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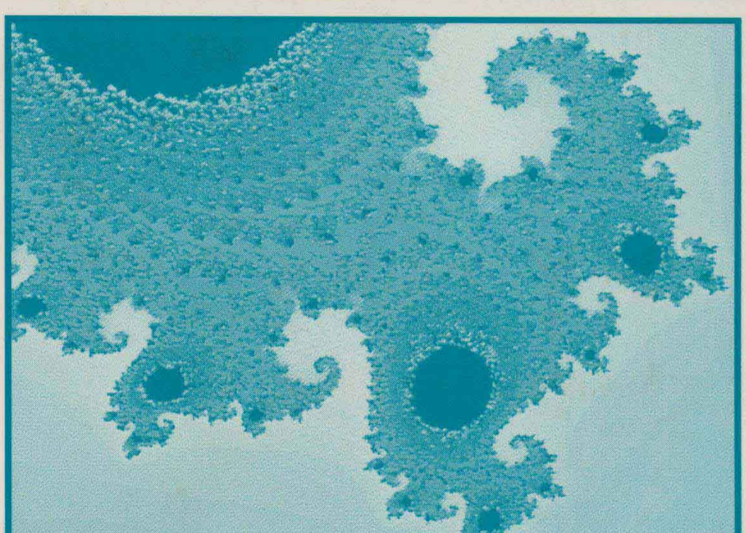
VOLUME 2

CROSS-FLOW INDUCED VIBRATION OF CYLINDER ARRAYS



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CROSS-FLOW INDUCED VIBRATION OF CYLINDER ARRAYS

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PREFACE

This symposium is the sequel to the successful 1984 and 1988 symposia on the same subject, co-sponsored by the same six Divisions of ASME (Applied Mechanics, Fluids Engineering, Heat Transfer, Noise Control and Acoustics, Nuclear Engineering, and Pressure Vessels and Piping) and by JSME, CSME (Canada), IMechE and IAHR. More truly international, this third symposium has papers from 16 countries, other than the U.S.A. and from four continents.

The proceedings of the 24 sessions (with more than 100 papers) devoted to the FIVN symposium will be published in the following set of bound volumes:

- Vol. 1 FSI/FIV in Cylinder Arrays in Cross-Flow
- Vol. 2 Cross-Flow-Induced Vibration of Cylinder Arrays
- Vol. 3 Flow-Structure and Flow-Sound Interaction
- Vol. 4 Acoustical Effects of FSI
- Vol. 5 Axial-and Annular-Flow-Induced Vibrations and Instabilities
- Vol. 6 Bluff-Body/Fluid and Hydraulic Machine Interactions
- Vol. 7 Fundamental Aspects of Fluid-Structure Interactions
- Vol. 8 Stability and Control of Pipes Conveying Fluid

In the Foreword of each volume will generally be found comments on the subject matter covered by the papers, and on trends and developments in the field. Comparing the papers in these proceedings to those in the 1984 and 1988 symposia, the increased sophistication in analytical and computational techniques is striking, with the use of nonlinear analysis and CFD methods becoming more widespread. Similarly, equally impressive are developments in experimental and data-manipulation techniques.

The four-yearly cycle of these symposia has been coordinated with the European ("Keswick") series of symposia, the last two having successfully taken place in the U.K. in late spring, at Bowness-on-Windermere in 1987 and Brighton in 1991; the next one is planned for 1995 in London. However, others would like to host such events: one was being planned for 1991, then 1992, in the USSR, before being overtaken by events; but the successor states are equally keen, although not for the immediate future. Another, on Aero-Hydro-Elasticity was held in 1989 in Prague, Czechoslovakia, and another is being proposed for 1994. China and Japan are very keen on hosting symposia or conferences on FSI and FIVN. Clearly, coordination is required to avoid having these conferences too close together. I propose that the ASME-based series of symposia should occasionally be held in the Asia–Pacific area [with co-sponsorship by JSME and CSME (China) and others], while the European series be held sometimes in Western Europe and sometimes in Eastern Europe (including Russia), if this distinction still matters a few years down the road. How feasible and successful this would be one cannot judge before trying it out. The state of global economics is a key factor.

I would like to thank the six divisions of ASME and the international sponsors for their support, the ASME meetings and publications staff (especially June Leach and Barbara Signorelli) for their excellent work and cheery cooperation, and the Session Developers and Co-Editors of these proceedings for their exceptionally conscientious work; finally, of course, the authors and reviewers of the papers, who are the most essential of all the elements that go together towards a successful symposium with high-quality papers. Special thanks to Professor K. T. Yang and Richard Gwaltney for giving us the extra sessions we needed and for arranging things so that we did not have parallel sessions — an important, but difficult undertaking. Special personal thanks go to Dr. S. S. Chen and Vrisseïs Mavrou-Païdoussis, my wife, for extra support and help with all kinds of symposium matters when I needed it most (when I was sick earlier in the year), and to Mary Fiorilli for her virtuoso typing of the mountains of paperwork such symposia neccesitate.

