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Inflation Risk, Payment Tilt, and the Design of Partially Indexed Affordable Mortgages

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This paper integrates two fundamentally important parameters into a theory of optimal mortgage design: the proportion of inflation risk borne by the lender/investor and the borrower and the amortization-graduation schedule for loan repayments. Equations are derived for a family of innovative mortgages, termed hybrid PLAMs, which offer advantages to borrowers and lenders over either the standard fixed rate mortgage (FRM) or the price level adjusted mortgage (PLAM). The superiority of the hybrid PLAMs lies in their ability to simultaneously *and* independently *accommodate differing degrees of inflation-risk sharing and payment affordability. Inflation-risk sharing is represented by an indexation parameter set over a continuum of values such that the FRM has zero index variability and the PLAM has unit index variability. Similarly, payment tilt is represented by a tilt parameter such that the FRM has zero tilt and the PLAM has unit tilt. We demonstrate that these two parameters are* independent *and can each be continuously varied in a two-dimensional family of self-amortizing mortgages. A specific hybrid PLAM can be designed to partition inflation risk in any proportion between the borrower and the lender and to simultaneously prescribe any level of payment tilt between the extremes of the FRM and PLAM. The behavior of representative hybrid PLAMs is simulated and compared to FRMs and PLAMs for three different inflation scenarios, one of which uses actual market data from the period of 1960–1990.*

During the last decade significant progress has been made in developing a new class of indexed mortgages (see, for example, Lessard and Modigliani 1975; Friedman 1980; McCulloch 1982, 1986; Houston 1988). Like the standard fixed rate mortgage (FRM), the indexed, or the price level adjusted, mortgage (PLAM) represents an extreme solution to the problems of risk sharing and payment tilt. With the FRM, the risk of unexpected inflation and the associated movements in interest rates are essentially allocated to the lender. Consequently, both expected inflation

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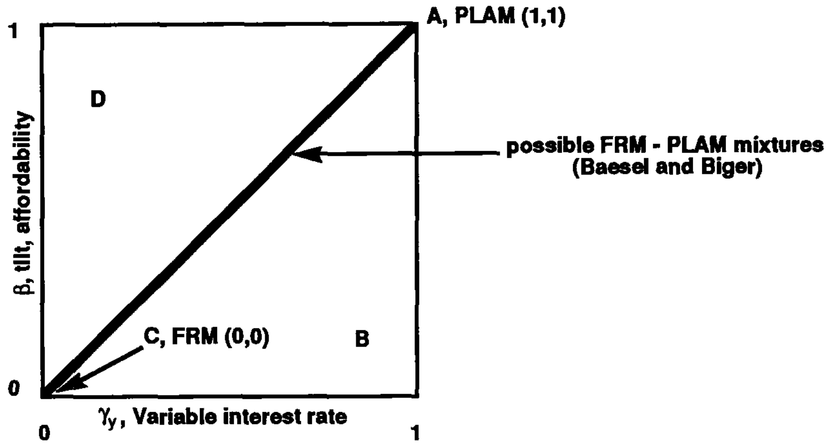
and an inflation-risk premium associated with unexpected inflation are incorporated into the fixed payment stream of the FRM. This leads to an affordability problem in periods of high and volatile inflation, because the high initial payments effectively bar many first-time home buyers from the market. Younger home buyers bear the heaviest burden of the increase in mortgage payments because their *current* incomes tend to be below average. The fixed payment of the FRM does not permit tailoring of the borrower's payments to expectations of rising income. On average, all households can expect higher future income during inflationary periods, and younger households, in particular, can usually look forward to above-average increases in future income streams. Unfortunately, home buyers are unable to take advantage of expected increases in future income when attempting to qualify for the relatively high fixed payments of the FRM. Hence, with the FRM, all of the inflation risk is allocated to the lender and, in nominal terms, there is no payment tilt. With the PLAM, all of the inflation risk is removed from the lender and, from a practical viewpoint, payment tilt is at a maximum.[1]

