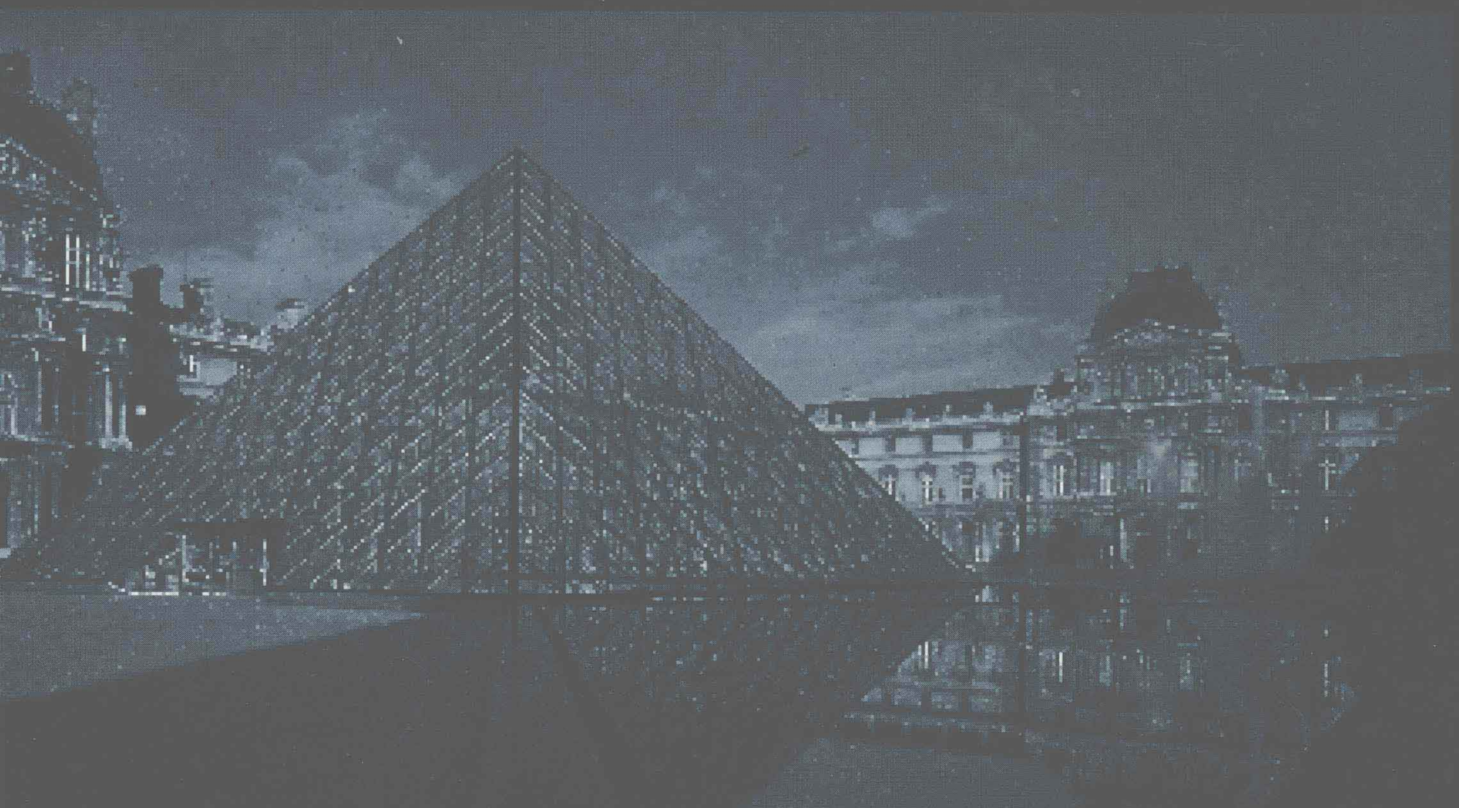
STUDENT MATHEMATICS HANDBOOK



CALCULUS

ERALD L. BRADLEY / KARL J. SMITH

STUDENT MATHEMATICS HANDBOOK

AND

INTEGRAL TABLE

FOR

CALCULUS

by Gerald L. Bradley and Karl J. Smith

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PREFACE

Calculus is probably the first mathematics course you have taken that is not self-contained in the sense that the material from previous mathematics courses is expected *without* specifically mentioning it in the textbook. This supplement is a convenient reference book to be used along with your textbook, *THE CALCULUS,* to remind you of those formulas or topics that you may have forgotten.

