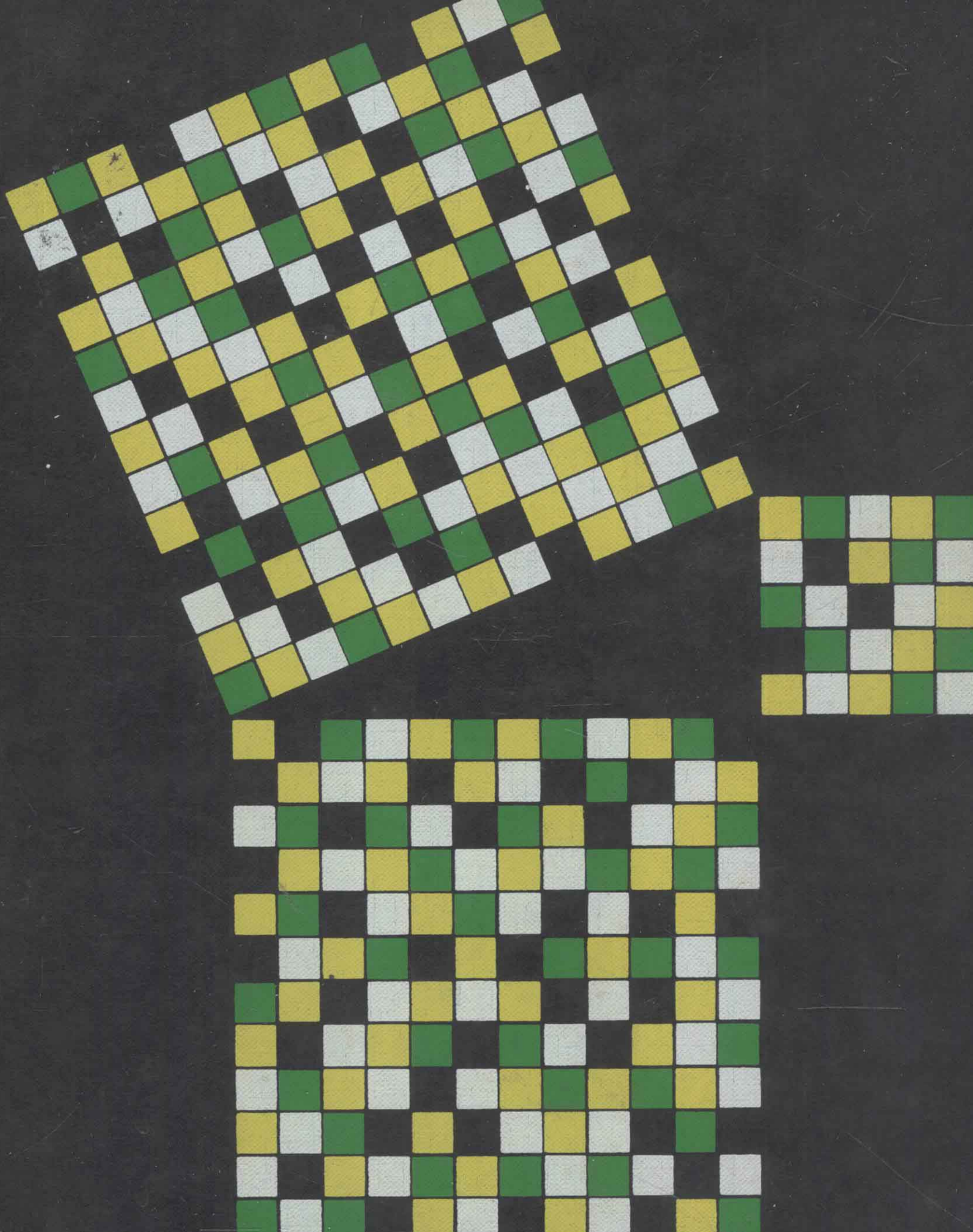

Plane Trigonometry

WALTER FLEMING DALE VARBERG

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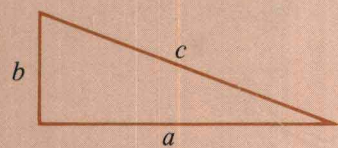
FORMULAS FROM GEOMETRY

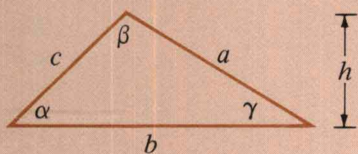
Triangles



Right Triangle

Pythagorean Theorem

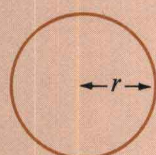
$$a^2 + b^2 = c^2$$



Any Triangle

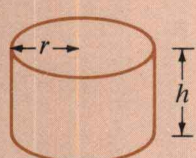
Angles	$\alpha + \beta + \gamma = 180°$
Area	$A = \frac{1}{2}bh$

