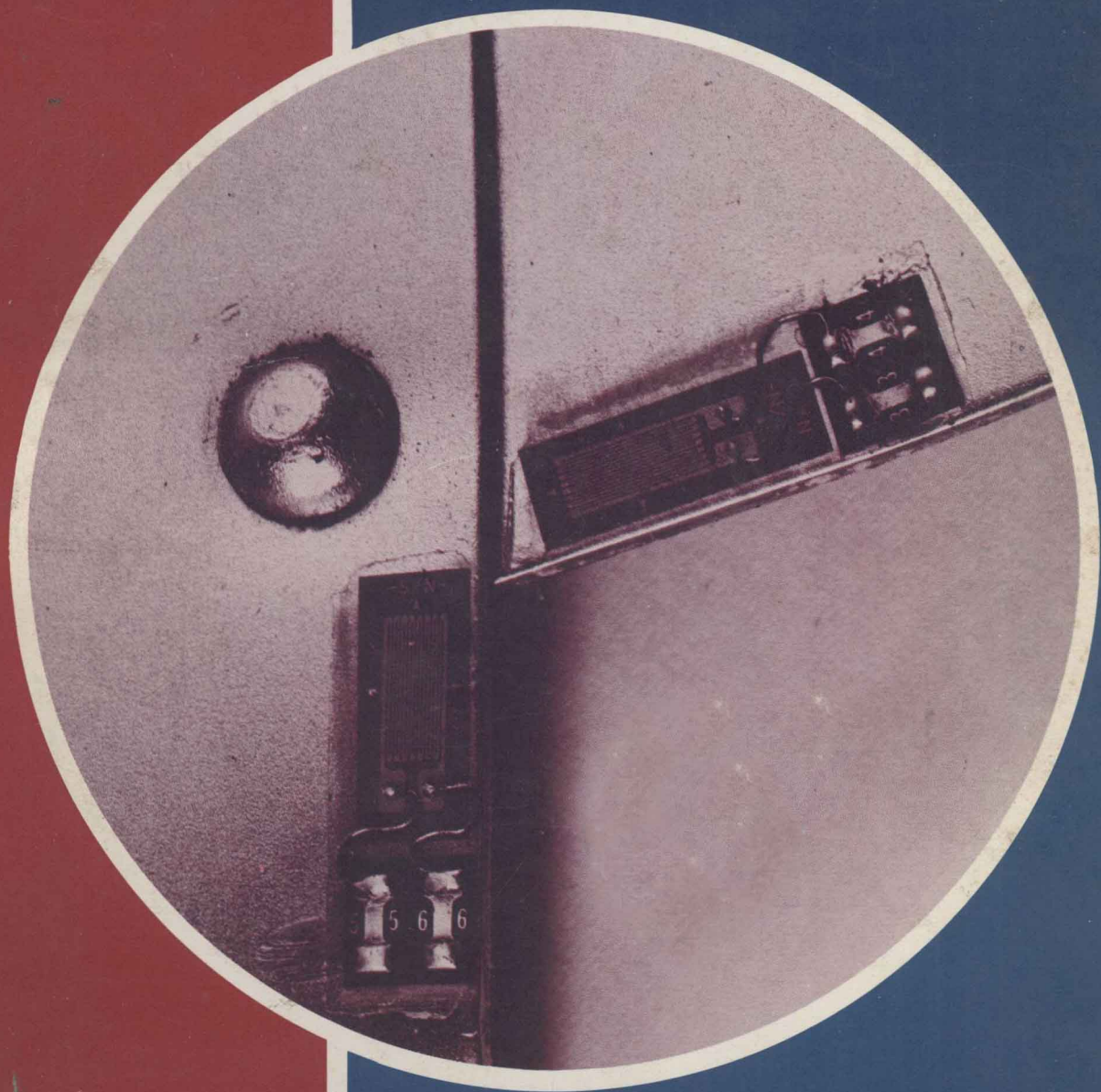


BELA I. SANDOR

# STRENGTH OF MATERIALS



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**STRENGTH  
OF  
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# PREFACE

This text is intended primarily for students who are taking their first course in strength of materials. Several sections will also be of interest to more advanced students and to engineers who are seeking introductions with perspective to some of the most important recent developments in this area. The following features make this book distinguishable from others in its category.