This handbook is organized so that it can be used in two ways. The first use is as a handbook to provide an easy reference manual providing a summary of terminology, formulas, and tables, not only of prerequisite mathematics, but also of the material covered in a standard calculus course. The second use is as a brief review of material assumed as a prerequisite for a course in calculus. This material is presented with examples, brief written exposition, and practice problems. The topics that are included with exposition and practice problems are sometimes missing from the background of many students who otherwise have the prerequisites for calculus. We remind the student that nearly everyone qualified to enroll in calculus has, for a variety of reasons, gaps in knowledge of prerequisite material, and the brief review in this handbook can help to bridge that gap. We provide this supplement to THE CALCULUS free of charge with the purchase of the textbook, in an attempt to help ensure your success in calculus. Use this book for reference, and as a handbook as you progress through the course.

In addition, new technology has changed the emphasis of many of the topics in a calculus course. One recent change is the acknowledgement of calculators and computers to help not only with the mechanics of algebra, but also with the mechanics of differentiation and integration in calculus. Outside of the academic environment, engineers and physicists tells us that using available technology, as well as tables of integration is by far more important than many of the esoteric topics they were taught in their calculus courses many years ago. For that reason, calculus books are evolving, and the emphasis is not on obscure esoteric topics, but rather practical knowledge which is a balance between application and theory. To capture this new emphasis, we see the need for you to have a complete integration table, so one has been provided in this handbook.

CONTENTS

CHAPTER 1

Review of Geometry

In this book we use the following variables when stating formulas: A = area, P = perimeter, C = circumference, S = surface area, and V = volume. Also, r denotes radius, h altitude, l slant height, b base, B area of base, θ central angle expressed in radians.

1.1 Polygons

CLASSIFICATION

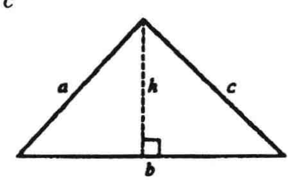
Type	Number of sides
triangle	3
quadrilateral	4
pentagon	5
hexagon	6
heptagon	7
octagon	8
nonagon	9
decagon	10
undecagon	11
dodecagon	12

TRIANGLES

$A = \frac{1}{2}bh$

$P = a + b + c$

The sum of the measures of the angles of a triangle is 180°.



Pythagorean theorem If angle C is a right angle, then $c^2 = a^2 + b^2$

45°-45°-90° triangle theorem For any right triangle with acute angles measuring 45° the legs are the same length, and the hypotenuse has a length equal to $\sqrt{2}$ times the length of one of those legs.

30°-60°-90° triangle theorem For any right triangle with acute angles measuring 30° and 60°:

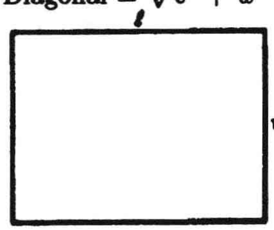
1. The hypotenuse is twice as long as the leg opposite the 30° angle (the shorter leg).
2. The leg opposite the 30° angle (the shorter leg) is $\frac{1}{2}$ as long as the hypotenuse.
3. The leg opposite the 60° angle (the longer leg) equals the length of the other (shorter) leg times $\sqrt{3}$.
4. The leg opposite the 30° angle equals the length of the other leg divided by $\sqrt{3}$.

