Introduction to Mathematics

Applied Academics

Fourth Edition

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Introduction to Mathematics-Applied Academics

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Preface

This text is to be used in conjunction with the Weeks, Handbook on the Texas Instruments, TI-34, Scientific Calculator.

Introduction to Mathematics-Applied Academics is designed for pre-transfer and professional technical programs. It is intended for those who require a basic knowledge of mathematics for use in their particular programs and professions. The general topics included are an arithmetic review, algebra, geometry, and trigonometry. The level of presentation is suitable for those whose backgrounds lack algebra or geometry or require a review of the basic topics. The concepts discussed in this book make it suitable for a course that can be used as preparation for more advanced work in mathematics or as a terminal course. This text has been designed primarily for formal courses, but it could be easily used by individuals wishing to follow a self-study program.

The examples and exercises include many applications from different fields of technology, such as electronics, mechanics, machine design, civil engineering, forestry, architecture, automotive engineering, physics, chemistry, computer science, and computer operations. These applications are intended to illustrate the use of mathematics. The technical material itself is not developed and no prior technical knowledge is required.

The content of this book is developed in an informal and intuitive manner. All important terms are carefully introduced, and basic concepts are developed in order to give the student an understanding of the material and how it relates to other topics. More detail has been included for those concepts which are generally found to be more difficult. The emphasis throughout the text is on the development of the mathematics necessary for technical work.

New Features of This Edition

- There is a new chapter (Chapter 20) dealing with complex numbers.
- There is a new section (14-4) that includes the use of determinants in solving systems of equations.
- There is a new section (14-5) describing algebraic solutions for systems of three equations with three unknowns.
- There is a new section (8-4) that covers factoring of the sum and difference of two perfect cubes.
- The sections in Chapter 7 have been rearranged, with a new section created.
- In Chapter 12, the first two sections have been combined into one, with greater emphasis placed on converting between logarithmic and exponential forms.
- This edition incorporates the use of a second color that enhances the design.
- A special margin symbol ▶ and diagrammed comments are used to identify and clarify the more difficult concepts.
- Each chapter begins with separate introductory remarks and an introductory problem. All of these introductory problems are related to the theme of aviation.
- The important formulas from the geometry chapters are summarized inside the back cover.
- At the end of each chapter, all chapter formulas are listed together for easy reference.
- There are now more than 6500 exercises, including approximately 1600 new exercises. That is an increase of 42%. Each exercise set is numbered according to the section it follows. Exercises are grouped so that there is generally an even-numbered exercise equivalent to each odd-numbered exercise.
- There are more than 700 examples, with 20% of them new. The examples and exercises include applications from the latest technological developments.
- The number of figures has been substantially increased so that there are now about 575 figures. This is an increase of 63% over the last edition.

Additional Features

- Stated problems appear in most sections. The solving of such problems makes the student more accustomed to them. The simplest forms of stated problems are found in the earlier chapters, and Chapter 5 devotes an entire section to the analysis of such problems.

- There is an appendix (Appendix B) that discusses the fundamentals of the BASIC programming language. That appendix includes fifteen programs, along with sample runs, many of which correspond directly to examples in the text.
- The examples are used to introduce concepts as well as to clarify and illustrate material already presented.
- Coverage of the use of calculators is extensive. Calculator usage is discussed throughout the text in appropriate sections, and Appendix A is a reference for most of the commonly used calculator keys.
- The order of coverage may be changed in several places without loss of continuity. Also, certain sections may be omitted without loss of continuity. Any omissions or changes of order will, of course, depend on the type of course and the completeness required.
- Review exercises are included at the end of each chapter. These may be used as a source of additional problems or for review assignments.
- The answers to all odd-numbered exercises are given at the back of the book. Included are answers to graphical problems and other types that are often left out of other textbooks.
- The instructor's manual for *Introduction to Mathematics-Applied Academics, 4th Ed.*, contains general comments and suggestions on the use of the material in each chapter and the answers to the exercises (both even and odd). This information could be helpful in preparing and designing the course to meet the needs of the students.

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A.J.W.
M.F.T.