## New Material

Strength of materials is an old subject, but it is now rejuvenated with important new developments that are caused by the appearance of new materials and the need to use even the known materials in unusually severe and critical situations. Furthermore, significant advances have been made in recent years in the mechanical testing of materials. These advances not only allow more rapid and more precise tests but also have led to the practical use of entirely new concepts in strength of materials. Designers who obtained their academic education a number of years ago must now enroll in special short courses to learn about the new ideas; otherwise they must glean the information from mountains of relevant but diverse publications. It is imperative that future designers be introduced to the new as well as to the classical fundamental concepts and methods of strength of materials. Following are the most important new items (order of appearance has no significance):

**1. *Permutations of signs of stress and strain.*** Every designer should know that in many practical situations all four combinations of the signs of stress and strain are possible (+ and +, - and -, + and -, - and +). This has serious implications in using experimental stress analysis in the design process where stresses are calculated from measured strains.

**2. Cyclic stress-strain curves.** The stress-strain response of many materials depends on the prior loading history. This means that the standard tensile stress-strain curve may be inadequate for predicting a material's behavior when the loads on it vary it time, which can be expected in most cases. Sustained efforts must be made to publicize the need for obtaining the cyclic stress-strain curves besides the tensile or compressive stress-strain curves. Numerous examples of stable and unstable mechanical behavior are given in this text, and these may be of considerable interest even to experienced designers.

**3. Modern failure criteria.** The classical failure theories have limited usefulness in practice because they cannot be applied readily to deal with members that have discontinuities. Furthermore, the full significance of a flaw or machined notch is not indicated by the theoretical stress concentration factor. Certain notched members can fail in the brittle manner even if they are made of intrinsically ductile materials. The introductory mechanics analysis of such members is within the scope of what students can and should learn in a modern elementary course in strength of materials. The concept and practical aspects of fracture toughness ( $K_{Ic}$ ) are also in this category.

**4. Composite materials.** Filament-reinforced composite materials such as boron-aluminum and graphite-epoxy are important in modern technology. The rule of mixtures and its limitations are presented for these. Sintered porous metals are also discussed. These can be made with wide ranges of density and strength.

**5. Modern testing equipment.** The capabilities of servo-controlled, electrohydraulic machines are discussed and demonstrated with several photographs.

### Old Material in Original Presentation

There are a number of original models and analogies throughout the text.

The use of Mohr's circle is extended to deal with optimum design. There are also examples of stress analysis for nonsteady loading.

The various concepts of material toughness are clarified and presented in perspective.

The concept of the absolute maximum shear stress is given special emphasis because it has significance in modern fracture analysis and control techniques.

### Mathematics

The reader should have a working knowledge of elementary calculus. The derivations of formulas are tailored to the average student.

### Units

The U.S. customary and the SI units are used alternately in the text and in the problems. Note that large numbers in the U.S. system use commas, but in the SI system a space is used instead of a comma. For example, 20,000 in. = 508 000 mm.

## Problems

There are approximately 160 examples worked out in the text. The solutions often show the patterns of engineering judgment in the modeling process (making assumptions) and in the evaluation of the answers.

In each chapter there are many simple, intermediate, and difficult (noted by asterisk, except in Chapter 15) problems for a wide range of experiences. Some of the difficult are open-ended, *multiple-answer problems*, which are the most common in the world. Although they are not universally liked many people consider them the most useful and the most interesting. These problems allow the student to practice the highest cognitive objectives of learning, which include analysis, synthesis, evaluation and choice, and strategies of problem-solving.

The readers should practice judgement in answering any problem. They should simply state whether the answer appears reasonable, unreasonable, or uncertain. The idea here is that anybody can make a mistake, but few people have the chance to check everything carefully. Well-educated guesses are important in guiding one to the correct answers.

Both systems of units are used in a few of the problems. This can be annoying, but real problems are this way sometimes, and perhaps can never be avoided completely.

Only a few of the project problems in Chapter 15 can be worked on in a given semester, at best. There are many ways of handling these problems, but generally they should be left for the second half or the end of the course. They are good for extra-credit work and for group projects. Students are encouraged to read Appendix B concerning the modeling process before trying to solve complex problems.