Recently, Baesel and Biger (1980) have suggested that lenders allow homeowners to finance part of their mortgage balance in an FRM and the remainder in a PLAM as an attempt to provide homeowners with the benefits of flexible indexation. However, this approach would be inflexible in terms of payment tilt once the proportions of FRM and PLAM financing were selected. This paper improves upon the suggestion advanced by Baesel and Biger to present a general mortgage instrument which incorporates parameters permitting (i) inflation-risk sharing in any proportion between the borrower and the lender and (ii) a wide range of amortization-graduation schedules for loan repayments (payment tilt). These innovative mortgages, called hybrid PLAMs, offer advantages to borrowers and lenders over either the FRM or the PLAM. The superiority of the hybrid PLAMs lies in their ability to *simultaneously* and *independently* accommodate differing degrees of inflation-risk sharing and payment affordability. The hybrid PLAMs offer borrowers and lenders unique combinations of affordability and inflation-risk sharing. This expands the choice of mortgage instruments. With a two-dimensional continuum hybrid PLAM (see Figure 1), borrowers, in effect, can select a FRM, a "pure" PLAM, a mixture of FRM-PLAM (as suggested by Baesel and Biger and as represented by the diagonal line in Figure 1) or mortgages with different combinations of payment tilt and indexation levels. For example, fixed

[1] Although it is possible to design mortgages with greater tilt than the PLAM, these mortgages are impractical because of the large amount of negative amortization and attendant default risk.

Figure 1 ■ A Two-Dimensional Continuum of PLAM-Like Loans



A PLAM - variable rate, variable payment - appropriate for young wage earner who expects to be able to afford higher payments and higher inflation risk.

B Variable rate with nearly constant payments. Fixed income retirees who want to speculate that lower future rates will allow their payments to drop.

C FRM - fixed rate, fixed payments - appropriate for fixed income retirees who want to assure that payments won't rise.

D GRM - affordable fixed - rate mortgage. Appropriate for the young wage earner who expects to be able to afford higher payments but wants to lock in a long - term low rate.

γ interest - how interest changes with unexpected rates, variability preserves present value

β tilt - how payments change with planned rates, tilt provides affordability but negative amortization

income retirees who want to speculate that lower future rates will allow their payments to drop may want to choose a combination of payment tilt and indexation levels as represented by point B in Figure 1 (i.e., a variable rate with nearly constant payments mortgage). Alternatively, young wage earners who expect to be able to afford higher payments but want to lock in a long-term, low rate may want to be at point D, etc.

The remainder of this paper is organized as follows: the next section discusses an informal rationale for the supply and demand of hybrid PLAMs; a review of formal models of borrower preference for indexation is provided in section 3; section 4 presents a model of hybrid PLAMs, which is followed by the derivation of a simple model of price risk in section

5; section 6 contains the simulation results for three inflation scenarios, including the actual inflation that occurred between 1960 and 1990; and section 7 provides concluding remarks and possible future extensions.

An Informal Rationale for the Supply and Demand of Hybrid PLAMs

The supply side of the mortgage market is composed of homeowners and commercial property owners.[2] We focus initially on homeowners. Paying a fee to transfer inflation risk to the lender (or ultimately the capital markets) may not make sense for many homeowners because of their ability to internally hedge inflation risk. Internal hedging occurs in a wealth portfolio whenever inflation (or deflation) simultaneously induces a gain (loss) in an asset and an offsetting loss (gain) in a liability or vice versa. Because homeowners issue mortgage liabilities against real assets, they have a naturally occurring opportunity to at least partly hedge inflation risk. Assume, for the moment, that inflation is perfectly neutral (i.e., that the prices of *all* goods and services rise or fall through time in perfect consonance with one "true" inflation index). In this case, real estate would be a perfect inflation hedge and it would make sense for homeowners to issue mortgages devoid of inflation risk (and inflation-risk premiums), such as the pure PLAM.[3] This ability of homeowners to internally hedge inflation risk is another reason why academics have long advocated PLAMs (see, for example, Lessard and Modigliani 1975; Friedman 1980; McCulloch 1982, 1986; Houston 1988).