Contents

CHAPTER ONE

Arithmetic

We live in an era of continued rapid technological growth. This technology provides us with dramatic results that affect and enhance all of our lives. Computers, the space program, and nuclear energy are but three of the major growth areas that constantly demand technological improvements. We now routinely use integrated circuits in calculators, televisions, cars, microwave ovens, and video cassette recorders. Lasers are routinely used in surveying and surgery. We have aircraft that are faster, more maneuverable, and able to carry heavier payloads with greater fuel efficiency. Our homes are more energy efficient. Our leisure activities are enriched by stereo color televisions, video cassette recorders, computer video games, and high-quality sound systems.

All of these developments depend on technical knowledge founded in mathematics. Mathematics is a necessary and critical component of contemporary technology. There is, of course, beauty in mathematics itself. But from a more practical viewpoint, an understanding of basic mathematics opens the door to the many different fields of technology. In this text we begin the development of the mathematics required for entry into these scientific and technical fields. This text includes examples and exercises from such areas as electrical and mechanical technology, civil engineering, solar engineering, physics, and computer science; however, successful solution of applied problems will *not* require a knowledge of the relevant field of application. In addition to the mathematics itself, this text is also designed to increase your familiarity with the metric system and the use of calculators.

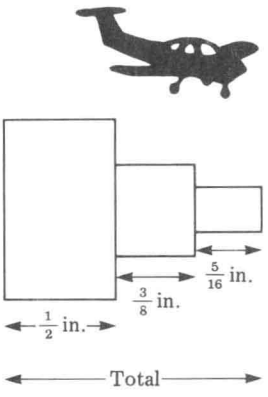


FIGURE 1-1

This text includes a wide variety of different exercises and examples that illustrate applications of the related mathematical concepts. In this chapter, for example, the applications refer to such topics as oil drilling, fuel-oil consumption, cassette tapes, satellites, electrical resistors, computers, gears, surveying, solar energy, chemistry, weather, architecture, and aviation. At the beginning of each chapter, we will present a problem that is related to aviation. Although these opening problems all relate to aviation, they involve a wide variety of different mathematical concepts. These problems will be solved in their respective chapters. Among the topics in Chapter 1 is the addition of fractions. If an aircraft firewall is $\frac{1}{2}$ in. thick and that thickness is increased by a $\frac{3}{8}$-in. washer and a $\frac{5}{16}$-in. bolt, what is the combined total thickness? See Fig. 1-1. This is only one of the many types of problems included in Chapter 1. This particular problem will be solved later in the chapter.

1-1 Addition and Subtraction of Whole Numbers

Much of the material in this chapter may be review. It is important to be skilled in this fundamental material so that the following chapters can be studied. Finding the solutions to most applied problems in science and technology involves arithmetical computations. To perform these computations, we use the basic arithmetical operations of addition, subtraction, multiplication, and division. These operations are essential to performing computations, and they are fundamental to the development of the various branches of mathematics itself. It is important that these operations be performed accurately and with reasonable speed. Although we assume that you are familiar with these operations, we shall include a brief discussion here for review and reference.

The most fundamental use of numbers is that of counting. *The numbers used for counting, the* **natural numbers,** *are represented by the symbols 1, 2, 3, 4, and so on. When we include* **zero,** *we have the* **whole numbers,** *which are represented by 0, 1, 2, 3, and so on.*

Any of the whole numbers can be written with the use of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if the actual *position* of a symbol in a given number is properly noted. This important feature used in writing numbers—that of *placing each symbol in a specified position in order to give it a particular meaning—is referred to as* **positional notation.**

EXAMPLE A In the number 3252, read as "three thousand two hundred fifty-two," the left 2 represents the number of hundreds and the right 2 represents the number of ones, because of their respective positions. Even though the symbol is the same, its position gives it a different value. Also, the 3 represents the number of thousands and the 5 represents the number of tens, because of their respective positions.

thousands	hundreds	tens	ones
3	2	5	2

hundreds	tens	ones
3	2	5

In the number 325, the 2 represents the number of tens and the 5 represents the number of ones. In the number 352, the 5 represents the number of tens and the 2 represents the number of ones. Even though the same symbols are used, the different positions result in different values. □

The process of finding the total number of objects in two different groups of these objects, without actually counting the objects, is **addition.** *The numbers being added are the* **addends,** *or* **terms,** *and the result is the* **sum.** In performing an addition we must be careful to ***add only like quantities,*** such as feet to feet or quarts to quarts. If we are adding or subtracting pure numbers without units like gallons or miles, we can add or subtract in the obvious way. But if the numbers are associated with units like gallons or miles, we cannot add or subtract unless the units are the same. This means that we cannot add 2 qt to 3 gal without first expressing both quantities in the same units, such as quarts. We cannot add 2 qt to 3 mi since there is no way to make the units agree.