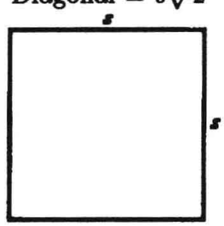
Equilateral triangle For any equilateral triangle:

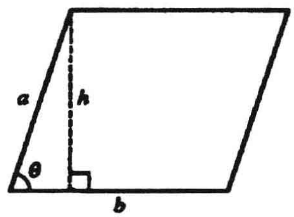
$\alpha = \beta = \gamma = 60°$ $\qquad A = \frac{1}{4}b^2\sqrt{3}$ $\qquad h = \frac{1}{2}b\sqrt{3}$

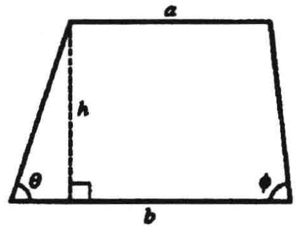
QUADRILATERALS

Rectangle	Square	Parallelogram	Trapezoid
$A = \ell w$	$A = s^2$	$A = bh = ab\sin\theta$	$A = \frac{1}{2}h(a+b)$
$P = 2\ell + 2w$	$P = 4s$	$P = 2a + 2b$	$P = a + b + h(\csc\theta + \csc\phi)$
Diagonal $= \sqrt{\ell^2 + w^2}$	Diagonal $= s\sqrt{2}$		





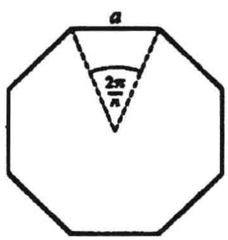




REGULAR POLYGON OF n SIDES

$A = \frac{1}{4}na^2\cot\frac{\pi}{n}$

$P = an$



central angle: $\frac{2\pi}{n}$

1.2
Circles

TERMINOLOGY

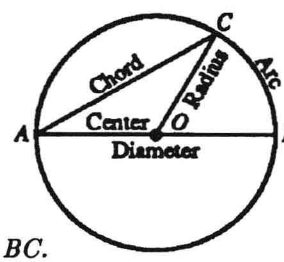


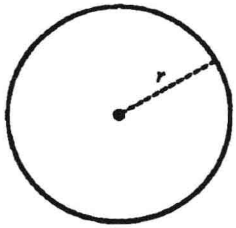
Figure 1.1

Circumference: distance around a circle.
Chord: a line joining two points of a circle.
Diameter: a chord through the center: AB is Figure 1.1.
Arc: part of a circle: BC, AC, or ACB in Figure 1.1. The length s of an arc of a circle of radius r with central angle θ (measured in radians) is $s = r\theta$.
To *intercept an arc* is to cut off the arc; in Figure 1.1, $\angle COB$ intercepts BC.
Tangent of a circle is a line that intersects the circle at one and only one point.
Secant of a circle is a line that intersects the circle at exactly two points.
Inscribed polygon is a polygon all of whose sides are chords of a circle. A regular inscribed polygon is a polygon all of whose sides are the same length.
Inscribed circle is a circle to which all the sides of a polygon are tangents.
Circumscribed polygon is a polygon all of whose sides are tangents to a circle.
Circumscribed circle is a circle passing through each vertex of a polygon.

BASIC FORMULAS

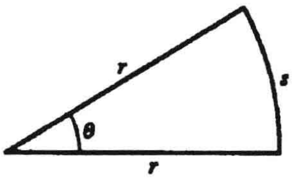
Circle

$A = \pi r^2$
$C = 2\pi r = \pi d$



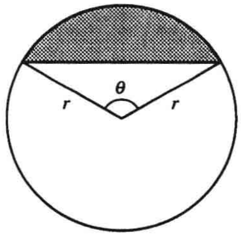
Sector

$A = \frac{1}{2}r^2\theta$
$s = r\theta$



Segment

$A = \frac{1}{2}r^2(\theta - \sin\theta)$



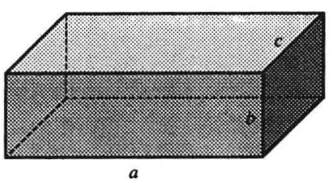
1.3

Solid Geometry

Rectangular parallelepiped (box)

$V = abc$

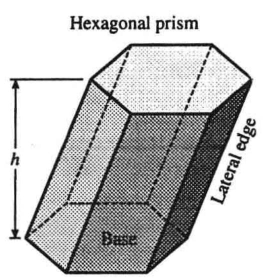
Diagonal $= \sqrt{a^2 + b^2 + c^2}$.



