ANSEL C. UGURAL

STRESSES

IN PLATES AND SHELLS

SECOND EDITION

Stresses in Plates and Shells

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Ansel C. Ugural
New Jersey Institute of Technology

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List of Symbols

A	area, constant
a, b	dimensions, outer and inner radii of annular plate
c	distance from neutral axis to outer fiber
D	flexural rigidity $[D = Et^3/12(1-\nu^2)]$
d	diameter, distance
$[D]$	elasticity matrix
E	modulus of elasticity
F	resultant external loading on shell element, concentrated force
f	frequency
G	modulus of elasticity in shear
g	acceleration of gravity (≈ 9.81 m/s^2)
h	mesh width, numerical factor, depth
I	moment of inertia
J	polar moment of inertia
k	modulus of elastic foundation, numerical factor, axial load factor for slender members in compression
$[k]$	stiffness matrix of finite element
$[K]$	stiffness matrix of whole structure
L	length, span
m	integer, numerical factor
M	moment per unit distance, moment, moment-sum $[M = (M_x + M_y)/(1+\nu)]$
$\bar{m}$	mass per unit area
M^*	thermal moment resultant per unit distance
M_x, M_y	bending moments per unit distance on x and y planes, moments
M_{xy}	twisting moment per unit distance on x plane
M_r, M_θ	radial and tangential moments per unit distance
$M_{r\theta}$	twisting moment per unit distance on radial plane
M_ϕ	meridional bending moment per unit distance on parallel plane
M_s	meridional bending moment per unit distance on parallel plane of conical shell
$M_{x\theta}$	twisting moment per unit distance on axial plane of cylindrical shell
n	factor of safety, integer, numerical factor
N	normal force per unit distance
N_{cr}	critical compressive load per unit distance
N^*	thermal force resultant per unit distance
N_x, N_y	normal forces per unit distance on x and y planes
N_{xy}	shear force per unit distance on x plane and parallel to y axis
N_r, N_θ	radial and tangential forces per unit distance

N_ϕ	meridional force per unit distance on parallel plane
$N_{\phi\theta}$	shear force per unit distance on parallel plane and perpendicular to meridional plane
$N_{x\theta}$	shear force per unit distance on axial plane and parallel to y axis of cylindrical shell
N_s	normal force per unit distance on parallel plane of conical shell
p	intensity of distributed transverse load per unit area or length, pressure, surface force per unit area
p^*	equivalent transverse load per unit area
P	concentrated force
Q	first moment of area
$\{Q\}$	nodal force matrix of finite element
Q_x, Q_y	shear force per unit distance on x and y planes
Q_r, Q_θ	radial and tangential shear forces per unit distance
Q_θ	shear force per unit distance on plane perpendicular to the axial plane of cylindrical shell
Q_ϕ	meridional shear force per unit distance on parallel plane
R	reactive forces
r	radius, radius of gyration
r, θ	polar coordinates
r_x, r_y	radii of curvature of midsurface in xz and yz planes
r_1, r_2	radii of curvature of midsurface in meridional and parallel planes, principal radii of curvature
s	distance measured along generator in conical shell
S	elastic section modulus
T	kinetic energy, temperature, torque
t	thickness
u, v, w	displacements in x, y, and z directions; axial, tangential, and radial displacements in shell midsurface
U	strain energy
U_o	strain energy density
V	shear force
V_x, V_y	effective shear force per unit distance on x and y planes
V_r, V_θ	radial and tangential effective shear forces per unit distance
W	work, weight
x, y, z	distances, rectangular coordinates
α	angle, coefficient of thermal expansion, numerical factor
β	angle, cylinder geometry parameter $[\beta^4 = 3(1-\nu^2)/a^2t^2]$, numerical factor
γ	shear strain, weight per unit volume or specific weight
γ_{xy}, γ_{yz}, γ_{zx}	shear strains in the xy, yz, and zx planes
$\gamma_{r\theta}$	shear strain in the $r\theta$ plane
δ	deflection, finite difference operator, numerical factor, variational symbol

