ENGINEERING APPLICATIONS OF FLUID MECHANICS

HUNSAKER & RIGHTMIRE

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PREFACE

This book has grown out of some 12 years' experience with a course in fluid mechanics for an undergraduate class in the mechanical engineering department of the Massachusetts Institute of Technology.

The controlling importance of flow phenomena in nearly every type of machine and process led to the conversion of a traditional course in hydraulics into a more fundamental treatment of the action of fluids generally. The authors were more anxious that the student understand flow phenomena than that he be familiar with details of many practical devices. They attempted to give unity to the subject by the careful development, at an adult level, of the mechanics of fluids and to provide interest and utility by condensed treatment of selected types of engineering application.

The course is essentially an introduction to a field of engineering science that includes important subjects for later professional study. An introductory course should lay an adequate foundation for advanced work and should, therefore, deal both with mathematical reasoning and with experimental results. Recent advances in the efficiency of machines and engines have resulted from a better understanding of their operation. This understanding requires not so much the determination of over-all performance in terms of a few pressure and temperature measurements as a detailed exploration of flow conditions.

In such investigations fluid mechanics is of first importance, both in machines actually handling fluid, such as compressors and turbines, and also in engines, where controlled flow of air, fuel, and combustion gases is essential.

The application of Newton's laws of mechanics to fluids with the aid of the tools of mathematics gives an exact description of flow phenomena for certain idealized cases but only an approximate description for many practical cases. The greatest difficulty comes in real fluids from the effect of friction and the resulting turbulence and separation from the boundary surfaces designed to guide the flow. When the type of flow is too confused to be postulated with any confidence, mathematical analysis from first principles is usually hopeless.

First the naval architect and later the aeronautical engineer have been brilliantly successful in the use of models to solve such complicated problems. For this, a theory of similitude is necessary to ensure that the model experiments shall predict full-scale performance under the desired conditions. In view of the increasing importance of controlled experiments in mechanical engineering, in this text the authors have gone into the theory of dimensions and physical similitude with some care. They believe that the student is entitled to a full explanation of the apparently simple rules for conducting model experiments.

The theoretical behavior of an ideal frictionless fluid has been discussed in order to introduce the student to basic concepts governing pressure and velocity distribution. While no real fluid is frictionless, in many engineering problems friction in the main flow may be neglected. At the same time, friction can be the controlling factor near solid boundaries, as Prandtl showed in his theory of the boundary layer. The student is expected to appreciate the simplicity and elegance of the mathematical treatment of the ideal fluid and also the validity and logic of the simplifying assumptions on which the treatment of frictional flow is based. For these reasons, the authors have given more weight to hydro-mechanics and boundary-layer theory than might be expected in an introductory text.

An explanation should perhaps be made of what may appear in the early chapters to be an overelaborate analysis of self-evident phenomena, such as the conditions of static equilibrium. A mathematical formulation of very simple physical relations is developed with some care to ensure familiarity with certain powerful methods of analysis needed later. For example, the device of a potential function is introduced as the potential energy of a gravity field, in anticipation of the later use of a velocity potential to describe a velocity field. Teaching experience indicates that the calculus needs restatement when applied to physical quantities.

The material in Chaps. I to VIII is essentially that presented during the first term of a two-term course in fluid mechanics. Parts of the remaining chapters are covered in the second term, with emphasis on topics in Chaps. X, XI, XII, XIV, XV, and XVII. The students for whom this material is intended have had 2 years of physics and mathematics, including differential equations, and 1 year of applied mechanics.

Since the student is taking a parallel course in thermodynamics at the same time, certain topics that might logically be included have been omitted. Furthermore, the class is also engaged in a sequence of laboratory exercises involving instrumentation, fluid measurements, and determination of the operating characteristics of many examples of machinery. While this book is designed to illuminate such laboratory work, it is in no sense a laboratory manual nor is it descriptive of current practice.