Prism

$V = Bh$

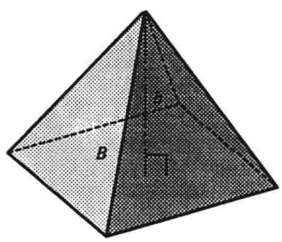
B is area of the base



Pyramid

$V = \frac{1}{3}Bh$

B is area of the base

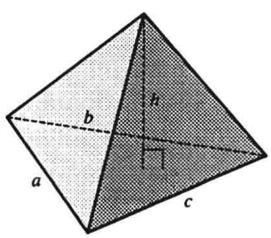


Tetrahedron

(A pyramid with a triangular base)

$V = \frac{1}{6}h\sqrt{s(s-a)(s-b)(s-c)}$

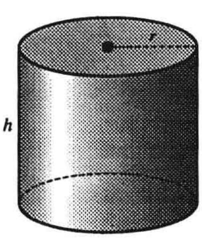
where $s = \frac{1}{2}(a + b + c)$



Right circular cylinder

$V = \pi r^2 h$
Lateral surface $= 2\pi rh$

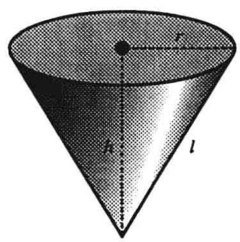
$S = 2\pi rh + 2\pi r^2$



Right circular cone

$V = \frac{1}{3}\pi r^2 h$
Lateral surface $= \pi rl$

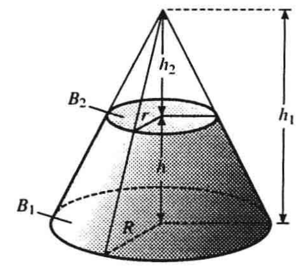
$S = \pi rl + \pi r^2$



Frustum of a right circular cone

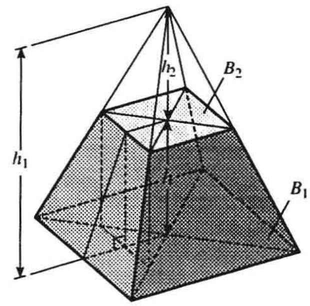
$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$ or

$V = \frac{1}{3}h(B_1 + \sqrt{B_1B_2} + B_2)$



Frustum of a pyramid

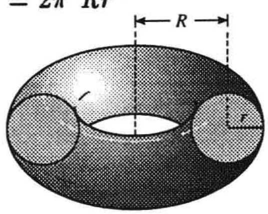
$V = \frac{1}{3}h(B_1 + \sqrt{B_1B_2} + B_2)$



Torus

$S = 4\pi^2 Rr$

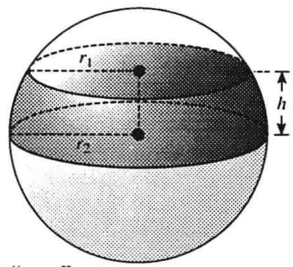
$V = 2\pi^2 Rr^2$



Spherical segment

$S = 2\pi r_2 h$

$V = \frac{1}{6}\pi h(3{r_1}^2 + 3{r_2}^2 + h^2)$



For the "cap"