Unfortunately, inflation is not perfectly neutral. Historically, national housing price indices have been positively, but less than perfectly, correlated with the CPI and other general price indices. Moreover, and far more importantly, *individual* homeowners cannot be assured of achieving *average* house price appreciation. There is considerable cross-sectional variation in housing price appreciation, not only regionally, but also in different neighborhoods of the same city.[4] Because of the non-neutral

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[2] Focusing on mortgages as financial instruments, we follow the convention of identifying borrowers as "suppliers" of mortgages. Lenders/investors (who supply *funds* to the mortgage markets) are demanders of mortgage instruments.

[3] Not *all* borrowers would wish to hedge inflation risk. Those who choose to speculate on unexpected inflation might prefer to pay inflation-risk premiums and issue FRMs.

[4] For evidence on the extent of cross-sectional variation in housing prices, see Tuccillo and Villani (1981), Blackley, Follain and Lee (1986), Case (1986), and Kiel and Carson (1990).

nature of inflation, it may generally benefit homeowners to hedge only a portion of inflation risk and to transfer the remainder to lenders/investors who are presumably more efficient in risk management (Hess 1984).

Commercial property owners are also potential beneficiaries of hybrid PLAMs. Long-term leases for commercial property typically provide for lease payments that are at least partially pegged to a price index. Because both the underlying property value and the associated stream of lease payments are, in a sense, inflation-adjusted, commercial property owners can also internally hedge inflation risk by using hybrid PLAMs. The benefits include lower initial mortgage payments, elimination of part or all of the inflation-risk premium and the ability to match the tilt of the payment stream to the tilt of the lease rental incomes. In summary, there should be considerable interest in the hybrid PLAM family from both homeowners and commercial property owners on the supply side of the mortgage market.

On the demand side, financial intermediaries, such as banks, insurance companies and pension funds, could utilize hybrid-PLAM investment portfolios to hedge new classes of indexed financial contracts. Banks could offer price-level adjusted deposits (PLADs) and insurance companies could offer indexed annuities, neither of which are currently offered in any significant amounts.[5] Although social security benefit payments are traditionally adjusted for inflation, the payments received by retirees from nongovernment pensions are generally not. This creates unnecessary hardship for many retired people. Pension funds with investments in hybrid PLAMs could provide a service of great social value by offering indexed retirement plans. In summary, the availability of hybrid PLAMs (or securities derived from these mortgages) would facilitate the marketing of indexed deposits, pensions and annuities with significant social benefits.

A Review of Formal Models of Borrower Preference for Indexation

Baesel and Biger (1980) developed a single-period model of borrower choice in which they derived borrower preferences for mortgage inflation indexation as a function of the characteristics of the homeowners' stream of labor income and the covariance between the real income stream and

[5] An exception is a type of 20-year noncallable collateralized bond, called Real Yield Securities (REALs), which has been underwritten and marketed in small amounts by Morgan Stanley and Company since 1988 (see Bodie 1990 for a more complete discussion).

the inflation variable. Starting with the principal assumption that borrowers are risk-averse in the mean-variance sense and concerned only with real terminal wealth, they analyzed mean-variance trade-offs under alternative income stream assumptions. Their results indicate that some homeowners will prefer zero indexation (FRM), some will prefer 100 percent indexation (PLAM) and the rest will prefer a degree of indexation between these extremes. In order to satisfy the demand for partially indexed mortgages, they suggest that lenders offer flexible mortgage packages in which homeowners can choose the proportion that will be financed in an FRM and the proportion that will be financed in a fully indexed PLAM.