$\{\delta\}$	nodal displacement matrix of finite element
ε	normal strain
ϵ	transmissibility
ε_x, ε_y, ε_z	normal strains in x, y, and z directions
ε_r, ε_θ	radial and tangential normal strains
ε_θ, ε_ϕ	normal strain of the parallel circle and of the meridian
θ	angle, angular nodal displacement
κ	curvature
λ	sphere parameter $[\lambda^4 = 3(1-\nu^2)a^2/t^2]$, numerical factor
ν	Poisson's ratio
Π	potential energy
ρ	density (mass per unit volume)
σ	normal stress
σ_x, σ_y, σ_z	normal stresses on the x, y, and z planes
σ_r, σ_θ	radial and tangential normal stresses
σ_ϕ	meridional normal stress on parallel plane
σ_1, σ_2, σ_3	principal stresses
σ_{cr}	compressive stress at critical load
σ_u	ultimate tension stress
σ_{uc}	ultimate compression stress
σ_{yp}	yield stress
τ	shear stress
τ_{xy}, τ_{yz}, τ_{zx}	shear stresses on the x, y, and z planes and parallel to the y, z, and x directions
$\tau_{r\theta}$	shear stress on radial plane and parallel to the tangential plane
τ_u	ultimate stress in shear
τ_{yp}	yield stress in shear
ϕ	angle, stress function, numerical factor, angle of twist
χ	change of curvature in shell
ω	natural circular frequency, rad/s

PREFACE

INTRODUCTION

The subject matter of this text is usually covered in one-semester senior and one-semester graduate level courses dealing with the *analysis of plates and shells, stress analysis, pressure vessels, analysis of thin-walled structures, advanced statics,* or *special topics in solid and structural mechanics.* As sufficient material is provided for a full year of study, the book may stimulate the development of courses in *advanced engineering design.* The coverage presumes a knowledge of elementary mechanics of materials.

The text is intended to serve a twofold purpose: to complement **classroom lectures** and to accommodate the needs of **practicing engineers** in the analysis of beam, plate, and shell structures. The material presented is applicable to aeronautical, astronautical, chemical, civil, mechanical, and ocean engineering; engineering mechanics; and science curricula. This volume attempts to provide synthesis and analysis that cut through the clutter and save readers time. It is hoped that clarity of presentation is maintained, as well as simplicity as permitted by the nature of the subject, unpretentious depth, an effort to encourage intuitive understanding, and a shunning of the irrelevant.

APPROACH

Emphasis is given to computer-oriented **numerical techniques** in the solution of problems resisting **analytical approaches**. The reader is helped to realize that a firm grasp of fundamentals is necessary to perform the critical interpretations, so important when computer-based solutions are employed. However, the stress placed upon numerical methods is not intended to deny the merit of **classical analysis**, which is given a rather full treatment. The volume attempts to fill what the writer believes is a void in the world of texts.

The book offers a simple, comprehensive, and methodical presentation of the principles of beam, plate, and shell theories and their applications to numerous structural elements, including domes, pressure vessels, tanks, and pipes. Theories of failures are employed in predicting the behavior of beams, plates, and shells under combined loading. Above all, an effort is made to provide a visual interpretation of the basic equations and of the means by which loads are resisted in shells and plates. A balance is presented between the theory necessary to gain insight into the mechanics and the numerical solutions, both so useful in performing stress analysis in a more realistic setting. Throughout the text, the author has attempted to provide the fundamentals of theory and application necessary to prepare students for more advanced study and for professional practice. Development of the physical and mathematical aspects of the subject is deliberately pursued.

The physical significance of the solutions, and practical applications, are given emphasis. With regard to application, often classical engineering examples are used to maintain simplicity and lucidity. The author has made a special effort to illustrate important principles and applications with numerical examples. A variety of problems is provided for solution by the student. *Answers to selected problems* are given at the end of the book. A **sign convention**, consistent with vector mechanics, is employed throughout for loads, internal force resultants, and stresses. This convention conforms to that used in most classical mechanics of materials and elasticity texts, as well as to that most often employed in numerical analysis of complex structures. The International System of Units (SI) is used.