EXAMPLE B If one container has a capacity of 4 qt and another has a capacity of 2 qt, we find the combined capacity of the two containers by adding

$$4 \text{ qt} + 2 \text{ qt} = 6 \text{ qt}$$ □

In Example B we used qt as the symbol for quart. When such units of measurement are designated by their appropriate symbols, and these symbols might be unfamiliar, they will be defined when used in this chapter. It is, however, assumed that you are familiar with the following units and their symbols:

Unit	*Symbol*	*Unit*	*Symbol*
inch	in.	gallon	gal
foot	ft	minute	min
yard	yd	pound	lb
mile	mi	quart	qt
		ounce	oz

A discussion of units and their symbols is found in the first section of Chapter 2.

When we are adding two whole numbers, we must take into account positional notation and add only those numbers with the same positional value. In this way we are adding like quantities.

EXAMPLE C When we add 46 and 29, we are saying

$$\begin{array}{r} 4 \text{ tens} + 6 \text{ ones} \\ 2 \text{ tens} + 9 \text{ ones} \\ \hline 6 \text{ tens} + 15 \text{ ones} \end{array}$$

Since 15 ones = 1 ten and 5 ones, we then have

$$6 \text{ tens} + 1 \text{ ten} + 5 \text{ ones} = 7 \text{ tens} + 5 \text{ ones}$$

We usually perform this addition as follows.

$$\begin{array}{r} {\scriptstyle 1} \\ 46 \\ +\ 29 \\ \hline 75 \end{array} \qquad 6 + 9 = 1 \text{ ten} + 5 \text{ ones}$$

where the 1 shows the number of tens "carried" from the ones column to the tens column. □

You should know the basic sums through $9 + 9 = 18$. If you are at all unsure of any of these, write them out so that you can review them. Being able to perform addition accurately and with reasonable speed comes from knowing the basic sums well; this takes practice.

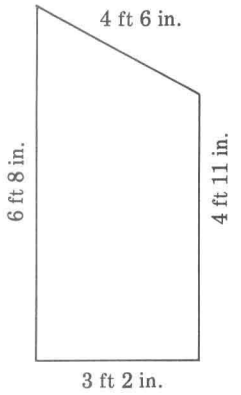
It is also wise to form the habit of checking your work. Several methods of checking addition are available. A simple and effective method is to add the columns in the direction opposite to that used in finding the sum originally.

EXAMPLE D If we find the sum of the indicated numbers by adding the columns downward, we can check the results by adding again, this time upward.

$$\begin{array}{r} {\scriptstyle 2\,2} \\ 327 \\ 582 \\ 695 \\ 419 \\ \hline 2023 \end{array} \quad \text{add} \qquad \begin{array}{r} {\scriptstyle 2\,2} \\ 327 \\ 582 \\ 695 \\ 419 \\ \hline 2023 \end{array} \quad \text{check}$$

□

The process of **carrying** *a number from one column to the next is necessary whenever the sum of the digits in a column exceeds 9.* A very similar situation occurs when we add distances expressed in more than one measuring unit. Consider the following example.



6 ft 8 in.
4 ft 6 in.
4 ft 11 in.
3 ft 2 in.
17 ft 27 in.

FIGURE 1-2

EXAMPLE E

The **perimeter** *of a plane geometric figure is the total distance around it.* Suppose a solar home has a window with a shape described by Figure 1-2. To find the perimeter of this window, we must add the indicated lengths. The result is 17 ft 27 in., but a more useful result is obtained if we use the fact that 27 in. = 2 ft 3 in. We can then state that the perimeter is 19 ft 3 in. In this addition process, we essentially "carried" 2 from the inch column to the foot column. □