Using similar analytical techniques, Statman (1982) extended the Baesel and Biger model to include an additional variable, home equity. In Statman's model, the covariance between a borrower's real home equity and the inflation variable also influences the choice of mortgage. Although the parameters of Statman's model are more complex (and more realistic), he also concludes that homeowners' preferences for mortgage indexation will be a continuum ranging from the zero indexation of the FRM to the full indexation of the PLAM.

In a related study of homeowners' *interest rate* risk preferences, Dokko and Edelstein (1991) employed von Neumann-Morgenstern expected utility maximization principles to derive estimates of demand for FRMs versus adjustable rate mortgages (ARMs). They found that (i) it would never be optimal for a homeowner to take full protection against interest rate risk (e.g., by using a callable FRM) and (ii) it is likely to be suboptimal for a homeowner to fully protect the lender against interest rate risk (e.g., by using an ARM). Thus, they also identify a demand for mortgages with flexible risk-sharing characteristics; in this case, a demand for proportional interest rate risk sharing. Since inflation risk is a major component of interest rate risk, their results can be viewed as supportive of those obtained by Baesel and Biger and by Statman.

In conclusion, existing theoretical models support the notion that a segment of homeowners would prefer to issue mortgages with variable indexation. The suggestion advanced by Baesel and Biger proposing that lenders allow homeowners to finance part of their mortgage balance in an FRM and the remainder in a PLAM is one approach to providing homeowners the benefits of flexible indexation. Once the proportions of FRM and PLAM financing were selected, however, this approach would be inflexible in terms of payment tilt. In contrast, hybrid PLAMs would allow homeowners (and commercial property owners) to simultaneously and independently select both the desired degree of indexation and the desired level of payment tilt, all in one mortgage.

The Model of Hybrid PLAMs

There are two fundamentally desirable characteristics which should be integrated into the design of alternative mortgages. First, they should enable parameterization of the proportion of inflation risk borne by the lender. That is, the mortgage equations should permit inflation-risk sharing in proportions agreed upon by the borrower and lender at the time of loan origination. Second, the mortgage equations should parameterize payment tilt; that is, they should permit a continuum of payment tilt ranging from the maximum tilt of the PLAM to the zero tilt of the FRM. The degree of tilt would also be agreed upon at the time of loan origination. Let γ measure the degree of indexation and β measure nominal payment tilt. Let the PLAM have unit indexation and payment tilt ($\gamma = \beta = 1$) and the FRM have zero indexation and payment tilt ($\gamma = \beta = 0$). The family of hybrid PLAMs has variable indexation and tilt described by the following boundary conditions

$$0 \leq \gamma \leq 1, \qquad 0 \leq \beta \leq 1,$$

where γ and β are *independently* selected at the time of loan origination. Mortgages are defined by two equations, one specifying the payment and one the balance at each point in time. We derive two generalized equations which parameterize γ and β. That is, the equations describe a two-dimensional continuum of mortgages with continuously variable payment tilt and variable degrees of indexation. The FRM ($\gamma = \beta = 0$) and the PLAM ($\gamma = \beta = 1$) are both (extreme) members of this family.[6]

Following Baesel and Biger (1980), we assume a modified version of the Fisherian model of interest rates, in which nominal mortgage interest rates are the sum of a constant real rate, expected inflation and an inflation-

[6] In an earlier paper, Harris and Page (1985) derive the equations for a family of mortgages with continuously variable payment tilt. They use the term "rate limited mortgage" (RLIM) to describe these mortgages. Although they are adjustable in terms of payment tilt, the RLIMs do not permit proportional inflation-risk sharing.

risk premium.[7] We assume that capital markets are efficient. To focus the analysis on inflation risk, we assume that all mortgages are in the same default risk class. For expository convenience, the resulting constant default-risk premium is subsumed in the real interest rate. The inflation-risk premium is defined as a simple linear function of the degree of indexation, and it implicitly includes the call-risk premium.[8]

The nominal mortgage interest rate at loan origination (contract rate) is then expressed as

$$r_0 = R + i_0 + \mathrm{IRP}(\gamma - 1) \qquad (1)$$

where R is the real rate of interest, i_0 is the current level of expected inflation, IRP is the inflation risk premium appropriate for the FRM and γ is the indexation parameter. Thus, the initial contract rate for the FRM will include the full amount of the inflation-risk premium, and the initial contract rate for the PLAM will exclude the inflation-risk premium. The initial contract rate is a constant.