Michael P. Païdoussis
Symposium Coordinator

FOREWORD

This volume contains 20 papers presented in the four sessions of the symposium devoted to *Cross-Flow-Induced Vibration of Cylinder Arrays.* The papers came from six countries: Canada (7 papers), France (5), USA (3), Portugal and France (2), Germany (2), and Japan (1). The geographic origins of the papers provide a representation of the current research activities on this subject around the world. In the previous symposium on this topic in 1988, it was noted that "fluidelastic instability of a group of cylinders in crossflow is one of the most debated and confusing topics....in recent history." Efforts at understanding fluidelastic instability of cylinder arrays in crossflow are continuing with the objective of developing better design guides for system components, to avoid this detrimental fluid/structure interaction phenomenon.

Various techniques are used to explore the physics of complicated coupled flow/cylinder interactions, including analytical models, experiments, and integrated analytical/experimental approaches. New and improved analytical and numerical models are presented to give better prediction capabilities. Numerical techniques, including finite-element, finite-difference and boundary-element methods, are used. Additionally, new or improved instrumentation and measurement techniques are employed to obtain new or more reliable experimental data. A wide range of parameters is covered, such as cylinder arrangements, including square and triangular arrays, and tube rows including straight and U tubes, cylinder pitch from 1.25 to 2.5, mass damping parameter of 2 to 500, and tests in air and water.

Most of the papers investigate cylinder/support interaction, because in practical system components such as heat exchanger tubes, the cylinders are loosely supported. The coupled flow/cylinder system with loose supports is a classical nonlinear fluid/structure system with many intriguing physical characteristics. A series of topics is considered, as follows:

- *Computer Simulation:* Different computer models are presented to predict cylinder response due to fluidelastic and turbulence buffeting.
- *Contact Force:* Measurement-device development, data analysis techniques, and characterization of contact forces are described; and loose support with circular oversize openings, antivibration bars, and baffle plates is employed. Other factors, such as Coulomb friction and stiffness, are also investigated.
- *Wear:* Many parameters affect the wear caused by cylinder/support interaction. The roles of different parameters such as preload due to steady fluid forces, clearance, and fluid in the clearance are studied.
- *Chaotic Vibration:* In a typical continuous nonlinear system, the response characteristics include periodic motion, quasiperiodic oscillation, chaotic vibration, and random vibration. Analytical and experimental studies are present to describe system behavior as measured by many different tools, including phase portraits, power spectral density, Poincaré maps, Lyapunov exponents, fractal di-

mensions, and bifurcation diagrams.

- *Mechanical Simulation:* Because fluid forces are difficult to quantify at this time, mechanical models are used to simulate motion-dependent fluid forces.

The theoretical models for fluidelastic instability continue to receive attention. A key element in predicting fluidelastic instability is motion-dependent fluid force. Approximate fluid dynamics is presented to compute the fluid force based on the pressure distribution function on the cylinder surface. An analytical model for fluidelastic instability that uses the wavy-wall channel model was originally developed for a single flexible cylinder, but has now been modified to improve its capabilities by using nonlinear lift and drag forces to provide better agreement with experimental data and for applications to multiple elastic cylinders. A parametric study is also presented for cylinder arrays with open lanes and nonuniform flow distribution with a stability model.

An interesting characteristic is the hysteresis behavior of fluidelastic instability, which is investigated analytically and experimentally to determine whether it is associated with systems with multiple flexible cylinders in the array only. Experimental data reconfirm that if it is associated with fluid-damping-controlled instability, fluid coupling between cylinders is not a necessary condition. To predict the hysteresis, nonlinear lift and drag forces are incorporated in the unsteady flow model (in particular, the dependence of fluid force on tube vibration amplitudes).

Among other factors considered is the question of how many tubes are needed to simulate the fluidelastic instability in practical cases. Depending on the instability mechanisms, for fluid-damping-controlled instability a single flexible cylinder will provide the basic characteristics. However, for fluid-stiffness-controlled instability, at least a two-degree-of-freedom system is needed. Additional analytical and experimental studies have been performed to reconfirm thsese conclusions.

This volume describes many elaborate analyses and tests developed to reveal the physical aspects of coupled flow/cylinder interactions. New characteristics are found, additional insights are given, more reliable experimental data are obtained, and better prediction methods are developed. However, it is fair to say that the subject remains as "baffling and confusing" a topic as it was in 1988. Until the detailed interaction process of the flow/cylinder interaction becomes well understood, the subject will continue to remain a fascinating topic.