The treatment of servomechanisms is included for use in a separate one-term elective course.

Acknowledgment is made of valuable help from those who have given the course, notably R. von Mises, C. B. Millikan, and H. Peters. The chapters on lubrication were largely written by J. T. Burwell. M. Raus-

cher helped with the material on jets, and C. E. Grosser with that on hydraulic transmissions. The authors, however, are responsible for the end product.

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CHAPTER I

INTRODUCTORY SURVEY

1.1. Definition. Fluid mechanics is the special branch of general mechanics that applies its fundamental principles to fluids. These principles are Newton's laws of motion, the conservation of energy, and the indestructibility of matter. Fluid mechanics can describe and predict the behavior of fluids to the extent that we know their physical properties and to the extent that practicable methods of applied mathematics can be found.

1.2. Objective. The primary objective of fluid mechanics for a mechanical engineer is twofold, (1) to explain the facts of experience by the deduction of general rules and (2) to apply such general rules to predict, at least to a practical approximation, the fluid phenomena involved in the performance of ships, airplanes, engines, compressors, turbines, pipe lines, and wherever a working fluid is involved in the operation of machinery. The rules of fluid mechanics are also fundamental to lubrication, convectional heat transfer, ballistics, oceanography, and meteorology, where important modern developments have taken place as a result of analysis of observed facts. In nearly all actual cases, some simplifying assumptions must be made as to the physical properties of the fluid or as to the character of the flow.

1.3. Fluids and Solids. The word "fluid" (from Latin *fluidus*) means a substance having particles that readily change their relative positions. Fluid refers, therefore, to both gases and liquids, as opposed to solids.

Solids and fluids behave differently under the action of an applied force. The force necessary to produce deformation of a solid depends mainly on the amount of the deformation, as, for example, in the case of a spring. The force must be maintained in order to hold the deformation. In the case of a fluid the force depends primarily on the speed with which a deformation is produced. Such force tends to vanish when the speed of deformation approaches zero. The increased resistance accompanying any increase in the speed of a solid body moving through a fluid illustrates this characteristic.

1.4. Viscosity. A fluid has the property of resisting deformation in proportion to the rate of deformation. Experiment shows that in a flooded journal bearing in which the shaft and bearing are concentric the following proportionality holds:

$$\frac{F}{2\pi aL} \sim \frac{V}{h}$$

where F is the tangential force acting on the lubricant at the surface of the shaft and the other quantities are shown in Fig. 1.1. Since it is known that a fluid does not slip at a boundary, the ratio V/h measures the rate of deformation of the oil in the bearing. The ratio $F/2\pi aL$ is the shear force

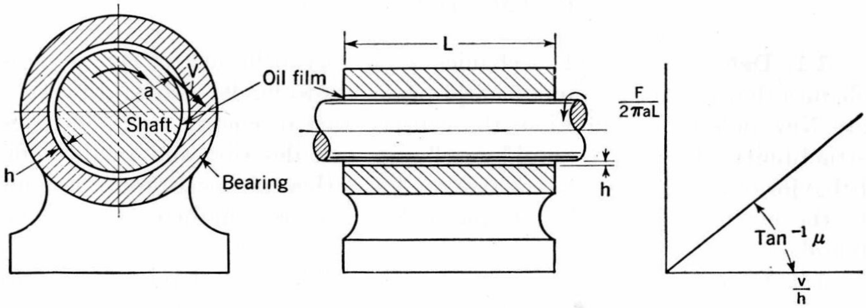


FIG. 1.1. — Oil viscosity causes shearing stress on the shaft of a flooded journal bearing.

per unit area, or shear stress exerted on the oil. The factor of proportionality between shear stress and rate of deformation is a property of the lubricating fluid and is called the "coefficient of viscosity" or, simply, "viscosity." The defining equation for viscosity of the lubricant in the journal bearing of Fig. 1.1 is

$$\text{Shear stress} = \frac{F}{2\pi aL} = \mu\frac{V}{h} \tag{1.1}$$

where μ is the viscosity. It is seen that the shear stress persists as long as the ratio V/h is greater than zero.