The time series of variable mortgage interest rates, r, is modeled as the original contract rate plus a proportion, γ, of the current level of unexpected inflation:[9]

$$r = r_0 + \gamma(i - i_0) \qquad (2)$$

[7] Although we believe it highly unlikely that inflation risk is completely and costlessly diversifiable in the capital markets, we should remind the reader that the existence of inflation-risk premiums is an unresolved empirical issue. In any case, the inflation-risk premium is negotiated at loan origination and constant throughout the term of the loan; hence, its magnitude does not affect our derivations.

[8] Borrowers can prepay (call) mortgages either to transfer ownership of the mortgaged property or because a period of unusually low interest rates makes it profitable to refinance the mortgage (a speculative call). Speculative call options exist in all U.S. mortgages. It is appropriate to include the call-risk premium as part of the inflation-risk premium, because the value of the call option to the homeowner (and hence its cost to the lender) is an increasing function of the variability of inflation. It follows that the magnitude of the inflation-risk premium will be proportional to the degree to which the mortgage is indexed to inflation. Our assumption of a simple linear relationship is convenient to illustrate the model; however, the inflation-risk premium could be any nonstochastic function of the indexation parameter.

[9] This follows from our assumption that inflation is a simple random walk.

Define the constant, k, as

$$k = r_0 - \beta i_0 \tag{3}$$

We prove below that the family of hybrid PLAMs has a constant payment rate given by k. From equation (3) it can be seen that the payment rate for the FRM ($\beta = \gamma = 0$) is identical to the FRM contract rate. By substituting equation (1) into (3), it can be seen that the payment rate of the PLAM ($\beta = \gamma = 1$) is simply the real rate of interest. In general, the family of hybrid PLAMs will have constant payment rates between these extremes.[10]

Our notation uses a subscript 0 for values established at loan origination. The absence of a subscript generally implies a time-dependent variable, except that β, γ and T are constants established at loan origination. The notation is summarized as follows.

- t = time in years.
- $t = 0$; the time of loan origination.
- $t = T$; the time of maturity, fixed at mortgage origination.
- C = mortgage payment at time t.
- k = payment rate.
- i = inflation rate at time t.
- i_0 = initial inflation level, also expected inflation for mortgages originated at time 0.[11]
- $i - i_0$ = unexpected inflation at time t.
- I = the level of the price index at time t relative to time 0.
- P = mortgage balance at time t.
- r = mortgage interest rate at time t.
- r_0 = the contract rate for newly originated mortgages.
- R = the real rate of interest, a constant.
- V = mortgage present value at time t.
- β = payment tilt parameter, constant throughout the loan.
- γ = indexation parameter, constant throughout the loan.

The mortgage interest rate for the PLAM varies with the inflation rate, and the payment increases with the full amount of realized inflation, both

[10] The contract and payment rates will be agreed upon by the borrower and the lender at mortgage origination in light of competitive market conditions and the borrower's selection of payment tilt and index variability parameters.