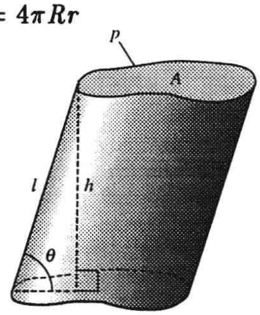
$S = 2\pi r_2(r_2 - h)$

$V = \frac{1}{3}\pi(r_2 - h)^2(2r_2 + h)$

Cylinder with a cross-sectional area A

$V = Ah; \quad S = p\ell + 2A$

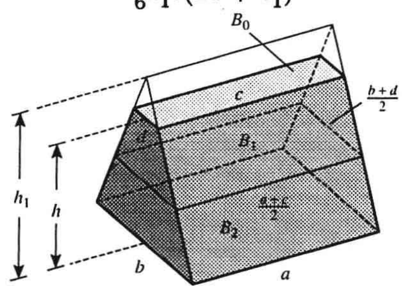
$S = 4\pi Rr$



Prismatoid, pontoon, wedge

$V = \frac{1}{6}h(B_0 + 4B_1 + B_2)$

$= \frac{1}{6}h_1 b(2a + c_1)$

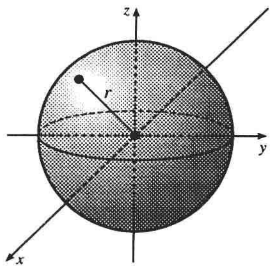


Quadric Surfaces

Sphere

$x^2 + y^2 + z^2 = r^2$

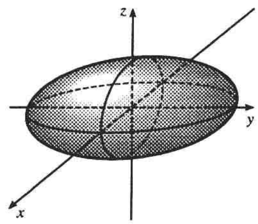
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$



Ellipsoid

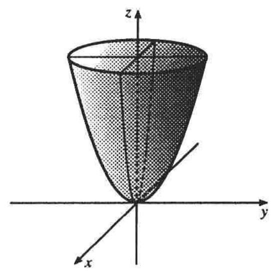
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$V = \frac{4}{3}\pi abc$



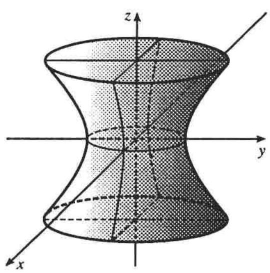
Elliptic Paraboloid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$



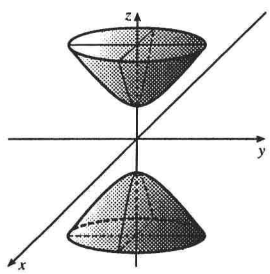
Hyperboloid of One Sheet

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



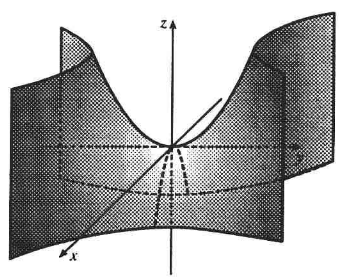
Hyperboloid of Two Sheets

$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



Hyperbolic paraboloid

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$



Oblate spheroid is formed by the rotation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its minor axis, b. Let ϵ be the eccentricity. $V = \frac{4}{3}\pi a^2 b$ $S = 2\pi a^2 + \pi \frac{b^2}{\epsilon} \ln\left(\frac{1+\epsilon}{1-\epsilon}\right)$

Prolate spheroid is formed by the rotation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its minor axis, a. Let ϵ be the eccentricity. $V = \frac{4}{3}\pi ab^2$ $S = 2\pi b^2 + \pi \frac{ab}{\epsilon} \sin^{-1}\epsilon$

1.4

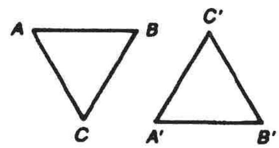
Congruent Triangles

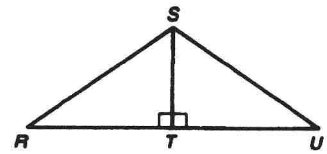
We say that two figures are **congruent** if they have the same size and shape. For **congruent triangles** ABC and DEF, denoted by $\triangle ABC \simeq \triangle DEF$, we may conclude that all six corresponding parts (three angles and three sides) are congruent. We call these **corresponding parts.**

EXAMPLE 1 **Corresponding parts of a triangle**

Name the corresponding parts of the given triangles.

a. $\triangle ABC \simeq \triangle A'B'C'$ **b.** $\triangle RST \simeq \triangle UST$





