INTRODUCTORY COLLEGE MATHEMATICS

HACKWORTH
and
HOWLAND

SAUNDERS SERIES IN MODULAR MATHEMATICS

Metric Measure

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PREFACE

Metric Measure

This book is one of the sixteen content modules in the Saunders Series in Modular Mathematics. The modules can be divided into three levels, the first of which requires only a working knowledge of arithmetic. The second level needs some elementary skills of algebra and the third level, knowledge comparable to the first two levels. *Metric Measure* is in level 2. The groupings according to difficulty are shown below.

Level 1	Level 2	Level 3
Tables and Graphs	*Numeration*	*Real Number System*
Consumer Mathematics	*Metric Measure*	*History of Real Numbers*
Algebra 1	*Probability*	*Indirect Measurement*
Sets and Logic	*Statistics*	*Algebra 2*
Geometry	*Geometric Measures*	*Computers*
		Linear Programming

The modules have been class tested in a variety of situations: large and small discussion groups, lecture classes, and in individualized study programs. The emphasis of all modules is upon ideas and concepts.

The conversion to the metric system by the United States makes *Metric Measure* an important module for all science and non-science majors. The module is a necessity for students enrolled in science or technology courses in which the metric system is used.

Metric Measure begins by presenting some of the history of measurement while showing some of the difficulties of developing adequate standards of measure and measurement systems. The organization of the metric system is followed by methods of converting measurements from one system or unit to another. The module deals with linear, area, volume, mass, liquid, and temperature measures.

In preparing each module, we have been greatly aided by the valuable suggestions of the following excellent reviewers: William Andrews, Triton College, Ken Goldstein, Miami-Dade Community College, Don Hostetler, Mesa Community College, Karl Klee, Queensboro Community College, Pamela Matthews, Chabot College, Robert Nowlan, Southern Connecticut State College, Ken Seydel, Skyline College, Ara Sullenberger, Tarrant County Junior College, and Ruth Wing, Palm Beach Junior College. We thank them, and the staff at W. B. Saunders Company for their support.

Robert D. Hackworth
Joseph W. Howland

NOTE TO THE STUDENT

Objectives:

Upon completing this unit, the reader is expected to be able to demonstrate the following skills and concepts:

1. An understanding of ancient measurement systems and the difficulties involved in using non-standardized units.

2. An ability to change measures from the inch-pound system to the metric system.

3. An ability to change measures from the metric system to the inch-pound system.

4. An ability to change measures in the metric system from one unit to another.

Three types of problem sets, with answers, are included in this module. Progress Tests appear at the end of each section. These Progress Tests are always short with only four to six problems. The questions asked in Progress Tests always come directly from the material of the section immediately preceding the test.

Exercise Sets appear less frequently in the module. More problems appear in an Exercise Set than in a Progress Test. These problems arise from all sections of the module preceding the Exercise Set. Part I problems in the Exercise Sets are specifically chosen to match the objectives of the module. Part II problems are Challenge Problems.

A Self-Test is found at the end of the module. Self-Tests contain problems representative of the entire module.

In learning the material, the student is encouraged to try each problem set as it is encountered, check all answers, and re-study those sections where difficulties are discovered. This procedure is guaranteed to be both efficient and effective.

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METRIC MEASURE

INTRODUCTION

A recent headline stated, "POLICE SEIZE KILO OF MARIJUANA." The feeling for the amount of marijuana that was seized based on the previous headline may have varied tremendously from person to person. Some people may have thought that a whole roomful of marijuana was found. Others may have had the impression that just a small container was seized. Possibly most readers had no idea how much a kilo weighed. Actually a kilogram weighs approximately 2.2 pounds. For many Americans, the experience of interpreting measurements given in metric units may occur sooner than people realize. The United States may soon adopt a new system of weights and measures, called the metric system, and drop the old system of measuring lengths in inches, feet, yards, and miles, and weights by ounces, pounds, and tons. Hopefully, the change will be gradual -- allowing time for people to learn to "think metric." There are good reasons for believing that the metric system will replace the inch-pound system in the United States. In 1975, the United States was the only industrialized nation in the world not using the metric system. Great Britain, the originator of the English inch-pound system of measure, has adopted metric units as its official system of weights and measures. Canada and Mexico, our neighbors to the north and south, have officially adopted the metric system. The countries of the European Common Market have gone so far as making an agreement to accept only those products that are in metric units and sizes after 1977. Foreign corporations that are building factories in the United States must have employees capable of working with metric units. American corporations currently using inch-pound measurements will have to build at least some of their products to metric units so they can meet world competition. As a country interested in exporting and importing vast quantities of material, the United States will need an increasing number of people with an ability to build and repair products produced to metric specifications. Consequently, the pressures of world trade will force the United States and its individual

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A recent bill in the Florida legislature proposed that the liter be used as the largest legal size allowable for the sale of alcoholic beverages. A liter bottle holds slightly more whiskey than a quart bottle.

businesses to deal with goods measured and designed in metric units. The scientific community in the United States is already using the metric system. Widespread, everyday use of the metric system cannot be far in the future.

