

Dynamics of Internal Gravity Waves in the Ocean

Yu. Z. Miropol'sky



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Dynamics of Internal Gravity Waves in the Ocean

by

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This book is dedicated
to the great worldwide
efforts of the numerous
army of theoretical and
experimental
researchers in the field
of physical
oceanography

Foreword

The author of this book, Yuri Zakharovich Miropol'sky, will never see it. An aggressive disease cut the life of this young and talented person. Almost up to the last hour of his short life he continued working on the manuscript.

Yuri Zakharovich Miropol'sky was born in June 1943. In 1966 he graduated from the Moscow Physical-Technical Institute. All of his scientific activity was connected with the Shirshov Institute of Oceanology of the Russian Academy of Sciences. Being already sick, he defended his professorial thesis in 1977.

The scientific interests of Prof. Miropol'sky were connected basically with two of the most important problems of oceanology—a theory of the upper quasi-homogeneous ocean layer and a theory of internal waves. One of the first nonstationary models of the upper quasi-homogeneous layer belongs to Prof. Miropol'sky and his colleagues. The authors of this model pointed out for the first time an important property of self-similarity of temperature distribution in the upper thermocline. A study of these problems was the last point in Prof. Miropol'sky's scientific life: on the basis of the revealed self-similarity he developed an elegant theory describing the seasonal evolution of the upper thermocline in the ocean.

Studies by Prof. Miropol'sky reflect almost all of the aspects of internal waves problems. He investigated the problems of effectiveness of the generation of internal waves by atmospheric pressure and by a wind field; questions of propagation of small amplitude internal waves from

local initial disturbances (the Cauchy–Poisson generalized problem) and of small amplitude waves in the presence of horizontal inhomogeneity of the density field in the ocean, as well as the influence of the fine vertical structure on the internal waves' propagation. A number of works contain a nonlinear theory of oceanic internal waves. There are the problems of the propagation of nonlinear stationary internal waves in shallow water and in thin stratified layers of the deep ocean. A nonstationary theory of weakly nonlinear internal waves built by Prof. Miropol'sky and his colleagues appeared to be extremely important.

Yu. Miropol'sky was very interested in experimental investigations, natural measurements, and data processing, and worked on numerous expeditions. He took part in the creation of interpretation methodology of measurements of temperature and velocity fluctuations in the ocean, which allowed one to obtain both dispersion relations and the most important statistical characteristics of the internal waves field. He studied in detail the fine structure of the temperature field and pointed out a number of its universal properties. These and other results are presented in the book.

Colleagues of Prof. Miropol'sky esteemed any possibility of scientific cooperation with him and loved him very much. As are many gifted persons, he was talented in several fields: he was highly educated, had an outstanding sense of humor, and a real charm. In the memory of all who knew him he will always be a bright, generous, and merry person.

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Preface

The book *Dynamics of the Internal Gravity Waves in the Ocean* by Yu. Miropol'sky considers modern theoretical methods of investigating wave motions, results of their application to internal waves problems, as well as ocean observational data. Nonlinear effects and their connection with the formation of oceanic fine structure of the density vertical distribution are analyzed. It interprets oceanic observational data and supplies methods of distinguishing internal waves and turbulence.

The book is meant for specialists in physics of the ocean, oceanography, geophysics, and hydroacoustics.

In writing the monograph the author wanted to create a systematic interpretation of the theoretical and the most practical experimental aspects of the dynamics of internal waves in the Ocean. The author, drawing the reader's attention first of all to the physical effects important from an oceanological standpoint, nevertheless was inspired by the beauty of a mathematical description. He distinguished ideas popular among oceanographers but unproved by theory or by experiment, from quantitative theoretical conclusions and observational data.

A secondary target set by the author, giving some detailed theoretical results, was the fast introduction of an inexperienced reader to the range of modern ideas and methods in the study of wave processes in dispersive media.

A number of books in the Western literature have considered internal gravity waves. Some fragmentary data on internal waves are summa-

alized in corresponding chapters of the books by Phillips *The Dynamics of the Upper Ocean* [299], by Turner *Buoyancy Effects in Fluids* [367] and by Monin, Kamenkovich and Kort *The Variability of the Ocean* [262], which present only qualitative pictures of some particular aspects of the problem. Despite the appearance of the detailed book by Whitham *Linear and Nonlinear Waves* [391], internal waves—due to their anisotropy and nonuniformity—present the non-trivial object of applying of these methods.