Solution

a.
$\overline{AB}$ corresponds to $\overline{A'B'}$
$\overline{AC}$ corresponds to $\overline{A'C'}$
$\overline{BC}$ corresponds to $\overline{B'C'}$
$\angle A$ corresponds to $\angle A'$
$\angle B$ corresponds to $\angle B'$
$\angle C$ corresponds to $\angle C'$

b.
$\overline{RS}$ corresponds to $\overline{US}$
$\overline{RT}$ corresponds to $\overline{UT}$
$\overline{ST}$ corresponds to $\overline{ST}$
$\angle R$ corresponds to $\angle U$
$\angle RTS$ corresponds to $\angle UTS$
$\angle RST$ corresponds to $\angle UST$

□

Line segments, angles, triangles, or other geometric figures are *congruent* if they have the same size and shape. In this section, we focus on triangles.

CONGRUENT TRIANGLES

> Two triangles are **congruent** if their corresponding sides have the same length and their corresponding angles have the same measure.

In order to prove that two triangles are congruent, you must show that they have the same size and shape. It is not necessary to show that all six parts (three sides and three angles) are congruent; if some of these six parts are congruent, it necessarily follows that the other parts are congruent. Three important properties are used to show triangle congruence:

CONGRUENT TRIANGLE PROPERTIES

SIDE-SIDE-SIDE (SSS)
If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.

SIDE-ANGLE-SIDE (SAS)
If two sides of one triangle and the angle between those sides are congruent to the corresponding sides and angle of another triangle, then the two triangles are congruent.

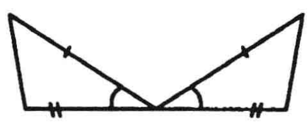
ANGLE-SIDE-ANGLE (ASA)
If two angles and the side that connects them on one triangle are congruent to the corresponding angles and side of another triangle, then the two triangles are congruent.

EXAMPLE 2 **Finding congruent triangles**

Determine if each pair of triangles is congruent. If so, cite one of the congruent triangle properties.

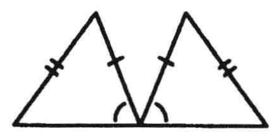
Solution

a.



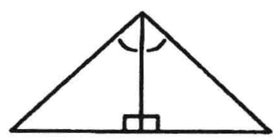
Congruent; SAS

b.



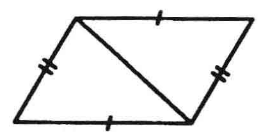
Not congruent

c.



Congruent; ASA

d.



Congruent; SSS

A side that is in common to two triangles obviously is equal in length to itself, and it does not need to be marked. □

In geometry, the main use of congruent triangles is when we want to know whether an angle from one triangle is congruent to an angle from a different triangle, or when we want to know whether a side from one triangle is the same length as the side from another triangle. In order to do this, we often prove that one triangle is congruent to the other (using one of the three congruent triangle properties) and then use the following property:

CONGRUENT TRIANGLE PROPERTY

Corresponding parts of congruent triangles are congruent.

1.5

Similar Triangles

It is possible for two figures to have the same shape, but not necessarily the same size. These figures are called **similar** figures. We will now focus on **similar triangles.** If $\triangle ABC$ is similar to $\triangle DEF$, we write

$$\triangle ABC \sim \triangle DEF$$

Similar triangles are shown in Figure 1.2.

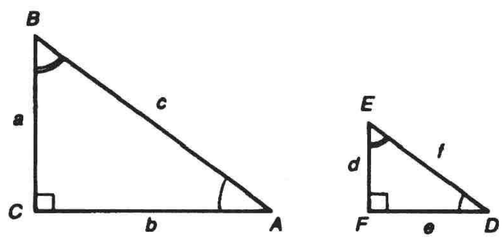


Figure 1.2 Similar triangles

You should note that congruent triangles must be similar, but similar triangles are not necessarily congruent. Since similar figures have the same shape, we talk about **corresponding angles** and **corresponding sides.** The corresponding angles of similar triangles are the angles that have the same measure. It is customary to label the vertices of triangles with capital letters and the sides opposite the angles at those vertices with corresponding lower-case letters. It is easy to see that, if the triangles are similar, the corresponding sides are the sides opposite equal angles. In Figure 1.2 we see that

$\angle A$ and $\angle D$ are corresponding angles;
$\angle B$ and $\angle E$ are corresponding angles; and
$\angle C$ and $\angle F$ are corresponding angles.