The economic well-being of the United States is an important factor in considering an adoption of the metric system, but another factor may ultimately be far more significant. The metric system is a far simpler system for almost every type of problem involving computation or arithmetic with measurements. There are two reasons for the simplified computations in the metric system. First, metric measurements are given in decimals instead of fractions or mixed numbers. Most people find calculations are easier with decimals than with fractions or mixed numbers. Second, changing units in the metric system is always a case of multiplying or dividing by 10, 100, 1000, etc. Such multiplication and division in a decimal system is relatively simple because it can be accomplished simply by moving the decimal point.

Progress Test 1

1. Which of the following seems to describe the height of a six foot tall man?

   a. 543 cm b. 1.8 m c. 96 mm

2. The girl with the hour-glass figure in a metric country would have measures :

   a. 91-55-91 cm b. 53-34-53 cm

   c. 15-9-15 cm d. 36-26-36 cm

3. Most people have trouble adjusting to change. Did you have trouble with problems 1 and 2 in confronting a new system of measures?

ANCIENT MEASUREMENT SYSTEMS

The history of measurement probably begins about 10,000 BC. There is evidence that the people of that time were using linear measurements. Linear measures are those used for lengths. Inches, feet, and miles are examples of linear measures in the inch-pound system. Centimeters, meters, and kilometers are examples of linear measures in the metric system. In the primitive cultures of the Neolithic Age (10,000 BC to 8,000 BC) linear measures were probably used to compare the lengths of spears, the sizes of animals killed, or the lengths of sticks in a lean-to shelter.

Measures of volume were developed when people learned to farm and cultivate food grains. Tribal chieftains probably set up measuring units of cups, pots, jars, or baskets to measure volumes. There is archeological evidence of devices for measuring volume dating back to 2050 BC. A mural found in an Egyptian tomb shows a copper cylinder for measuring oil and another cylinder for measuring wheat.

Ancient linear measuring units apparently were based on comparisons with parts of the body. One of the most interesting, complete reports written on ancient linear measures was prepared by John Quincy Adams in 1821 when he was United States Secretary of State. Adams listed and described the following linear measures:

cubit -- the distance from the top of the middle finger to the tip of the elbow.

ell -- twice the cubit or the distance from the top of the middle finger to the middle of the breast when the arm is extended horizontally to the side.

fathom-- twice the ell or the distance between the tips of the middle fingers when the arms are extended horizontally.

span -- one-half a cubit.

palm -- one-third a cubit.

finger-- one-fourth a cubit.

foot -- one-sixth of a fathom.

Three interesting facts are illustrated by the list of ancient measuring units. First, the description of the ell makes the tacit assumption that the distance from the elbow to the middle of the breast is equal to one cubit. Second, each of these units is based upon the size of the measurer's body. Third, the subdivision of units was in halves, thirds, fourths, and sixths.

The use of weights and measures preceded recorded history. The origin of weights and measures was attributed to the gods; the Egyptians to their god Theuth and the Greeks to their god Mercury.

Measuring lengths through the use of body parts is a natural method that almost every person has attempted. The fisherman who holds his hands apart to show the "big one that got away" has a long tradition of body-part-measurers behind him. There are certainly advantages to carrying one's measuring stick as part of one's body, but there are also distinct disadvantages. The fellow with long arms has a different set of measuring units than a shorter person. Sometimes that may be an advantage, sometimes a disadvantage, but always it means there is poor comparability.

The subdivision of ancient units into halves, thirds, fourths, and sixths has been carried over to our present inch-pound system. Today, most foot rulers are marked off into twelfths (inches), half-inches, quarter-inches, and eighth-inches. Pounds are divided into sixteenths (ounces). Pints are half-quarts and each pint is divided into sixteenths (ounces again). Apparently these divisions are the result of the subdivisions used in ancient measuring units, but they depart drastically from a base ten number system which is built upon multiples or divisions of 10.

Although the cubit may have been the first unit of linear measuring and some people consider it sacred because Noah supposedly used it in building his ark, the foot became the basic linear unit in the cultures surrounding the Mediterranean from 2000 BC until the advent of the metric system in the 1800's AD. Nevertheless, the foot was a non-standardized unit. Each small geographic area had its own "foot" in terms of length. The table below shows some of these different "foot" measurements.