[11] Given our assumption that inflation is a random walk, this is also the best estimate of expected inflation over the life of the loan.

expected and unexpected. In the hybrid PLAM, the mortgage interest rate will vary with a proportion, γ, of unexpected inflation $(i - i_0)$. We show that a payment stream that increases in part with a proportion, β, of expected inflation and with a different proportion, γ, of unexpected inflation will exactly solve the variable rate cash flow equation. We call this mortgage the hybrid PLAM. Equation (4) describes such a cash flow, increasing with the proportion, β, of expected inflation and the proportion, γ, of unexpected inflation.

$$C = C_0 \exp\left(\beta i_0 t + \int_0^t \gamma(i - i_0)\, dt\right) \tag{4}$$

where C_0 is the initial payment set to amortize over T. The first term within the exponential assures an expected increase or tilt to the payments of the fraction β of expected inflation. The second term under the integral shows future changes to the payments to compensate for future unexpected changes in the interest rate. The form of equation (4) will become clear in the next few steps as it exactly cancels out the future unknown inflation for the integrating factor which will be necessary to solve the cash flow equation.

The tilt parameter sets the initial expected payment increase schedule. The integral within the exponential accumulates the actual inflation into the future. When the average future inflation equals the expected inflation, the two terms with γ cancel and the payment increases with βi_0. When the average future inflation turns out to be different than the expected inflation, the payments increase more or less rapidly in order to compensate for the difference. Thus, the tilt parameter β prescribes how much payments are planned to increase relative to the expected inflation i_0. When actual inflation varies from i_0, future payments will differ from the planned payments by a degree set by γ.

The differential cash flow equation for the principal with variable interest of equation (2) and the payment cash flow for equation (4) is

$$\frac{dP}{dt} - (r_0 + \gamma(i - i_0))P = -C_0 \exp\left(\beta i_0 t + \gamma \int_0^t (i - i_0)\, dt\right) \tag{5}$$

Equation (5) is solved by multiplying both sides by the integrating factor, which includes both γ and the exponential inflation integral. When both sides of equation (5) are multiplied by this intregrating factor, the left side becomes a perfect differential and can be directly integrated. When the right side of equation (5) is multiplied by the integrating factor, the integral terms with the future inflation exactly cancel out. This explains the complex form of the variable payment of expression (4) and allows the hybrid PLAM to be amortized exactly over T, independent of accumulated unexpected inflation. Using the initial principal of P as the boundary condition, the following expression for the hybrid PLAM principal is obtained:

$$P = \left[P_0 - \frac{C_0}{r_0 - \beta i_0} [1 - e^{-(r_0 - \beta i_0)t}] \right] \exp \left[(r_0 - \gamma i_0)t + \gamma \int_0^t i \, dt \right] \tag{6}$$

At time T, the principal goes to zero. Thus, the solution of C_0 is independent of the future inflation scenario, and with $(r_0 - \beta i_0)$ as the rate, k, the initial payment is

$$C_0 = \frac{kP_0}{1 - e^{-kT}} \tag{7}$$

Expressions (4) and (6) can be further simplified by noticing that the exponential of the integral of gamma times inflation is just the price index raised to the gamma power. Thus,

$$\exp\left(\gamma \int_0^t i \, dt \right) = I^{\gamma} \tag{8}$$

where I is the price index relative (e.g., the current level of the CPI divided by the CPI at time zero). By substituting equations (7) and (8) into (4) and by rearranging terms, we obtain the equation for the variable payment of the hybrid PLAM:

$$C = C_0 \exp((\beta - \gamma) i_0 t) I^{\gamma} \tag{9}$$

By substituting equations (7) and (8) into (6) and by rearranging terms, we obtain the equation for the hybrid PLAM principal:

$$P = P_0 \left(1 - \frac{1 - e^{-kt}}{1 - e^{-kT}} \right) (\exp(r - \gamma i_0)t) I^{\gamma} \tag{10}$$

Both equation (9) for the variable payments and equation (10) for the outstanding principal contain the price index raised to the power γ. The future payments, as a function of future outstanding balance, can be solved by eliminating I^{γ} from equations (9) and (10). The result is

$$C = \frac{(r_0 - \beta i_0)P}{1 - e^{-(r_0 - \beta i_0)(T-t)}} = \frac{kP}{1 - e^{-k(T-t)}} \qquad (11)$$