We thank the authors for their willingness to share their expertise and experience with the scientific community. As in the case of the 1984 and 1988 symposia, the "fire fighters" in the worldwide battle to reduce the damage caused by fluidelastic instability are the major contributors to this volume. We also appreciate the reviewers for their efforts in providing many constructive comments that have enhanced the quality of the papers.

S. S. Chen
D. A. Steininger
M. P. Païdoussis

CONTENTS

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A STUDY ON AN APPROXIMATE THEORY OF FLUIDELASTIC VIBRATION OF TUBE ARRAY: PART 1 – BASIC CONCEPT OF FLUIDELASTIC FORCE AND INSTABILITY OF TUBE ROWS

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ABSTRACT

This study is on the behavior of the so-called fluidelastic vibration of a tube array caused by cross flow. The theory is based on an approximate fluid dynamics, where the fluid force acting on a tube is computed with the pressure distribution on the tube surface and is associated with the movement of the tube itself. At first, the onset of the instability based on the equivalent linearization method has been discussed with the comparison of the experimental data. Then, the estimation of the separation point of boundary layer reveals an important role to the fluidelastic instability. Finally, the unstable vibration of the tube array is analyzed with a time-history calculation of equation of motion for the tube.

NOMENCLATURE

$a_1 \sim a_4$	Coefficients in the equation (17)
$b_1 \sim b_4$	Coefficients in the equation (17)
$C_p(\theta)$	Normalized pressure distribution around the surface of a tube
C_{pb}	Normalized pressure at far field
C_{po}	Variation range of $C_p(\theta)$
C_s	Damping factor of a tube
$[C_s]$	Damping matrix of a tube
D	Tube diameter
f	Frequency of a tube
$\{F\}$	Force vector acting on a tube
F_D	Fluid force in the in-flow direction
F_L	Fluid force in the cross flow direction
g_i	Fluid force in the in-flow direction acting on i-th tube
h_i	Fluid force in the cross flow direction acting on i-th tube
$k_e(y_o)$	Equivalent stiffness for displacement y_o
K	Constant for an instability
K_s	Stiffness of a tube
$[K_s]$	Stiffness matrix of a tube
m	Mass of a tube per unit length
M	Mass of a tube
$[M_F]$	Added mass matrix
$[M_s]$	Mass matrix of a tube
Re	Reynolds number
T	Pitch of tube array

u_k	Displacement of k-th tube in the in-flow direction
U	Gap Flow velocity
U_c	Critical flow velocity
U_∞	Approach flow velocity
v_k	Displacement of k-th tube in the cross flow direction
y	Displacement of a tube in the cross flow direction
y_o	Maximum displacement of a tube
α_{ik}, α_{ik}', α_{ik}''; β_{ik}, β_{ik}', β_{ik}''; σ_{ik}, σ_{ik}', σ_{ik}''; τ_{ik}, τ_{ik}', τ_{ik}''	Fluid force coefficients related to acceleration, velocity and displacement of the tube in each direction
δ	Logarithmic decrement factor of a tube
θ_s	Angle of separation point
$\lambda_e\ (y_o)$	Equivalent damping for displacement y_o
μ	Ratio of pressure decrease at separation point
ρ	Mass density of fluid
ϕ_s	Phase lag term between tube displacement and the separation point
ϕ_s	Real phase lag between tube displacement and the separation point
ψ	Phase angle of a tube displacement ($=\omega t$)
ω	Circular frequency
ω_o	Circular natural frequency of a tube

1. INTRODUCTION

The unstable oscillation of tube arrays caused by cross flow, named as "fluidelastic vibration" by Connors in 1970 [1], has been investigated by a number of researchers. Connors assumed the fluid force which is a linear function of the displacement of the tube causes a negative damping effect when the flow velocity is greater than a certain level. His assumption is classified as the fluid-stiffness theory according to Chen [2], however, the following criterion obtained by Connors is very simple and useful for design use even today, because this criterion is basically derived from the energy balance between the work by fluid force and the dissipation energy by tube motion.