Culture	Approximate Measurement in Inches
Sumeria	10.50
Greek	12.14
Phoenicia	10.98
Carthage	11.08
Sardinia	11.17
Rome	11.30

As the table clearly shows, the foot varied from culture to culture. The variation was probably due to the size of particular individuals who established the measuring unit. The number of different units was limited only by the number of chieftains for the separate villages, fiefdoms, or nations. Those differences are understandable for a time in history when distances greatly inhibited travel and trade. More surprising perhaps is the fact that even today the United States Coast and Geodetic Survey uses a foot that varies from the standard by 0.000 06 foot.

Measurements for weights were established by 8300 BC. An Egyptian balance from that time has been found which evidently was used for weighing precious metals. Because weight measures originally were intended for grains such as wheat and barley, the "grain" became the unit of weight for precious metals. Today the "grain" is still used by pharmacists, drug manufacturers, scientists, and the government. The amount of gold equal to the United States dollar is measured in grains. Of course, the grain as it is used today is quite accurate and standardized, but the ancient measurement of "grain" varied with the rainfall, grain type, richness of the soil, and the moisture content on a particular day. The shrewd trader may have realized that on hot, humid days the grain was wet and heavy whereas in times of drought or low humidity the grain was light and dry.

Progress Test 2

1. What fraction of a cubit is a foot?

2. Hector and Homer, from two different villages, weighed a gold ring, each man using his own grain for a unit of weight. Hector said the ring weighed 18 grains, but Homer said the ring weighed 24 grains. Which man's grain had the lightest weight per kernel?

3. Suppose that Alitta, a village chieftain, set up the following set of measures for measuring wine.

   cup - the basic unit, jar - ten times as large as a cup

   firkin - ten times as large as a jar

   baryl - ten times as large as a firkin

   a. 3 jars would equal _____ cups.

   b. 3 firkins would be equivalent to _____ cups

c. 3 baryls would be equivalent to _____ cups.

d. True or false. 1.5 firkins equal .15 baryl.

EFFORTS TO STANDARDIZE UNITS

Most of the history of man's measurements is marked by the use of natural non-standardized units such as the length of body parts or the weight of grains of barley. Natural units that were dependent upon a particular time, place, or individual would vary in size. There could be no uniformity of measure with these natural units. The lack of standardized units common to all people who used them was recognized by the Egyptians 4000 years ago. The Egyptians attempted to overcome the lack of standardized units and ease the problems of computation about 2000 BC.

The Egyptians established a basic unit of linear measure. From this basic unit other measuring units were developed. The Egyptians constructed relationships into their measuring system. Other measuring systems of that time, and the present inch-pound system, first named the units and then tried to find relationships between them.

Unfortunately, the Egyptian system was ignored by other cultures and declined in use with the rise of the Roman Empire. The Romans used a natural base measurement system based on twelve subdivisions to a unit. For example, there were 12 unciae (ounces) to a libra (pound). As the Romans conquered nations they made no attempt to force their entire system on the other cultures, but the Romans did insist that the basic units be named by the Roman names and subdivided into twelve units. Therefore, in the Roman Empire every basic unit of weight was called a pound and divided into twelve ounces. Although the pound varied radically in weight from England to Judea, it always contained 12 ounces.

At the end of the 9th Century, Charlemagne, famous for the Magna Carta, attempted to standardize systems of weight and measure throughout his Empire. Two stories are told of the source of his linear foot which was 12.789 3 inches long. One said it was actually the length of Charlemagne's foot. The other story said that it was exactly one-half of the Arabic cubit (established by the Caliph from the "Arabian Nights").

Although the Charlemagne foot existed as the French standard until displaced by the metric system, his edicts on measurements had limited effect. Local war lords ignored them, preferring to use their own personal standards instead. Peasants living far from the seats of power continued to use the units they had inherited from their ancestors. From Charlemagne's time the number of measuring units seemed to constantly expand. An indicator of the number of units used in England alone is a dictionary of the units used from the 12th Century to the 19th Century. The dictionary contains 184 pages of unit names ranging frôm alna to butt to dicker to pokke to werkop. As an example of the changing standard for just one unit, a barrel contained 32 gallons in the 13th century and was equal to 4 firkins of 8 gallons each. In 1688 it contained 34 gallons, but by 1803 it was changed to 36 gallons. The ale barrel had a capacity of 32 gallons, but a barrel of herrings only contained 30 gallons. Barrels of salmon were the largest at 84 gallons. A barrel of wine held 31-1/2 gallons after 1750. In Ireland, the barrel held 40 gallons of grain. Since the size of the gallon varied just as much as the barrel, it is indeed difficult to know how much a barrel really held.