Side a ($\overline{BC}$) is opposite $\angle A$, and d ($\overline{EF}$) is opposite $\angle D$, so we say that a corresponds to d;
b corresponds to e; and
c corresponds to f.

Even though corresponding angles are the same size, corresponding sides do not need to have the same length. If they do have the same length, then the triangles are congruent. However, when they are not the same length, we can say they are *proportional.* As Figure 1.2 illustrates, when we say the sides are proportional, we mean

$$\frac{a}{b} = \frac{d}{e} \qquad \frac{a}{c} = \frac{d}{f} \qquad \frac{b}{c} = \frac{e}{f}$$

$$\frac{b}{a} = \frac{e}{d} \qquad \frac{c}{a} = \frac{f}{d} \qquad \frac{c}{b} = \frac{f}{e}$$

SIMILAR TRIANGLES

Two triangles are **similar** if two angles of one triangle have the same measure as two angles of the other triangle. If the triangles are similar, then their corresponding sides are proportional.

EXAMPLE 1 — **Similar triangles**

Identify pairs of triangles that are similar in Figure 1.3.

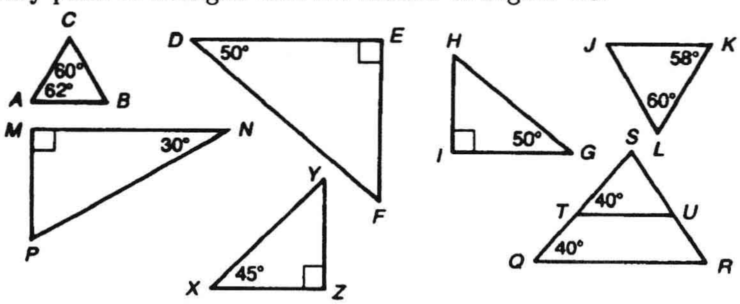
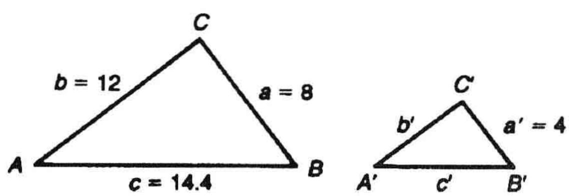


Figure 1.3

Solution $\triangle ABC \sim \triangle JKL$; $\triangle DEF \sim \triangle GIH$; $\triangle SQR \sim \triangle STU$. □

EXAMPLE 2 — **Finding Unknown lengths in similar triangles**

Given the similar triangles below, find the unknown lengths marked b' and c'.



Solution Since corresponding sides are proportional (other proportions are possible), we have

$$\frac{a'}{a} = \frac{b'}{b} \qquad\qquad \frac{a}{c} = \frac{a'}{c'}$$

$$\frac{4}{8} = \frac{b'}{12} \qquad\qquad \frac{8}{14.4} = \frac{4}{c'}$$

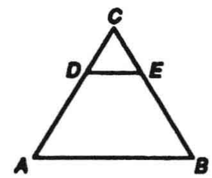
$$b' = \frac{4(12)}{8} \qquad\qquad c' = \frac{14.4(4)}{8}$$

$$= 6 \qquad\qquad = 7.2$$

□

EXAMPLE 3 **Finding a perimeter using similar triangles**

In equilateral $\triangle ABC$, suppose $DE = 2$ and is parallel to $\overline{AB}$ as shown at right. If $\overline{AB}$ is three times as long as DE, what is the perimeter of quadrilateral $ABED$?