$$\frac{U_C}{fD} = K\left[\frac{m\delta}{\rho D^2}\right]^{1/2} \tag{1}$$

There are a number of data for the coefficient K in equation (1) and some hypotheses have been proposed to explain the fluidelastic vibration. The most general expression for the fluidelastic force is given by Chen [2] as in the followings:

$$g_i = -\frac{\rho\pi}{4}D^2\sum_{k=1}^{N}\left(\alpha_{ik}\frac{\partial^2 u_k}{\partial t^2} + \sigma_{ik}\frac{\partial^2 v_k}{\partial t^2}\right) + \frac{\rho U^2}{\omega}\sum_{k=1}^{N}\left(\alpha'_{ik}\frac{\partial u_k}{\partial t} + \sigma'_{ik}\frac{\partial v_k}{\partial t}\right) + \rho U^2\sum_{k=1}^{N}\left(\alpha''_{ik}u_k + \sigma''_{ik}v_k\right) \tag{2}$$

$$h_i = -\frac{\rho\pi}{4}D^2\sum_{k=1}^{N}\left(\tau_{ik}\frac{\partial^2 u_k}{\partial t^2} + \beta_{ik}\frac{\partial^2 v_k}{\partial t^2}\right) + \frac{\rho U^2}{\omega}\sum_{k=1}^{N}\left(\tau'_{ik}\frac{\partial u_k}{\partial t} + \beta'_{ik}\frac{\partial v_k}{\partial t}\right) + \rho U^2\sum_{k=1}^{N}\left(\tau''_{ik}u_k + \beta''_{ik}v_k\right) \tag{3}$$

These equations contain both the fluid-stiffness effect and the fluid-damping effect in the fluid force coefficient terms, α', α'', $\cdots$, which can be regarded as the first-order approximation terms of the Taylor expansion of the fluid force with the displacement and the velocity of the tube [3]. Some tests have been done to obtain these coefficients and the calculated results have shown a reasonable agreement with the experimental data with respect to the critical flow velocity.

On the other hand, the fluidelastic force is considered to be an unsteady force having a time delay effect [3] between the force and the tube motion. However, when the time delay effect had to be introduced to the above fluid coefficients, some experimental data might be needed to estimate it for each case because there is a gap between the physical explanation and the engineering formula as Eq.(2).

From this experience, authors have decided to re-study the fluidelastic instability from the different viewpoint, where the physical insight to the fluid dynamics has great importance to solve the fluidelastic phenomenon as likely as Lever and Weaver [4]. The final purpose of our study is not the determination of the formula for stability boundary but is the estimation of the unstable response, including the response by two-phase mixture flow. Since it may be a long way to complete this study, we have tried to make up a basic frame of the theory which will be useful and must be flexible for the above purpose. But the basic concept of the theory concerning the stability boundary will be introduced in this paper.

This theory has been found to be similar to that of Roberts [5], where an approximate model for the unsteady fluid force has been proposed to explain the existence of an unstable oscillation of a tube in a staggered tube array in in-flow direction. His work had been published some years before the Connors' paper, however, the basic idea for this kind of instability might be useful even today. It is because the physical approach should be the same as his work.

There are some differences between this paper and his as will be explained later, however, we have again to recognize that the physical insight might be important to start a new work. Here, it is the estimation of the response after instability. This paper shows only the beginning of our theory, because it would take a great volume [6]. The further study should be published in the sequential paper.

2. THEORY

2.1 Insight to Fluidelastic Vibration

Since many papers have been published concerning the fluidelastic instability [8] and a large number of tests have been done, the study of the fluidelastic vibration should start for deducing the general trend from those experimental results. Summarizing these experimental results, the following features could be pointed out:

(1) This instability can be observed even in a row of tubes, however, it cannot occur in a single tube.
(2) A single flexibly-supported tube can be unstable if it is in a tube array. This means that the energy transformation from fluid flow to the tube movement may be made through a jet-like flow and may have an important role.
(3) The geometry of the tube array affects the stability boundary. Besides this effect, the position of the tube flexibly supported in a tube array can change the critical flow velocity [9].
(4) The viscosity of the fluid seems to have small effect on the critical flow velocity. This is based on the fact that the increase of the temperature shows almost no effect on stability [10].

From the above informations, the important mechanism for the instability might be considered as the following: One is the fluid force induced by the jet-like flow along a narrow path between the tubes, which has been regarded as the dominant mechanism by Lever and Weaver [4]. The other is the movement of separation point along the tube surface.

The fundamental concept of the fluidelastic vibration in this paper is that the fluid force acting on a tube in a tube array may change its magnitude according to the movement of the tube and that the change in fluid force is closely related with the movement of the separation point combined with the jet-like flow effect on the fluid force. Then the next problem is how to evaluate the magnitude of the fluid force and the movement of the separation point relating to the motion of the one of tubes in an array.

2.2 Estimation of Unsteady Fluid Force

To make the problem simple, the following assumptions have at first been introduced:

(1) The fluidelastic vibration is assumed to occur only in the lift direction. This assumption has been widely employed by the published papers [11], [12] and [13].
(2) Tubes in an array oscillate in the out-of-phase mode in the lift direction as Fig.1. This mode is usually observed at the onset of the unstable vibration as likely as Lever and Weaver [4] indicated.