Specialists in this scientific field have to rely basically on what has been written on this in parts of more general text books by Tolsoy *Wave Propagation* [361], by LeBlond and Mysak *Waves in the Ocean* [201], by Yih *Stratified Flows* [403], some passages in text books by Lighthill *Waves in Fluids* [212], which focuses on the theory of internal wave generation and propagation, *Atmosphere–Ocean Dynamics* by Gill [100], which focuses on the role of internal waves in atmospheric and oceanic processes, and by Craik *Wave Interactions and Fluid Flows* [55] discussing the topics of wave interaction phenomena in fluids at rest and shear flows.

Few books dealing entirely with internal waves are known: *Interne Wellen* by Krauss (in German) [179], *Internal Gravity Waves* by Olbers [277] and Munk *Internal Waves* [268], the latter is more like an extensive literature review, as well as the papers [83], [95], [210].

However, all books published in this topic in the Western literature view internal waves only as special cases within a more general framework, so that their different aspects are covered only partially.

Since the point at which a sufficiently complete state of the physical theory of internal waves appeared in journals and comprehensive experimental data were accumulated, there has been a need for writing this monograph, in which a wide knowledge on the internal ocean waves is generalized and systematized.

The book starts with a systematic statement of principles of linear (Part 2) and nonlinear (Part 3) internal waves theory after a brief description of principles of the Ocean's thermohydrodynamics (Part 1). It gives a clear notion of the internal waves amongst other oceanic wave motions and of the degree of approximations permissible for the theoret-

ical description of the internal waves. Unfortunately the limited content of the monograph has not allowed us to give as detailed an account of the available experimental data (Part 4) as the author desired.

Nonlinear effects in internal waves propagation (Part 3) are studied by the Hamiltonian formalism, though many of the results in the nonlinear theory of internal waves are obtained by different asymptotic decompositions directly from the initial hydrodynamic equations. Application of the Hamiltonian formalism method gives the theory of nonlinear internal waves a required generality and clarifies the connection between the theory of internal waves and the general physical theory of wave propagation in dispersive media. From the methodical viewpoint, using the Hamiltonian formalism in the present monograph is described thoroughly in Chapter 7 in the terms of some simple mathematical procedures.

Of course, the book cannot capture all of the numerous works dealing with internal waves. Therefore the author has tried to give the most detailed information only on those aspects of the physics of internal waves which were based on his personal experience or which he considered especially important. The other problems are supplemented by a quite detailed bibliography.

Some of the results presented were obtained by the author in collaboration with A.I. Leonov, A.G. Voronovich, B.N. Filyushkin, R.E. Tamsalu and Yu.D. Borisenko. Chapter 7, containing the method of the Hamiltonian formalism in the internal waves theory, and sections 9.1, 9.4, 10.1–10.3 were written by A.G. Voronovich. Experimental data and the references on the main topics were updated by the interpreter.

Acknowledgments

The translator of this book is grateful to Dr. Leo Maas (Netherlands Institute for Sea Research, The Netherlands) and to Dr. Bruno Voisin (Laboratoire des Écoulements Géophysiques et Industriels, CNRS–UJF–INPG, France) for the initiation of this translation and their permanent further assistance.

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Introduction

Investigations in the field of internal gravity waves has occupied one of the central places in oceanology in recent years. The topicality of the problems is caused first of all by propagation of internal wave motions in the entire depth of the World Ocean, and, in this connection, by their important role in all oceanic dynamic phenomena.

The practical importance of internal waves is evident for hydroacoustics, submerged navigation, hydrobiology, hydro-optics and marine mud formation. Particular recent interest in internal waves study has been caused by their great, and perhaps decisive, role in the processes forming horizontal and vertical exchange in the Ocean. In this connection investigations of internal gravitational waves, their appearance, propagation, and especially problems of unsteadiness and breaking, become decisive for understanding ocean dynamics.

The main factor forcing internal waves in the Ocean is a stable stratification according to wave density increase towards the gravity direction, i.e., from the upper surface to the ocean's bottom. Such stratification stability takes place everywhere in the Ocean and is connected first of all with the free water surface heated by the solar radiation. Only polar zones and temperate latitudes in winter time, where cooling from above is possible, may be considered as exceptions. But even in these cases neutral and stable stratification is observed. The sea water density ρ depends not only on the temperature T but also on the salinity s , which is why, besides thermodynamic processes, evaporation and atmospheric