The Asian countries also followed the natural tendency to use familiar objects as measurement standards.

One ancient Asian country, Thailand, had an interesting way of organizing linear units. In Thailand, small linear units were expressed in terms of hairs, louse eggs, and grains of rice. In early times, one atom would be one-sixty-fourth of a hairbreadth, eight atoms made up one molecule, and eight molecules would be a hairbreadth. Eight hairbreadths were equal to one louse egg, eight louse eggs became one louse, eight lice would be one grain of rice, two grains of rice (why two instead of eight?) made one krabiad, and four krabiads were equal to one fingerbreadth. The Thai people may have used these relatively small linear measurements in the construction of their exquisitely engraved gold and silver jewelry.

One of the familiar nursery rhymes may have resulted from a King's tax on a 17th century unit of volume. Charles I placed a tax on the jack or jackpot, a pot used to hold honey, wine or milk. As a result, the size of the jack was reduced. Half a jack made a gill. There is reason to suspect that the rhyme about Jack and Jill was a protest against the jack tax. Gill or Jill were synonyms for sweetheart. When Jack fell down, Jill came tumbling after.

Progress Test 3

1. Romans had to be familiar with unciae and libras. If 14-1/2 libras are converted into unciae, how many unciae were obtained?

2. 32 unciae = ? libras.

3. According to the Thai measurement system, a line as long as a grain of rice would be _____ hairbreadths long.

4. If a design on a Thai ring was 1 krabiad wide, how many atoms wide would it be?

THE BIRTH OF THE METRIC SYSTEM

As varied and confusing as the chaotic multitude of measurement units used by ancient cultures may seem, the difficulties did not bother most people because of the lack of mobility at that time. Most people never traveled as much as 50 miles from their birthplace. A brewer in Birmingham did not care that a gallon in Hamburg was a different size because he never dealt with the brewers in Hamburg anyway. Traders both tolerated and profited from the different measuring systems when exchanging one unit of merchandise for another. The only people who were starting to see the advantage of a well organized system of weights and measures based on some natural phenomenon were the scientists. This group, exchanging ideas, problems, and solutions on an international scale, was greatly hampered by the lack of common measurement standards. The scientists also wanted a measurement system that facilitated computations.

As chaotic as measurement systems were in England, Europe, and Asia, they were no better in newly established America. Like other countries, the United States, in its early years, used standards for measurement that were natural. Consequently, they varied in size from town to town. In George Washington's time, the length of three barleycorns was equivalent to one inch. Since the length of three barleycorns will vary from farm to farm, one inch in South Carolina might have varied significantly from the length of an inch in Boston.

Thomas Jefferson, in 1784, proposed two workable plans for systems of weights and measures, but the plans did not achieve Congressional support.

One of Jefferson's proposals was to "define and render uniform and stable the existing system...to reduce the dry and liquid measures to corresponding capacities by establishing a single gallon of 270 cubic inches and a bushel of eight gallons or 2 160 cubic inches."

Jefferson's second proposal was interesting because it had some of the features of the metric system. He proposed "to reduce every branch to the same decimal ratio already established for coin and thus bring the calculations of the principal affairs of life within the arithmetic of every man who can multiply and divide plain numbers." Jefferson defined a natural linear unit as "a cylindrical rod of iron of such length as in latitude 45^0 in the level of the ocean and in a cellar or other place, the temperature of which does not vary through the year, shall perform its vibrations in small and equal arcs in one second of mean time." The effects of gravity, air pressure, and temperature were considered by Jefferson's definition. Therefore, it could be duplicated wherever the conditions of latitude, altitude, and temperature could be met.

The other units in Jefferson's system were derived from the basic linear unit. A foot was related to the distance that an iron rod swings in one second. An inch would be one-tenth of a foot. A line would be one-tenth of an inch. A point would be one-tenth of a line. Longer units were: a decade which was ten feet; a rood which was ten decades; a furloung which was ten roods; and a mile which was ten furloungs. Jefferson defined an ounce as "the weight of a cube of rainwater of one-tenth of a foot; or... the thousandth part of the weight of a cubic foot of rain water, weighed at standard temperature."

Sharing the fate of so many worthy ideas, Jefferson's proposals died in the Congress. Only his proposal to decimalize the currency was adopted.