As a result, for the mode pattern in Fig.1, a tube moves in the manner of mirror-image like having a mirror plane in the middle of the gap to the adjacent tubes. Then, the magnitude of the fluid force acting on the tube, which is modeled as a quasi-static force, might be approximated as follows if we use some published data and introduce a simplification to them.

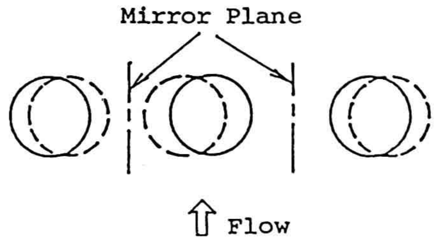


Fig.1 Out-of-phase Mode Pattern of Instability

Okamoto et al. [14] have investigated the pressure distribution on the surface of the single tube in a jet-like flow path surrounded by the parallel walls. They gave the nondimensional pressure distribution $C_p(\theta)$ as shown in Fig.2. This figure has been confirmed by Bearman et al. [15] and Muraoka [16], and this type of pressure distribution might be a typical pattern even in the case of tube bundle (See Nishikawa et al. [17] and Kobayashi [18]). This type of the pressure distribution can be computed by a potential flow theory where the separation point should be assumed as Parkinson et al. [19] and Shiina [20] did, however, the nondimensional pressure distribution of $C_p(\theta)$ is briefly assumed as shown in Fig.3 in this paper.

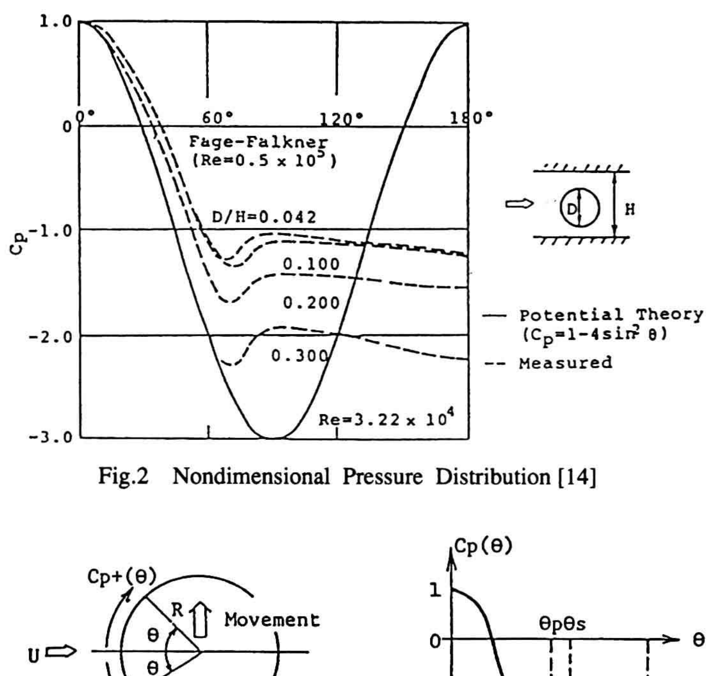


Fig.2 Nondimensional Pressure Distribution [14]

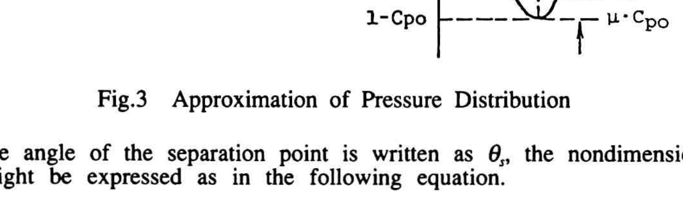


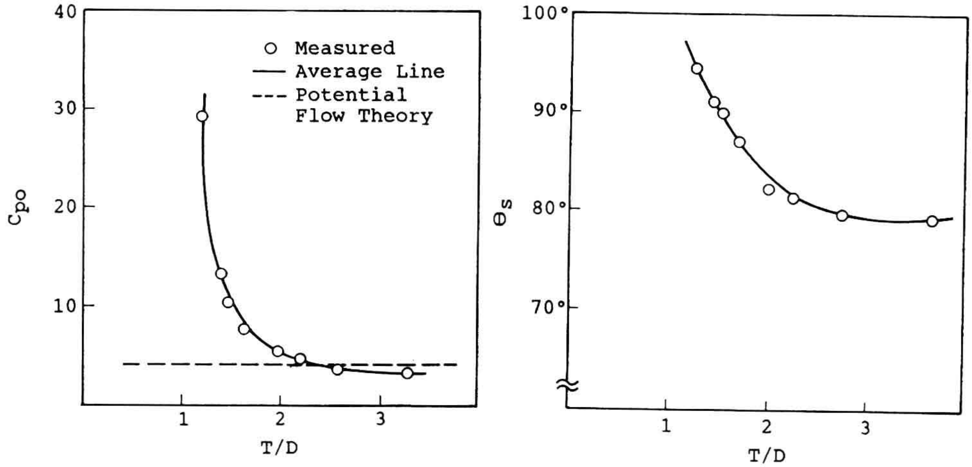
Fig.3 Approximation of Pressure Distribution