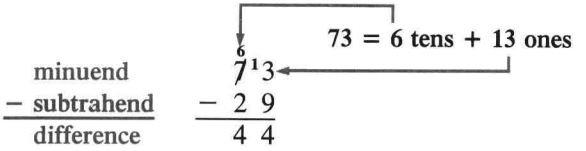
We must often determine how much greater one number is than another. This leads to the operation of **subtraction,** the inverse of addition. *Subtraction consists of reducing the number from which the subtraction is being made (the* **minuend***) by the number being subtracted (the* **subtrahend***). The result is called the* **difference.**

EXAMPLE F

If we wish to subtract 29 from 73, we find that for the number of ones involved, we are to reduce 3 by 9. When we consider natural numbers, we cannot perform this operation. However, if we "borrow" 10 ones from the tens of 73, the subtraction amounts to subtracting 2 tens + 9 ones from 6 tens + 13 ones. We can show the subtraction as

$$\begin{array}{r} 7 \text{ tens} + 3 \text{ ones} \\ -\ 2 \text{ tens} + 9 \text{ ones} \end{array} \quad \text{or} \quad \begin{array}{r} 6 \text{ tens} + 13 \text{ ones} \\ -\ 2 \text{ tens} + 9 \text{ ones} \end{array}$$

Using the second form, we see that the result is 4 tens + 4 ones, or 44. The usual form of showing the subtraction is



Here the small 1 shows the number of tens borrowed, and the small 6 shows the remaining tens after the borrowing. Borrowing in subtraction is essentially the opposite of carrying in addition.

We can check subtraction by adding the difference and the subtrahend. The sum should be the minuend. This follows directly from the meaning of subtraction. The check of the preceding subtraction is as follows:

$$\begin{array}{rl} 29 & \text{subtrahend} \\ +\ 44 & +\ \text{difference} \\ \hline 73 & \text{minuend} \end{array}$$

□

In many subtraction problems it is necessary to borrow more than once. It might be necessary to borrow from the tens and then again from the hundreds or thousands. The next example illustrates this type of subtraction.

EXAMPLE G The subtraction 8203 − 4659 is shown as follows:

$$\begin{array}{r} {}^{7}\not{8}\,{}^{1}\not{2}\,{}^{1}\not{0}\,{}^{1}3 \\ -\ 4\ 6\ 5\ 9 \\ \hline 3\ 5\ 4\ 4 \end{array} \longleftarrow 8203 = 7 \text{ thousands} + 11 \text{ hundreds} + 9 \text{ tens} + 13 \text{ ones}$$

Here we see that it was necessary to borrow from the tens, although there were initially no tens to borrow from. We had to borrow first from the hundreds and then from the tens. Finally, it was necessary to borrow from the thousands to complete the subtraction in the hundreds column. □

Although the additions and subtractions discussed in this section can be easily and quickly performed on a calculator, you must understand these basic arithmetic operations and be able to perform them mentally. A good understanding of all the basic operations is important to comprehending algebra and other topics we will develop. Only after you understand these operations and can perform them accurately should you consider using a calculator. Many gross errors have been made with calculators because they have been used improperly. We discuss the use of calculators in Section 1-11.

Exercises 1-1

In Exercises 1 through 12, add the given numbers. Be sure to check your work.

1. 36 29 87	**2.** 45 89 37	**3.** 627 83 524	**4.** 433 612 109	**5.** 446 915 992 67	**6.** 809 826 278 548
7. 8028 4756 4803 3823	**8.** 7695 4803 986 7375	**9.** 3873 9295 4082 399 7646	**10.** 989 3216 4807 736 9297	**11.** 30,964 9,877 92,286 5,547 965	**12.** 87,657 93,984 57,609 8,726 92,875

In Exercises 13 through 24, perform the indicated subtractions. Check your work.

13. 8704 − 3102	**14.** 5162 − 2041	**15.** 873 − 292	**16.** 921 − 224	**17.** 8305 − 7356	**18.** 2006 − 1197
19. 36,047 − 26,249	**20.** 32,105 − 22,116	**21.** 40,165 − 9,586	**22.** 10,906 − 9,928	**23.** 290,078 − 194,396	**24.** 872,110 − 682,324

In Exercises 25 through 32, perform the indicated operations. Be sure to express answers with the proper units.

25. 5 ft 8 in. + 2 ft 6 in. **26.** 6 ft 10 in. + 5 ft. 8 in. **27.** 8 lb 13 oz + 9 lb 3 oz