Circles



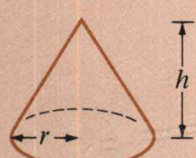
Circumference	$C = 2\pi r$
Area	$A = \pi r^2$

Cylinders



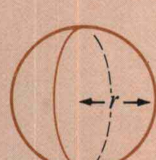
Surface Area	$S = 2\pi r^2 + 2\pi rh$
Volume	$V = \pi r^2 h$

Cones



Surface Area	$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
Volume	$V = \frac{1}{3}\pi r^2 h$

Spheres



Surface Area	$S = 4\pi r^2$
Volume	$V = \frac{4}{3}\pi r^3$

FORMULAS FROM ALGEBRA

Exponents

$$a^m a^n = a^{m+n}$$
$$(a^m)^n = a^{mn}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Radicals

$$(\sqrt[n]{a})^n = a$$
$$\sqrt[n]{a^n} = a, \text{ if } a \geqslant 0$$
$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Logarithms

$$\log_a MN = \log_a M + \log_a N$$
$$\log_a (M/N) = \log_a M - \log_a N$$
$$\log_a (N^p) = p \log_a N$$

Factoring Formulas

$$x^2 - y^2 = (x-y)(x+y)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$
$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$
$$x^2 + 2xy + y^2 = (x+y)^2$$
$$x^2 - 2xy + y^2 = (x-y)^2$$
$$x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3$$

Binomial Formula

$$(x+y)^n = {}_nC_0 x^n y^0 + {}_nC_1 x^{n-1} y^1 + \cdots + {}_nC_{n-1} x^1 y^{n-1} + {}_nC_n x^0 y^n$$

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Complex Numbers

Multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Polar form: $a + bi = r(\cos\theta + i\sin\theta)$ where $r = \sqrt{a^2 + b^2}$

Powers: $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Roots: $\sqrt[n]{r}\left[\cos\left(\dfrac{\theta + k \cdot 360^\circ}{n}\right) + i\sin\left(\dfrac{\theta + k \cdot 360^\circ}{n}\right)\right]$ $k = 0, 1, 2, \cdots, n-1$

Plane Trigonometry

Plane

WALTER FLEMING

Hamline University

DALE VARBERG

Hamline University

To our children

Robert, Joel, Karen, Thomas

Preface

Mathematicians argue long and hard about how trigonometry should be taught. Traditionalists plead for introducing the subject as it was done historically, in terms of angles and right triangles. That simple setting, they say, gives trigonometry its clearest and most intuitive foundation and allows an immediate study of significant practical problems.

Who needs triangles or even angles? ask the modernists. It is the analytic properties of the trigonometric functions together with the oscillatory character of their graphs that are important. And the quickest and surest way to get to these features is via the unit circle.

Having at one time or other fought bravely in each camp, we have finally chosen the middle ground. After an introductory section on right-triangle trigonometry, trigonometric functions for angles and numbers are developed simultaneously, based on the unit circle.

There is another argument on the horizon. The recent mass production of inexpensive, reliable scientific calculators means that we could dispense with the voluminous tables that have been an integral part of trigonometry since the days of Hipparchus and Ptolemy. Why burden people and books with these relics of the past? Well, why?

It is hard to think of a good reason other than the obvious one that not every student owns one of the little electronic wizards. To accommodate all potential users, short tables for angles measured in both degrees and radians are included. Almost all problems can be done using either these tables or a calculator. Some that are significantly easier on a calculator are marked with a [C] ; they can be omitted without loss of continuity.

Opening Displays Each section begins with a challenging problem, a historical anecdote, a famous quotation, an important result, or a cartoon. These devices are designed to spark curiosity about the section. They are always closely related to what follows.

Informal Writing Style The text avoids the ponderous theorem-proof style of most mathematics books, presenting the material in simple terms illustrated by carefully chosen examples.