Solution

$\triangle ABC \sim \triangle DEC$ so $\triangle DEC$ is equilateral. This means that $\overline{CE}$ and $\overline{DC}$ both have length 2; thus, $\overline{EB}$ and $\overline{AD}$ both have length 4. The perimeter of the quadrilateral is

$$|\overline{AB}| + |\overline{BE}| + |\overline{DE}| + |\overline{AD}| = 6 + 4 + 2 + 4 = 16$$ □

Finding similar triangles is simplified even further if we know the triangles are right triangles, because then the triangles are similar if one of the acute angles has the same measure as an acute angle of the other.

EXAMPLE 4 **Using similar triangles to find an unknown length**

Suppose that a tree and a yardstick are casting shadows as shown in Figure 1.4. If the shadow of the yardstick is 3 yards long and the shadow of the tree is 12 yards long, use similar triangles to estimate the height of the tree if you know that angles S and S' are the same size.

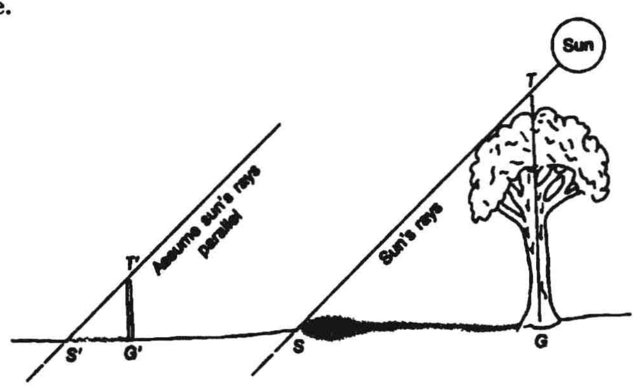


Figure 1.4

Solution

Since $\angle G$ and $\angle G'$ are right angles, and since S and S' are the same size, we see that $\triangle SGT \sim \triangle S'G'T'$. Therefore, corresponding sides are proportional.

$$\frac{1}{3} = \frac{h}{12}$$

$$h = \frac{1(12)}{3}$$

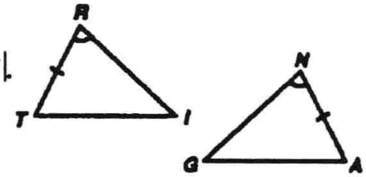
$$= 4$$

The tree is 4 yards tall. □

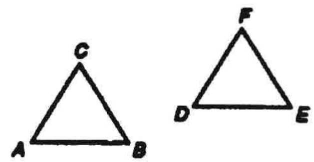
1.6 PROBLEM SET

A

1. In $\triangle TRI$ and $\triangle ANG$ shown at right, $\angle R \simeq \angle N$ and $|\overline{TR}| = |\overline{AN}|$. Name other pairs you would need to know in order to show that the triangles are congruent by
   a. SSS **b.** SAS **c.** ASA

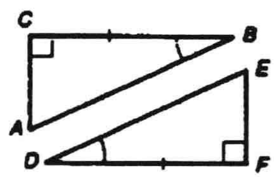


2. In $\triangle ABC$ and $\triangle DEF$ shown at right, $\angle A \simeq \angle D$ and $|\overline{AC}| = |\overline{DF}|$. Name other pairs of would need to know in order to show that the triangles are congruent by
   a. SSS **b.** SAS **c.** ASA

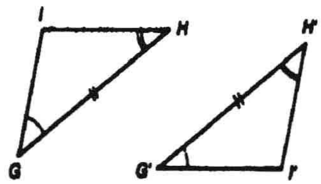


Name the corresponding parts of the triangles in Problems 3–6.

3.



4.



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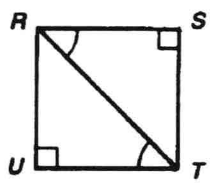
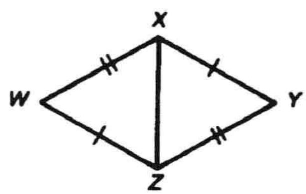
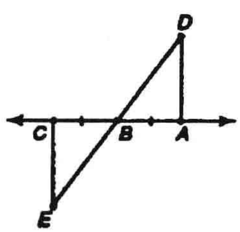


6.



In Problems 7–10, determine if each pair of triangles is congruent. If so, cite one of the congruent triangle properties.

7.



8.