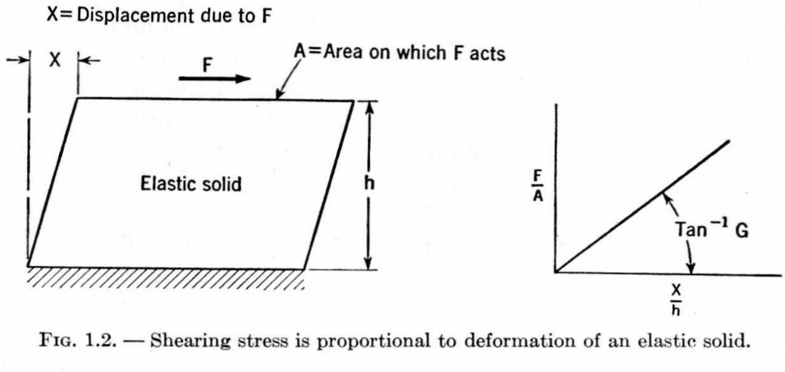


FIG. 1.2. — Shearing stress is proportional to deformation of an elastic solid.

Many substances, like metals, become true fluids when heated, while quasi solids, like jelly, pitch, and grease, may behave like fluids of very high viscosity.

For an elastic solid deformed by a shear force F the following equation holds:

$$\text{Shear stress} = \frac{F}{A} = G\frac{x}{h} \tag{1.2}$$

where G is a property of the solid called the "shear modulus" and the other quantities are shown in Fig. 1.2. The analogy between Eqs. (1.1) and (1.2) is obvious.

1.5. Plasticity. Metals heated just short of the melting point or strained beyond the elastic limit no longer behave as elastic solids but deform continuously under a constant load. This phenomenon of creep is characteristic of the plastic state of many solids. A metal in the plastic state is still polycrystalline and is not a fluid until melted. Hot metal in the rolling mill is plastic; so is cold metal under the very great local forces involved in wire drawing, press-forming, or cold-riveting. A yielding material that does not resist shear in proportion to the rate of shear is sometimes called a "non-Newtonian liquid." Heavy grease at low temperatures may exhibit a definite yield point and act like a plastic rather than like a true liquid.

The methods of fluid mechanics are ordinarily inapplicable to substances in the plastic state.

1.6. Perfect Fluid. Since real substances may partake of the fluid state to various degrees depending on conditions of temperature and pressure, application of the principles of fluid mechanics can be greatly simplified if the analysis is first made for an imaginary perfect fluid that is nonviscous, *i.e.*, for one that is subject to no shearing stress during motion. The assumption of such a frictionless fluid, under many circumstances, leads to very fair approximations to the behavior of real fluids of low viscosity, such as, for example, air, water, and alcohol.

It should be noted that in problems of statics, since there is no motion, it is unnecessary to postulate a perfect fluid because the effect of viscosity is nil.

1.7. Liquids and Gases. A fluid may be either a gas or a liquid. A gas completely fills the region within given boundaries regardless of the amount of gas enclosed, while a definite amount of liquid is required to fill a given region. A smaller amount of liquid will fill only part of the region, a free surface being formed as one boundary. In the words of Sir Oliver Lodge: "A solid has volume and shape, a liquid has volume but no shape, a gas has neither."

The most important difference between a liquid and a gas, from the viewpoint of fluid mechanics, is the fact that a liquid, like a solid, is practically incompressible under ordinary conditions, while a gas can be readily compressed. When the change in density of a gas is small, however, it can often be treated as an incompressible fluid to a good approximation.

A vapor is a gas that condenses to a liquid under lowered temperature or increased pressure. The pressure at which vapor begins to condense at any given temperature is called the "vapor pressure" or "saturation pressure." This pressure forms a limit below which laws applicable to gases cannot be used. Strictly speaking, all gases are vapors, since they can be condensed under extreme conditions. Vapors, however, behave like true gases under conditions sufficiently far removed from the saturation pressure.