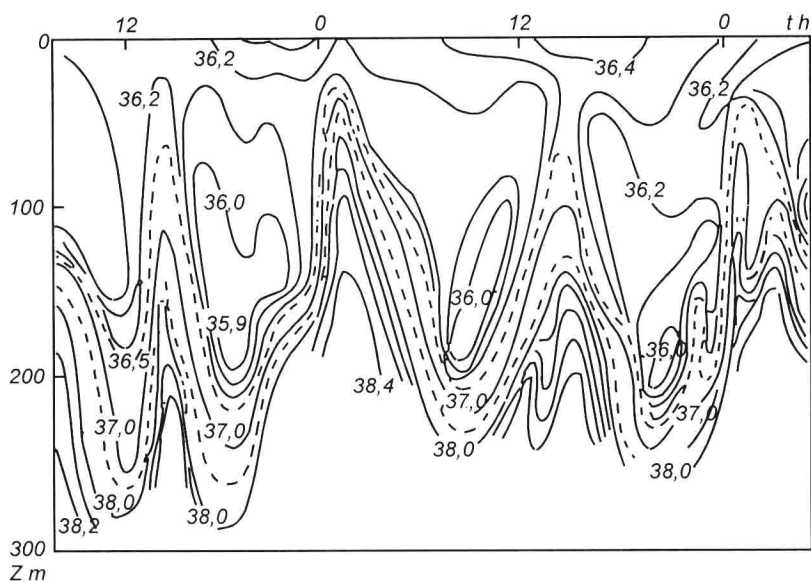


Figure 0.1. Oscillations of the salinity (‰) in the Straits of Gibraltar, 16–18 May 1961 [26].

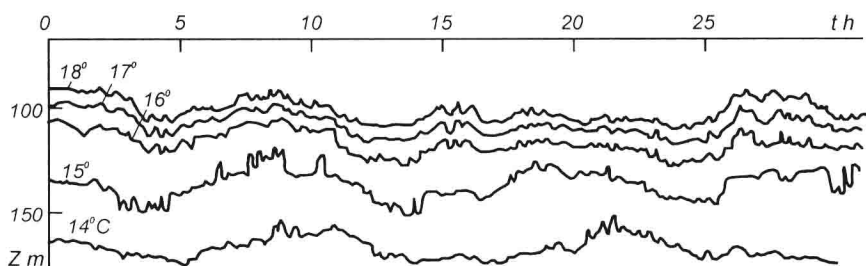


Figure 0.2. Depth variation of isotherms in the Atlantic Ocean [249].

precipitation processes also take part in the stratification formation. Nevertheless, thermal processes are evidently dominant.

Let us consider that some disturbance has appeared in a fixed stable stratified medium and has displaced liquid particles there from their mechanical equilibrium state. Then, under influence of the gravity and of the buoyancy forces aimed at the equilibrium state restoration, os-

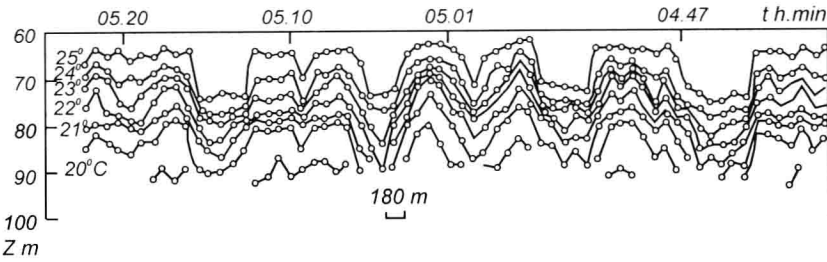


Figure 0.3. Depth variations of isotherms in the Atlantic Ocean caused by short period internal waves [276].

cillations in the fluid volume will start. Those oscillations present a phenomenon of internal waves propagating everywhere in the Ocean.

The parameters of ocean waves vary within a rather wide range. Low frequency internal waves may have lengths of tens and hundreds of kilometers, and propagation velocities up to several meters per second. Short period internal waves with periods from 5–10 min to 2–5 hrs. have lengths from several hundred meters to kilometers and propagation velocities of the order of several tens of centimeters per second. Amplitudes of low frequency internal waves may reach 100 m (Fig. 1). Short period internal waves evidently have amplitudes lower than 10–20 m.

Internal waves observed in the Ocean have different shapes. Long enough internal waves may be almost sinusoidal (Fig. 2). Shorter internal waves frequently have a shape different from sinusoidal and preferably propagate as wave trains (Fig. 3). Sometimes, especially in shallow water regions or in the coastal zone, solitary internal waves are observed (Fig. 4).

As a rule internal waves, having maximum amplitudes at some depth, cause only insignificant displacements at the free surface. But though these displacements are small they are not zero: seismographic study at the Arctic ice revealed oscillations with periods typical for internal waves (Fig. 5). Internal waves passing inside the ocean water are also confirmed by horizontal lines and highlights seen from satellites and airplanes (see, e.g., [6]).