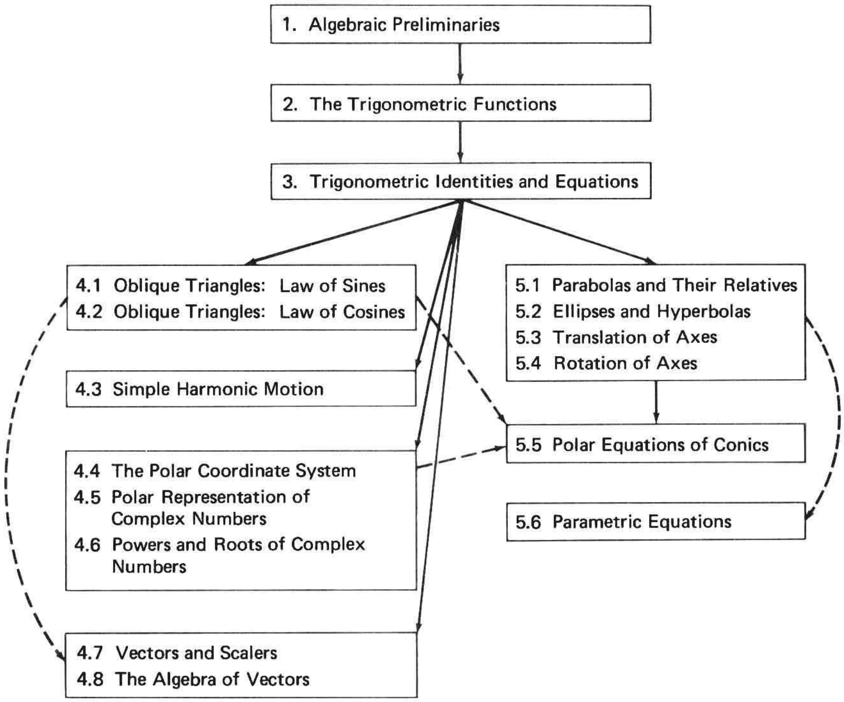
Problem Solving Problem solving is one of the principal tasks of mathematics, so the text poses good problems and gives clear guidance in how to solve them. There are approximately 1350 problems, including numerous word problems, for students to solve. Applications from several disciplines, especially science and engineering, are given. A separate section, "Answers to Odd-numbered Problems," appears at the end of the book.

Examples There are many examples in the text, but they are not placed exclusively within the discussion. A few examples are used to highlight the major points of each section. Then, in the problem set, several more examples that are designed to elucidate and expand upon the text are added. Each of these examples is accompanied by a number of related exercises for the student. To avoid the old danger that students will adopt the cookbook approach to problem solving, following the example without thinking, each problem set concludes with miscellaneous problems designed to test all the skills needed in the section and perhaps to challenge the very best students.

Conic Sections A novel feature of this text is the inclusion of a final chapter on the conic sections. The cartesian equations (including rotation of axes), the polar equations, and parametric forms are all developed; each of these ideas uses trigonometry in a significant way.

Flexibility The first chapter, "Algebraic Preliminaries," will be review for those students who have had a substantial algebra course. It can be covered rapidly or omitted. Then come two chapters, "The Trigonometric Functions" and "Trigonometric Identities and Equations," which form the core of any trigonometry course. These are followed by "Applications of Trigonometry," from which the instructor may choose to suit her or his taste. Finally, there is an optional chapter, "The Conic Sections." Chapters 2 through 4 are identical with the corresponding chapters in the authors' *Algebra and Trigonometry*, except for the addition of two sections on vectors. A Dependence Chart follows at the top of page xiii.

DEPENDENCE CHART



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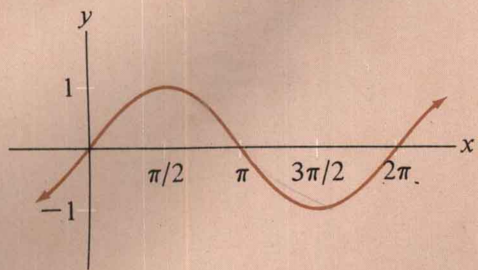
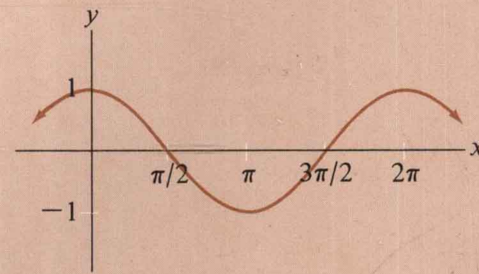
WALTER FLEMING
DALE VARBERG