When the angle of the separation point is written as θ_s, the nondimensional pressure distribution might be expressed as in the following equation.

$$C_p(\theta)=\begin{cases}1-C_{po}\sin^2\left[\dfrac{\pi}{2\theta_p}\cdot\theta\right] & (0\le\theta\le\theta_s)\\ 1-(1-\mu)C_{po} & (\theta\ge\theta_s)\end{cases} \tag{4}$$

where, $\theta_p=\dfrac{\pi\theta_s}{2\sin^{-1}\left(\sqrt{1-\mu}\right)}$

The variation range of the pressure, C_{po}, can be changed according to the position of the tube in a narrow path. C_{po} can be regarded as a function of the tube displacement, X. The experimental data obtained by Hiwada et al. [21] for $Re\approx10^4$ and for some values of T/D including $T/D\approx0.667$ can be depicted in Fig.4 (a). This empirical formula (4) may have a similar shape to the Roberts' [5], however, the equation (4) might be easier to estimate the effect on the stability boundary than the Roberts'.



(a) Variation Range of Pressure Distribution

(b) Relation between Tube Gap and Separation

Fig.4 Parameters for Pressure Distribution [21]

To give the complete set of equation (4) from the above test results, the variation range of the nondimensional pressure distribution, C_{po}, can be approximated as a function of the tube displacement by using the fourth order of least-square method,

$$C_{po}(y)=277.6\left(\frac{y+D}{T}-0.25\right)^4+3.1 \tag{5}$$

when the C_{po} in Fig.4 (a) is depicted by an approximation formation. This is rather accurate in the vicinity of $(y+D)/T\approx0.667$. The magnitude of the fluid force can be calculated by integrating the pressure distribution around the surface of the tube, however, before doing it, another parameter included in the above equation has to be examined, because this kind of fluid force might be regarded to produce the stiffness-controlled instability as the displacement mechanism [22]. And there is another type of instability called fluid-damping one, which is regarded as the velocity mechanism. The following formulas are mainly to introduce the unsteady effect of the fluidelastic force, despite the fact that the estimation is closely related to that of the separation point.

Equation (4) shows that the pressure distribution is also the function of the separation point θ_s, and thus the unsteady fluid force is considered to be associated with the movement of θ_s in relation to the tube response. Authors have obtained two test results with respect to the separation point θ_s. One is the data [23] for a single tube which was sinusoidally oscillated in the lift direction, and another is the one [24] for a tube arranged in an array which was sinusoidally oscillated in the out-of-phase mode in the lift direction as in Fig.1.

(1) Data by a single tube: Nagata et al. [23] studied the fluid force acting on a single tube oscillating in the cross flow direction. They found that the smallest separation angle can be observed when the tube moves to the separation point and just before the displacement of the tube becomes the maximum, where the separation is about 40 degrees in advance to the displacement.

From this observation, the separation point θ_s might be considered as a function of the tube displacement, however, it should be the function of the tube velocity and Reynolds number, et al.. The experimental data by Hiwada et al. [21] can be depicted in Fig.5 as the relationship of the separation point to D/T, despite data was obtained from a static experiment.

The separation point θ_s can also be expressed as a function of the displacement as follows using the result in Fig.4 (b), while the separation angle is estimated to be θ_s=80° in y=0, because this formula might be physically useful for the subcritical flow range of single tube.

$$\theta_s(y,\dot{y}) = 52° + 40°\left(\frac{y+D}{T}\right) + \bar{\theta}_s(\dot{y}) \tag{6}$$

Here, the term, $\bar{\theta}_s(\dot{y})$, is introduced to express the velocity-dependent term of the separation point. If the term can be approximated as a linear function of the tube velocity, it is expressed as in the following equation, because we have the fact that the variation range of the separation point is 11.5° when $\dot{y}/f{\cdot}D$ equals to 1.26 according to the test of Nagata et al. [23].

$$\bar{\theta}_s(\dot{y}) = 57.3°\left(\frac{\dot{y}}{\omega D}\right) \tag{7}$$

However, the motion of the separation point can be assumed to have some phase lag, ϕ_s, to the tube displacement, which can be expressed as Δt in Eq.(8).

$$\theta_s(y,\dot{y},t) = 52° + 40°\left(\frac{y(t)+D}{T}\right) + 57.3°\frac{\dot{y}(t-\Delta t)}{\omega D} \tag{8}$$

(2) Data by a tube array: Hara [24] showed that the separation point moves with the progressive phase angle of 20° to of 30° to the displacement of the tube arranged in an array as shown in Fig.5, where the tube would become unstable at the flow velocity if the tube had the same natural frequency as that of the forced sinusoidal excitation.