Thus, the hybrid PLAM payment is calculated by inserting the nonstochastic rate, k, into the annuity equation (11). Thus, k is truly a payment rate, and all hybrid PLAMs are *constant* payment-rate loans. In contrast, adjustable rate mortgages are *variable* payment-rate loans. They function by inserting the variable interest rate into the annuity equation (11). The result is rapidly varying payments needed to stay on the amortization curve. The hybrid PLAM inserts a constant payment rate into the annuity equation, which generates smoothly varying payments partially indexed to inflation. Harris and Page (1985) showed that a fully variable mortgage interest rate could be paid with a constant payment rate. We have further demonstrated that a partially variable mortgage interest rate can be paid with a constant payment rate.

Equations (9) for the payment and (10) for the principal define the family of fully amortized hybrid PLAM loans with the two independent parameters β and γ specifying payment tilt and degree of indexation, respectively. Table 1 displays a two-dimensional array of representative hybrid PLAMs as values of γ and β are varied over some reasonable and interesting values, and equation (9) is modified to determine the corresponding payment formulas. In this table, the variable i represents the inflation expected at the beginning of the loan, which was represented by i_0 in previous expressions. The columns show the variation with the level of indexation, γ. The rows show varying degrees of payment tilt. The pure PLAM and the standard FRM are special cases which anchor the corners of this array.

A Simple Model of Price Risk

The FRM suffers radical swings in value as future interest rates experience unexpected variations over the life of the mortgage (price risk). The value of indexed instruments is much more stable. In order to make comparisons, it is necessary to have a simple valuation model that captures the main features of both fixed and floating rate valuations in rate variation scenarios. Our use of calculus to derive Hybrid PLAMs assumes a continuously varying interest rate that can be simultaneously set and changed. This leads to pricing by present value rather than a more de-

Table 1 ■ Hybrid PLAM payment formulas.

Tilt β	$\gamma = 1$ Lender has No Inflation Risk	$\gamma = 1/2$ Borrower and Lender Share Inflation Risk	$\gamma = 0$ Borrower has No Inflation Risk
Index Tilt $\beta = 1$	$\frac{(r-i)P}{1-e^{-(r-i)T}} I$ PLAM	$\frac{(r-i)P}{1-e^{-(r-i)T}} e^{.5it} I^{.5}$	$\frac{(r-i)P}{1-e^{-(r-i)T}} e^{it}$
Half Tilt $\beta = 1/2$	$\frac{\left(r-\frac{i}{2}\right)P}{1-e^{-[r-(i/2)]T}} e^{-.5it} I$	$\frac{\left(r-\frac{i}{2}\right)P}{1-e^{-[r-(i/2)]T}} I^{.5}$	$\frac{\left(r-\frac{i}{2}\right)P}{1-e^{-[r-(i/2)]T}} e^{.5it}$
No Tilt $\beta = 0$	$\frac{rP}{1-e^{-rT}} e^{-it} I$	$\frac{rP}{1-e^{-rT}} e^{-.5it} I^{.5}$	$\frac{rP}{1-e^{-rT}}$ FRM

Formulas are based on equation (9). Here, i represents initial expected inflation, which is i_0 in the previous derivations.

tailed continuous time valuation model. Our present value calculations assume that changes in the inflation rate are a simple random walk and that the term structure of interest rates is flat (but not deterministic). Our present value formulation will not address the following, which are left to future studies: (i) averaging the valuation over a continuum of all possible rate variations, (ii) including the time lags between the setting and changing of interest, and (iii) accounting for mortgage-specific differences in the values of borrowers' call and default options.

The present value V of a debt instrument at time t is the sum of the cash flows at each future time t' discounted by the expected cost of funds, as given by

$$V_t = \int_t^T dt' C(t') e^{-r(t'-t)} \tag{12}$$

where $C(t')$ is the expected future payments and r is the current contract rate for newly originated mortgages and the discount rate appropriate for present value calculations, given by

$$r = r_0 + \bar{i} - i_0$$