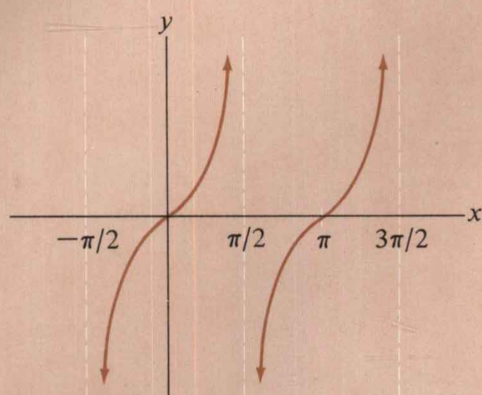
GRAPHS OF TRIGONOMETRIC FUNCTIONS



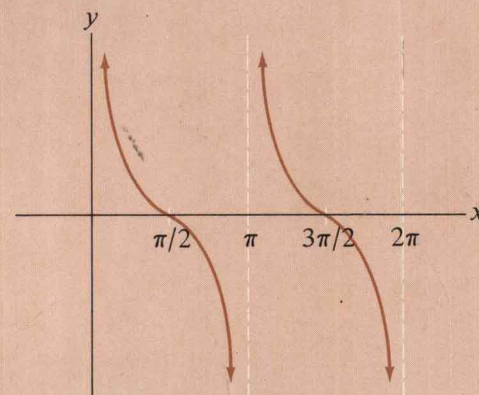
$y = \sin x$



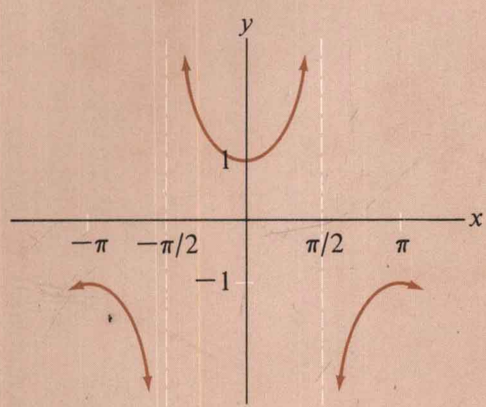
$y = \cos x$



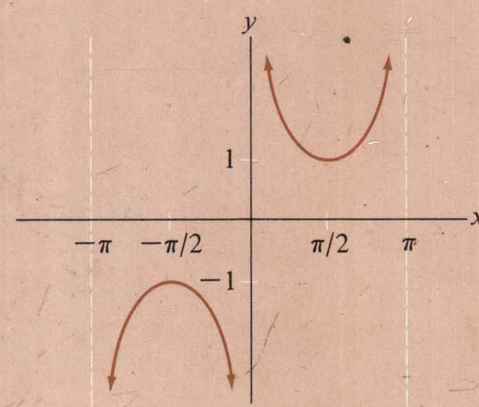
$y = \tan x$



$y = \cot x$



$y = \sec x$



$y = \csc x$

FORMULAS FROM TRIGONOMETRY

Basic Identities

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t}$$

$$\sin^2 t + \cos^2 t = 1 \qquad 1 + \tan^2 t = \sec^2 t \qquad 1 + \cot^2 t = \csc^2 t$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - t\right) = \cos t \qquad \cos\left(\frac{\pi}{2} - t\right) = \sin t \qquad \tan\left(\frac{\pi}{2} - t\right) = \cot t$$

Odd-even Identities

$$\sin(-t) = -\sin t \qquad \cos(-t) = \cos t \qquad \tan(-t) = -\tan t$$

Addition Formulas

$$\sin(s + t) = \sin s \cos t + \cos s \sin t \qquad \sin(s - t) = \sin s \cos t - \cos s \sin t$$

$$\cos(s + t) = \cos s \cos t - \sin s \sin t \qquad \cos(s - t) = \cos s \cos t + \sin s \sin t$$

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t} \qquad \tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

Double Angle Formulas

$$\sin 2t = 2 \sin t \cos t$$

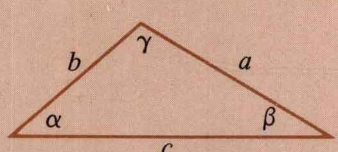
$$\cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t = 2\cos^2 t - 1$$

$$\tan 2t = \frac{2 \tan t}{1 - \tan^2 t}$$

Half Angle Formulas

$$\sin\frac{t}{2} = \pm\sqrt{\frac{1 - \cos t}{2}} \qquad \cos\frac{t}{2} = \pm\sqrt{\frac{1 + \cos t}{2}} \qquad \tan\frac{t}{2} = \frac{1 - \cos t}{\sin t}$$

Laws of Sines and Cosines



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

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"On the whole, algebra is a means rather than an end. It is the spelling, grammar, and rhetoric of most mathematics, but it is not literature."

Morris Kline

CHAPTER ONE

Algebraic Preliminaries

1-1 Numbers and Number Systems

1-2 Pocket Calculators

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1-5 Functions and Their Inverses