## Motivation

Last, but not least, an aim of this text is to motivate the students. Most of the topics are introduced in ways that make them interesting and reasonable to explore. Some people consider this unimportant when the subject is intrinsically interesting. There is evidence from educational research, however, that affective learning (which involves interest, attitude, value, and emotion) should be nurtured to achieve the best results in educating students in any area. One could say on the basis of common sense that the most successful athletic coaches do much more for their athletes than help in muscle building and in the acquisition of techniques (and most athletes are basically more motivated than the average student in any average course). It is hoped that this book will be accepted as an enthusiastic assistant coach might be.

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# ACKNOWLEDGMENTS

At least a few of the people who have contributed to this book must be mentioned. Professor JoDean Morrow of the University of Illinois has influenced my thinking in numerous ways since about 1960. He is the grandfather-author in general and the creator of several of the most interesting problems in particular. Dr. R. W. Landgraf of the Ford Scientific Research Staff has provided guidance through his published and unpublished work in the use of recently introduced mechanical properties of materials. William Garver gave invaluable assistance in working out many examples and in critically evaluating the whole manuscript. Mrs. M. Lynch and Mrs. E. Schultz were excellent and patient typists.



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# INTRODUCTION

Strength of materials has been of interest to humans since prehistoric times. All solid materials have limits to their strengths, and a qualitative understanding of these has always been essential in making useful tools, weapons, machines, vehicles, and structures. War machinery is of special historical interest because Archimedes was deeply involved with the concepts and practical problems of equipment used in warfare. He was spectacularly successful in the systematic application of a few fundamental principles of mechanics. According to an ancient historian, he developed the novel technique of dropping hooks on an attacking Roman ship, quickly raising the ship out of the water, and dropping it back. It appears that the Romans could not occupy Syracuse by force even after a long siege because Archimedes applied his knowledge and creative ability in the defense of the city (see Chapter 15, Problem A).

Among the numerous great achievements of Archimedes, one item must be mentioned in introducing this text. He formulated and communicated to later generations the concept of equilibrium conditions involving forces and moments. The concept was refined later, and now it is essential in analyzing the strength of materials.

Through most of the Middle Ages there was a gradually growing need for more knowledge in using materials, but engineers and architects relied on specific experiences and intuition instead of generally applicable analysis. To show this state of affairs and to provide a perspective for the important developments that were to follow, consider some of the efforts by Leonardo da Vinci concerning materials technology and design. One of his most ambitious projects was the design and creation of the largest equestrian statue of all time. The horse alone was to be 24 ft high. At first Leonardo boldly contemplated a rearing horse of magnificent

beauty, but he was realistic in his final plans. He knew how strong bronze was, and he understood the problem of stability for a rearing horse (the total weight of the statue would have been about 200,000 lb). But he must have realized that his simple tension tests of wires did not provide enough information for the design of the most critical parts of the statue, the rear legs. Thus, he resigned himself to the idea of supporting three legs of the horse, which must have been a very painful decision because that was how all other artists made their equestrian statues. He could have gambled and won, or he could have lost disastrously. The knowledge necessary to ensure success simply was not available at the time.

The first substantial steps toward understanding the strength of materials were made by Galileo Galilei (1564–1642). He experimented extensively and made keen observations of the origins of fractures. Interestingly, the area of strength of materials was one in which Galileo made a serious error, as will be shown later. In spite of his mistake, he made such important contributions that now he is regarded as the founder of mechanics of materials as a subject for rational investigation. This little known fact is worthy of thought by both students and teachers.

Since Galileo's time, the subject has received increasing attention. It is a cornerstone of modern technology, and it will remain important in the future. There are diverse examples in this text to show the universal applicability of knowledge of strength of materials.

# 1

## STRESS

The words *stress* and *strain* are used frequently but they denote different things to different people. The meanings are most specific to engineers who deal with the strength or deformability of materials. A number of special definitions of stress and strain are needed by engineers, and the most important of these will be presented in this text. Sometimes it is necessary to be careful and precise in using the appropriate definition in technical communication. For example, normal stress, shear stress, local stress, elastic strain, and plastic strain all have their own particular meanings.