The expression defining the small lateral deflection of the midplane of a thin plate is formulated in two ways. The first utilizes the fundamental assumptions made in the customary *theory of beams and plates*. The second is based upon the differential equations of equilibrium for the *three-dimensional stress*. The former approach, which requires less mathematical rigor but more physical interpretation, is regarded as more appealing to the engineer, and is equally used in the case of thin shells. Emphasized also are the energy aspects of plate and shell bending and buckling because of the importance of *energy methods* in the solution of many real-life problems and in modern computational techniques. Because of the introductory nature of this book, the classical approaches requiring extensive mathematical background are not treated.

Recent publications dealing with shell-bending theory include analytical presentations generally valid for any shell under any kind of loading. These formulations usually necessitate the employment of tensor notation, vector analysis, and a system of curvilinear coordinates. The theory introduced in this text is a special case of the above. The equations governing shells are developed only to the extent necessary for solving the more usual engineering problems. The finite element method is applied to treat plates of nonuniform thickness and arbitrary shape as well as to represent shells of arbitrary form subjected to variable loading.

Most chapters are substantially self-contained. Hence, the order of presentation can be smoothly altered to meet an instructor's preference. It is suggested, however, that Chap. 3 be studied first. The remaining chapters may be taken in any sequence except that Chaps. 12 and 13 should be read before Chaps. 14 and 15. Numerous alternatives are possible in making selections from the book for two single-semester courses. The chapters have been arranged in a sequence compatible with an orderly study of the analysis of plates and shells. *Part I* and a number of sections marked with an asterisk (*) are optional and can be omitted. Some chapters are carefully integrated by means of cross-referencing.

NEW TO THIS EDITION

This second edition of *Stresses in Plates and Shells* seeks to preserve the objectives and emphasis of the previous edition. It is extended to include stresses in bars and beams, analysis of composite plates and shells, and some other additional topics described below. A major effort has been made to provide a more comprehensive and modern text. The concept of "design to meet strength requirements" as those requirements relate to individual members, elastic design criteria, basic design process, and design factor of safety are briefly discussed. The examples and problems that appear throughout the book illustrate the close link between analysis and design.

Practical stress analysis is usually done using widely available and highly sophisticated software. This demands that engineering courses emphasize more the *fundamentals* and the basis of the *computer-oriented methods*. To this end great care is taken to explain the **three aspects of solid mechanics problems**, basic assumptions made in developing engineering theory of typical structural members, and the implications and limitations of the method.

Included are the *new materials* dealing with fundamental principles of stress analysis; engineering materials; exact theory of plates; plates in the form of a sector; plates with variable flexural rigidity; long rectangular plates; rectangular plates on elastic foundation; thermal stresses in plates; rectangular orthotropic plates; laminated composite plates; circular panels; dynamic stresses in plates; folded structures; shells of general shape; spherical shells; laminated composite cylindrical shells; approximate and energy methods for plates and shells; and tables of area characteristics, material properties, units, and beam and plate stresses and deflections.

The entire text has been reexamined and many major improvements are made throughout by elimination, rearrangement, and the addition of Chaps. 1 and 2 and Secs. 3.5, 3.9, 3.11, 3.13, 4.4, 4.7, 6.5, 7.7, 7.10, 8.6, 8.9, 8.10, 9.3, 10.5, 12.10, 12.11, 13.10, 13.12, 15.5, 15.7, 15.8, 15.9, 15.11, 15.12, and App. B. Some sections are expanded to improve upon previous expositions. The temptation to increase greatly the material covered is resisted. However, it is considered desirable to add a *large* number of new real-life examples and problems. Most problems can be readily modified for in-class tests. References, listed at the end of each chapter, provide readily available sources where additional information can be obtained.

This text offers a wide range of about 100 fully worked-out illustrative examples; 275 problem sets, many of which are drawn from engineering practice; a multitude of formulas and tabulations of beam, plate, and shell theory solutions from which direct and practical design calculations can be made; analyses of plates and shells made of isotropic as well as composite materials under ordinary and high temperature loadings; numerical methods amenable to computer solution; and applications of the formulas developed and of the theories of failures to increasingly important structural members.

SUPPLEMENT

The book is accompanied by a **Solutions Manual**, available to instructors. It is written and class-tested by the author, and features complete solutions to all problems in the text.

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