowledgments

We would like to thank the many people who have helped us at various stages of revising this text. The encouragement, criticism, and suggestions that have been offered have been invaluable to us. Special thanks go to Dipendra Bhattacharya, Steve Gendler, Susie Lanier, Carol Nessmith, Sahib Singh, and Steve Wise for their contributions to this project.

For their reviews of draft manuscript and the many helpful comments that were offered, we would also like to thank Mitzi Chaffer, Central Michigan University; Florence Chambers, Joliet Junior College; R. D. Derderian, Providence College; Edward Gade, University of Wisconsin at Oshkosh; Dauhrice Gibson, Gulf Coast Community College; Betty L. Hawkins, Shoreline Community College; Rose Marie Kinik, Sacred Heart University; Vincent Koehler, Trinity College of Vermont; Lee H. LaRue, Paris Junior College; James Loats, Metro State College of Denver; Janice C. Lyon, Tallahassee Community College; Theodore F. Moore, Mohawk Valley Community College; James R. Nolan, Siena College; Todd D. Oberg, The University of Montana; Gordon Shilling, University of Texas at Arlington; Michel Smith, Auburn University; and Diane Van Nostrand, University of Tulsa.

We would also like to express our appreciation to the editorial staff at D. C. Heath for their continued enthusiasm and support.

Ronald J. Harshbarger James J. Reynolds

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