The idea of basing linear units on objects such as a person's foot was abandoned by the French scientists in the 1790's. At that time Talleyrand, a powerful leader in the French National Assembly, proposed that the Academy of Sciences be instructed to study a universal system of weights and measures for France.

The scientists in the Academy insisted that a nonvariable base be used as a standard for linear measure. After considering the distance a pendulum swings in a second and a section of the length of the equator, the Academy decided on a section of the arc of the meridian through Dunkirk and Barcelona on the Mediterrenian Sea. A meridian is a circle around the earth perpendicular to the equator. After the distance from Dunkirk to Barcelona was measured with a great deal of difficulty over a seven year period, that distance was used to fix the length of the meter at one ten-millionth of the distance from the North pole to the equator. The meter was one of the units defined in the new "metric system." Three platinum and several iron bars were constructed by the French to serve as physical representations of the meter. If another country wanted to use the meter as its official standard, it had to have a bar constructed according to the length of the bars in Paris. This awkward and time consuming procedure was necessary to the standardization of the meter.

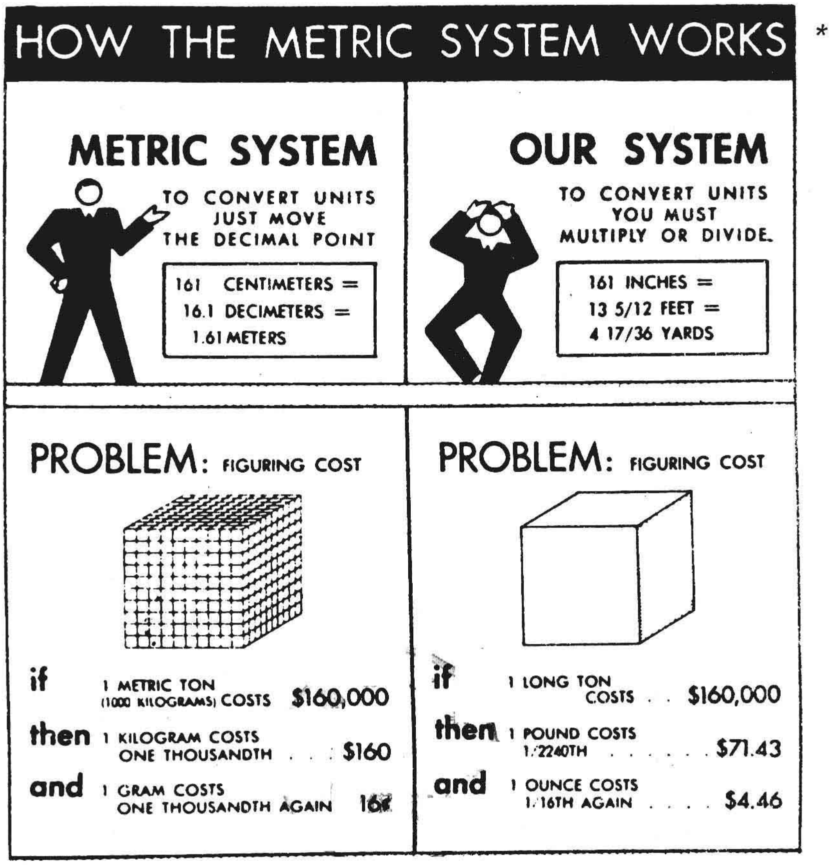
All the other linear units determined by the Academy were multiples or divisions of the meter by powers of 10. A kilometer is 1 000 times a meter and a centimeter is 1/100 of a meter. Therefore this new system was a decimal system: moving the decimal in a measure created larger or smaller units.

The table below shows the prefixes established for the linear multiples and divisions of the meter.

Myriameter	10 000 meters
Kilometer	1 000 meters
Hectometer	100 meters
Dekameter	10 meters
Meter	1 meter
Decimeter	0.1 meter
Centimeter	0.01 meter
Millimeter	0.001 meter

Each unit in the table above is 10 times longer than the unit in the next line below. A decimeter is 10 times as long as a centimeter. $0.1 = 0.01 \times 10$. A dekameter is 10 times longer than a meter.

Conversions are not so easy in the English system of weights and measures. The figure on the opposite page compares the conversion difficulties of the metric and inch-pound systems.



Progress Test 4

1. The length of a king's girdle was once used as the basis for the length of a yard. What is wrong with using such a length as a basis for a unit measurement?

2. Place the following units in order from the shortest to the longest length. Centimeter, kilometer, meter, hectometer, millimeter.

3. A kilometer is __________ meters.

4. A kilometer is __________ hectometers.

5. A millimeter is __________ of a centimeter.

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