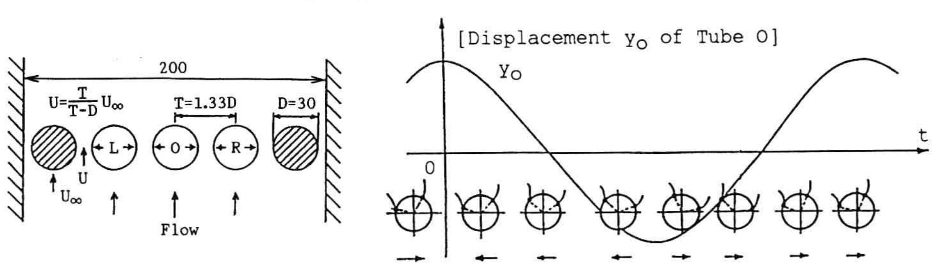


Fig.5 Movement of Separation Point with Tube Motion
(from visualization test by Hara [24])

The Hara's test data shows that when $\dot{y}/f{\cdot}D$ is 0.42 and the variation range of the separation point is 25°. Since the steady separation point θ_s is 130° based on the average value of his test data, that is a bit different for a single tube, the following equation for the separation point θ_s can be derived instead of (8) for the case (2).

$$\theta_s(y,\dot{y}) = 102° + 40°\left(\frac{y(t)+D}{T}\right) + 374°\frac{\dot{y}(t-\Delta t)}{\omega D} \tag{9}$$

As for the coefficient in this equation, the third term in the right hand, 374° should be smaller, as an order of 150°, for example, which is discussed later.

From the above discussion, when the nondimensional pressure distribution $C_p(\theta)$ can change according to the direction of the tube response, the pressure distribution around the tube surface should be determined in each surface, $C_{p+}(\theta)$ and $C_{p-}(\theta)$. Then the lift force, F_L, and the drag force, F_D, might approximately be computed by the following integrals.

$$\begin{cases} F_D = \frac{1}{2}\rho U_\infty^2 \int_0^\pi [C_{P+}(\theta) + C_{P-}(\theta) - 2C_{Pb}]\cos\theta \cdot Rd\theta \\ F_L = \frac{1}{2}\rho U_\infty^2 \left[\int_0^\pi C_{P+}(\theta)\sin\theta \cdot Rd\theta - \int_0^\pi C_{P-}(\theta)\sin\theta \cdot Rd\theta \right] \end{cases} \tag{10}$$

This gives the time-history unsteady fluid force in relation to the tube movement since C_{p+} & C_{p-} are time-dependent.

From Eq.(10), the first approximation of the fluid forces F_D and F_L can be derived using Taylor's expansion.

$$\left.\begin{aligned} F_D &= F_{Do} + \frac{\partial F_D}{\partial y} y + \frac{\partial F_D}{\partial \dot{y}} \dot{y} \\ F_L &= F_{Lo} + \frac{\partial F_L}{\partial y} y + \frac{\partial F_L}{\partial \dot{y}} \dot{y} \end{aligned}\right\} \tag{11}$$

Comparing Eq.(11) with Chen's equations (2) and (3), the following equations can be deduced.

$$\left.\begin{aligned} \frac{\partial F_D}{\partial y} &= \rho U^2 \sigma'', \quad \frac{\partial F_D}{\partial \dot{y}} = \frac{\rho U^2}{\omega^2} \sigma' \\ \frac{\partial F_L}{\partial y} &= \rho U^2 \beta'', \quad \frac{\partial F_L}{\partial \dot{y}} = \frac{\rho U^2}{\omega^2} \beta' \end{aligned}\right\} \tag{12}$$

However, there is a difficulty in evaluating analytically $\partial F_D/\partial y$, $\partial F_D/\partial \dot{y}$, $\partial F_L/\partial y$ and $\partial F_L/\partial \dot{y}$ at $y=\dot{y}=0$, so a direct solution to the equation (10) has been employed in this paper.

