Quantum Optics

Miguel Orszag

Quantum Optics

Including Noise Reduction, Trapped Ions, Quantum Trajectories, and Decoherence

With 75 Figures
and 79 Problems with Hints for Solutions

Springer

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To all those who gave me their support and wisdom.

My parents

Ladislao Orszag (1911–1984)

Isabel Posa (1908–1999)

and

Martita.

Preface

This graduate text originated from lectures given by the author at the Universidad Católica de Chile in Santiago, as well as at the University of New Mexico. Also, material has been drawn from short summer courses given in Rio de Janeiro and Caracas.

Chapter 1 is devoted to some basic ideas of the interaction of radiation and matter, starting from Einstein's ideas of emission and absorption, and ending with elementary laser theory.

The quantum mechanical description of the atom–radiation interaction is dealt with in Chap. 2, and includes Rabi's oscillations and Bloch's equations.

Chapter 3 contains the basic quantization of the electromagnetic field, while Chaps. 4, 5 and 6 study special states of the electromagnetic field and the quantum theory of coherence.

The Jaynes–Cummings model, which describes in a fully quantized manner, the atom–radiation interaction, is studied in Chap. 8, along with the phenomena of collapse and revival. We also introduce the dressed state description, which is useful when studying resonance fluorescence (Chap. 10).

Real physical systems are open, that is, one must always consider dissipative mechanisms, including electromagnetic losses in cavity walls or atomic decay. All these effects can be considered in great detail, by studying system–reservoir interactions, leading to the master and Fokker–Planck equations. These reservoirs can also be phase dependent, an effect that can modify the decay rate of an atom (Chap. 9). As we mentioned before, Chap. 10 is entirely devoted to resonance fluorescence, and the study and observation of photon antibunching.

The invention of the laser, in the 1960s, opened up a new area of research, baptized quantum optics. This discovery spurred the growth of new research fields such as non-linear optics and non-linear spectroscopy. First the semiclassical theory, and then the quantum theory of the laser was well developed by the late 1960s. The quantum theory of the laser, based on the master equation and the Langevin equation approach, is extensively treated in Chaps. 11 and 12, respectively. We have also added some more recent material, including the micromaser and the effect of pump statistics, as a form of noise reduction scheme. Although pump statistics did not play any role in the original laser theory, recent experiments and theoretical calculations have shown that one could reduce considerably the photon number fluctuations if one is careful enough in pumping the atoms in an orderly way.

We further study quantum noise reduction in correlated emission lasers and the generation of squeezed states, typically from a parametric oscillator. These subjects are studied in Chaps. 13 and 14 respectively. In Chap. 14 we also introduce the input–output theory, which is very appropriate for describing the parametric oscillator and other non-linear optical systems.

Quantum phase, which started with Dirac, is a controversial subject, even today (Chap. 15). Most of the time optical experiments deal with direct or indirect measurement of a phase. For this reason, I felt it was important to include it in this book, even if it may not be a mature subject.

The last five chapters deal with more recent topics in quantum and atom optics. The Monte Carlo method and the stochastic Schrödinger equation (Chap. 16) are recent tools that have been used to attack optical problems with losses. Theoretically, it takes a different point of view from the more traditional way via the master or Fokker–Planck equations, and it is convenient for practical simulations.

Measurements in optics, and physics in general, play a central role. This was recognized early in the history of quantum mechanics. We introduce the reader to the notions of quantum standard limits and quantum non-demolition measurements (Chap. 18). A detailed example is studied in connection with the QND measurement of the photon number in a cavity. Continuous measurements are also studied. A related subject, decoherence (Chap. 20), is quite relevant for quantum computing. This intriguing phenomenon is connected with dissipation and measurement.

Finally, although a bit outside the scope of quantum optics, we have included the topics of atom optics (Chap. 17) and trapped ions (Chap. 19). These are fast-growing areas of research.

Over the years I have collaborated with many colleagues and students, who directly or indirectly contributed to this work; in particular: G.S. Agarwal, Claus Benkert, Janos Bergou, Wilhelm Becker, Luiz Davidovich, Mary Fuka, Mark Hillery, T. Kist, María Loreto Ladron de Guevara, Jack. K. McIver, Douglas Mundarain, L.M. Narducci, Ricardo Ramírez, Juan Carlos Retamal, Luis Roa, Jaime Röessler, Bernd Rohwedder, Carlos Saavedra, Wolfgang Schleich, Marlan O. Scully, D.F. Walls, Herbert Walther, K. Wodkiewicz, Nicim Zagury, F.X. Zhao, Sh.Y Zhu. I thank them all.

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Finally, last but not least, I would like to thank my wife Marta Montoya (Martita) for her love and constant support during this project.

Santiago, April 1999 *Miguel Orszag*

No te escapes
Ahora
Ma ayudarás. Un dedo,
una palabra,
un signo
tuyo
y cuando
dedos, signos, palabras
caminen y trabajen
algo
aparecerá en el aire inmóvil,
un
solidario sonido en la ventana,
una estrella en la terrible paz nocturna,
entonces
tu dormirás tranquilo,
tu vivirás tranquilo:
será parte
del sonido que acude a tu ventana,
de la luz que rompió la soledad.

From: Odas Elementales, Pablo Neruda [P.1]

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1. Einstein's Theory of Atom–Radiation Interactión

In 1917, Einstein formulated a theory of spontaneous stimulated emission and absorption, based on purely phenomenological considerations [1.1]. His results paved the way for an understanding, in a qualitative way, of the basic ingredients of the atom–radiation interaction, and could be useful for describing the processes of absorption, light scattering by atoms, stimulated emission in a variety of laser and maser systems, etc. This happened after Planck found that the spectral distribution of blackbody radiation could be explained by quantizing the energy [1.2], and Einstein had explained the photoelectric effect [1.3] by postulating the existence of energy packets that were later called *photons*.

Einstein's theory is based on reasonable postulates, which will be justified more rigorously later, when we treat the same problem using quantum mechanics. The present arguments are of a heuristic nature [1.1]:

> 'Recently I found a derivation of Planck's radiation formula which is based upon the basic assumption of quantum theory and which is related to Wien's original consideration: in this derivation, the relationship between the Maxwell distribution and the chromatic black-body distribution plays a role. The derivation is of interest not only because it is simple, but especially because it seems to clarify somewhat the at present unexplained phenomena of emission and absorption of radiation by matter. I have shown, on the basis of a few assumptions, about emission and absorption of radiation by molecules, which are closely related to quantum theory, that molecules distributed in temperature equilibrium over states, in a way which is compatible with quantum theory, are in dynamic equilibrium with Planck's radiation. In this way, I deduced in a remarkably simple and general manner, Planck's formula.' [1.4]

1.1 The A and B Coefficients

We assume a closed cavity with N identical atoms, with two relevant bound-state energy levels, which we shall label E_b and E_a and which are quasi-