Stress is the description of the intensity of force; strain is the description of the intensity of deformation. The two are related in most cases. It always should be remembered that all solid materials deform under the action of any force. There are no perfectly rigid bodies, even though such an assumption is often useful.

The rational basis for evaluating and comparing the mechanical performances of different materials is called *stress analysis*, which encompasses the analysis of strains. This chapter introduces the concept of stress; the deformability of solids can be generally ignored at this point.

### 1-1. STRESS

A good way to learn about stresses and fully appreciate the significance of several basic concepts in stress analysis is to consider a glued joint. Imagine that two pieces of flat metal are glued together with an adhesive whose strength properties are known. Assume that the finished piece is rectangular, as shown in Fig. 1-1a, but there is a choice of the angle  $\theta$  of the seam.

A reasonable question is, “what is the optimum angle  $\theta$  to avoid failure of



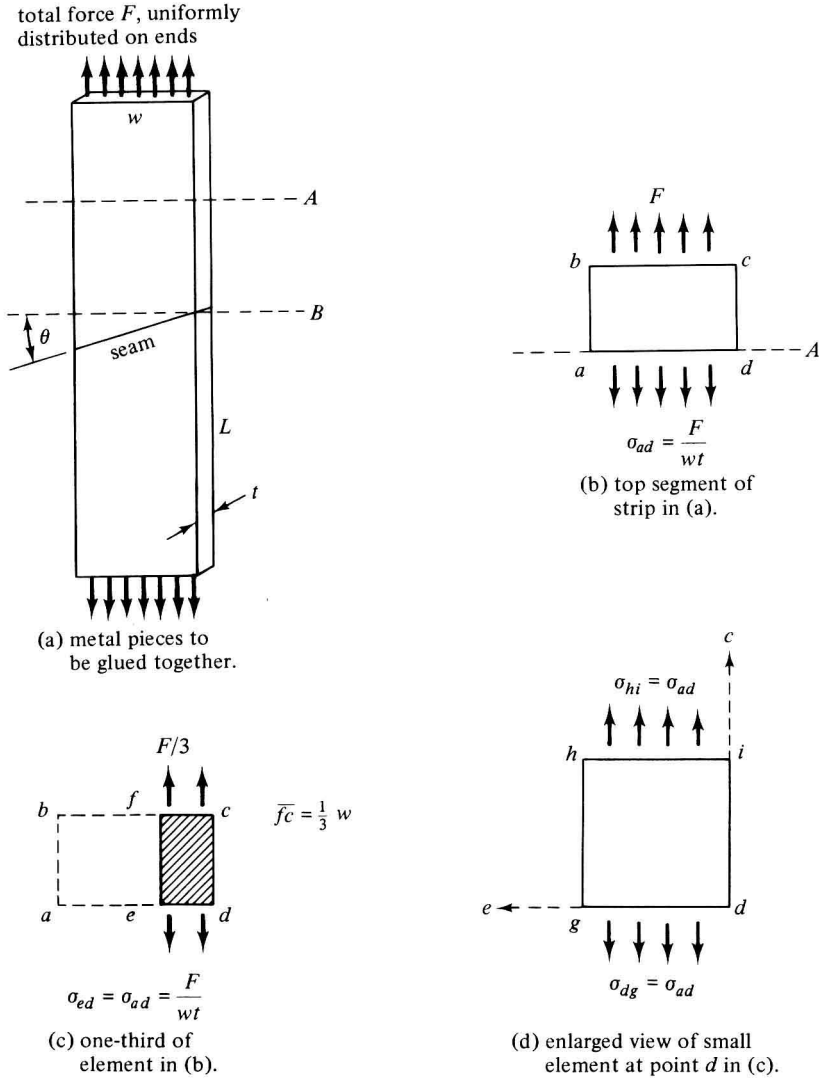


Fig. 1-1. (a) Metal pieces to be glued together. (b) Top segment of strip in Fig. 1-1a. (c) One-third of element in Fig. 1-1b. (d) Enlarged view of small element at point  $d$  in Fig. 1-1c.

the seam under the given conditions?” The answer to this can be found by determining the maximum values of the internal forces in the material and the directions in which these forces are acting and then relating these to the strength properties of the glue. Doing these steps for the first time will go rather slowly. Later, with a little experience, the answer can be found rapidly.