2.3 Deduction of Instability Boundary Criterion

The equation of motion of the tube in the cross flow direction can be expressed in general formula as follows:

$$M\ddot{y} + C_s\dot{y} + K_s y = F_L(y, \dot{y}, t) \tag{13}$$

Although this is a nonlinear equation, the response of the tube can be assumed to be sinusoidal.

$$y = y_o \cos\psi \tag{14}$$

Then Eq.(13) can lead to an approximation for an equivalent damping $\lambda_e(y_o)$ and the equivalent stiffness $k_e(y_o)$ [25], and will be rewritten as:

$$M\ddot{y} + \lambda_e(y_o) \cdot \dot{y} + k_e(y_o) \cdot y = 0 \tag{15}$$

where, $\lambda_e = C_s + \frac{1}{\pi y_o \omega_o} \int_0^{2\pi} F_L(y_o \cos\psi, -y_o \omega_o \sin(\psi - \phi_s)) \sin\psi d\psi$

$$k_e = K_s - \frac{1}{\pi y_o} \int_0^{2\pi} F_L(y_o \cos\psi, -y_o \omega_o \sin(\psi - \phi_s)) \cos\psi d\psi$$

Instead of the above method, Roberts [5] has introduced the method of Kryloff and Bogoliuboff to solve the equation (13). If the damping term in Eq.(15) becomes negative, the flutter type instability may occur, which gives the following instability criterion.

$$\frac{U_\infty}{fD} = K\left[\frac{m\delta}{\rho D^2}\right]^{1/2} \tag{16}$$

where, $K = 2\pi/\sqrt{\beta}$

$$\beta = \frac{D}{4y_o}\int_0^{2\pi}\left[\int_0^{\pi} C_p\left(y_o\cos\psi, -y_o\omega_o\sin(\psi-\phi_s)\sin\theta d\theta\right) - \int_0^{\pi} C_p\left(-y_o\cos\psi, y_o\omega_o\sin(\psi-\phi_s)\sin\theta d\theta\right)\right]\sin\psi d\psi$$

The coefficient β can be calculated by using Eq.(10) and the above experimental values from Eqs.(6) to (9). However, the experimental coefficients in Eqs.(5), (8) and (9) should be examined and modified if it is needed by the comparison with the experimental data at the critical flow velocity. Thus the following parameters a_1, a_2, a_3, a_4, b_1, b_2, b_3, and θ_s which might be equal to $\Delta t \cdot \omega_o$ are defined.

$$\begin{cases} C_{po}(y) = a_1\left(\dfrac{y+D}{T} - a_2\right)^{a_3} + a_4 \\ \theta_s(y,\dot{y}) = b_1 + b_2\left(\dfrac{y+D}{T}\right) + b_3\dfrac{\dot{y}(t-\Delta t)}{\omega_o D} \end{cases} \tag{17}$$

In addition, Eq.(13) can be expressed in general form as

$$[M_s]\{\ddot{X}\} + [C_s]\{\dot{X}\} + [K_s]\{X\} = \{F\} \tag{18}$$

and can be solved by a numerical integral. This gives us the response of the tube after it becomes unstable, which will be shown in the sequential paper.

In addition, the equation (13) seems to be derived as the one degree of freedom model, however, the effect of the movement of the adjacent tubes has been introduced when some empirical parameters have been estimated with the data of the tube array as likely as the equation (9).

3. EVALUATION OF PARAMETERS

In this section, the parameters in Eq.(17) are to be evaluated by using some test results [1][12][26]. Fig.6 shows the in-line tube array tested by air and water cross flow [12]. One tube in the middle of the array was 260mm in length and flexibly supported with a slender bar of 6.8mm in diameter as a cantilever beam. Other tubes were rigidly supported. Major specifications of the test equipment are shown in Table 1.

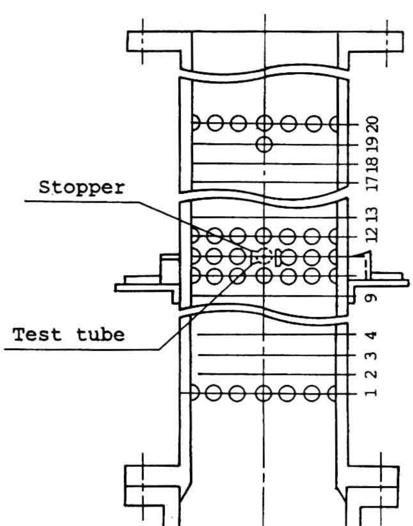


Fig.6 Test Tube Array

Table 1 Specification of Test Equipment

Test Tube			
Diameter [D] :	19.05		mm
Pitch [T] :	27		mm
Length [L] :	174		mm
	In Air	In Water	
Frequency [ft]	22.5Hz	19.0Hz	
Damping Ratio [ht]	0.39%	0.77%	
Stopper			
Frequency [ft]:	95.0 Hz		
Damping Ratio [ht]:	2.3 %		
Gap [δ]:	0～7.95mm		
Tube Array : In-line 5 x 20			

The flexible tube was equipped with a stopper at the center position of the tube for the cross flow direction, where the gap was able to be arranged at an arbitrary value. In this paper, the gap was set 8mm.