In this study of fluid mechanics the treatment will ordinarily be restricted to practically incompressible fluids, with consideration given to compressibility only in special cases.

1.8. Concepts. The behavior of fluids is extremely difficult to describe or to observe in detail because there are no separate elements to be seen. In the mechanics of solid bodies we deal with separate entities of known dimensions and motions. A fluid, however, is continuous throughout a space, though its motion may be different at every point. Pressure, density, and temperature may vary throughout the space. Sometimes the flow seems to be regular, as if in layers, or laminae, and at other times it is confused and turbulent. Under some conditions large whirls or eddies occur.

Analysis is hopeless unless we can distinguish the conditions that are associated with the various types of behavior. Our method will be first to observe characteristic situations and then to devise simple idealized conditions that give an approximation of what has been observed. This method leads to a further classification, not as to the properties of the fluid itself, but rather as to the conditions under which the fluid is acting. Certain concepts will be required to define these conditions.

1.9. Continuity. The most conspicuous feature of fluids is that they generally exist and move as a continuous body of substance without voids. We postulate for analysis continuous flow such that at no place is fluid created or destroyed. We, in effect, affirm conservation of matter. Furthermore, we observe that in general, as in a river, the velocity, pressure, temperature, and density vary continuously from point to point at a given instant of time but at a given point may vary with time if the flow is not steady.

1.10. Continuum. This observation leads to the idealized conception of a continuum in which the quantities characterizing the flow, such as velocity, pressure, and density, are continuous functions of time and position. It then remains to consider under what conditions such a continuum represents our experience and also what meaning to attach to such ideas as pressure, density, and velocity at a point. When we examine these questions later, we shall conclude that gas in a vacuum tube contains too few molecules to be treated as a continuum. We shall see also that, under some conditions of flow, velocity is not a continuous function of position but may change suddenly at a so-called "surface of discontinuity" which separates, for example, a jet of air from the surrounding atmosphere.

1.11. Statics. We may greatly simplify our analysis if we first confine our attention to fluid at rest. It is common knowledge that liquids come to equilibrium with less dense fluid on top. We also know that the barometric pressure on a mountain peak is less than in a valley and that a deep-sea diver is subjected to greater pressure as he goes deeper. Furthermore, the phenomenon of convection, due to heating lower layers of fluid, is a rupture of a previous condition of stable equilibrium. From the principles of statics we can explain why a ship floats, why balloons rise in the air, and why other phenomena concerned with the stability of fluid at rest occur as they do.

1.12. Dynamics. Since flowing fluid is subjected to dynamic forces due to the motion, the distribution of pressure and density in the continuum, as determined by static conditions, can be greatly modified by the motion. By the principle of the conservation of energy we can predict the interchanges between kinetic and potential energy as a flow proceeds, and by means of a special form of Newton's second law of motion we can predict the dynamic forces exerted by the fluid.

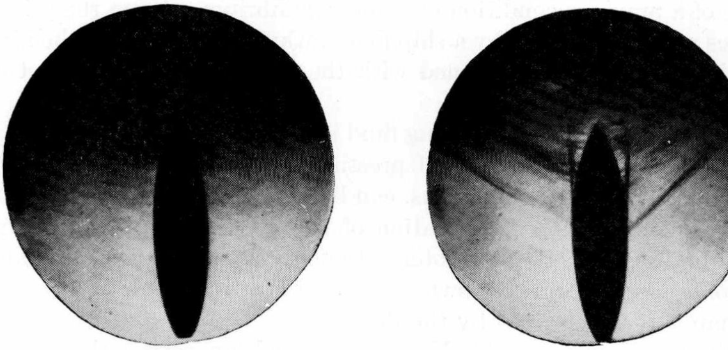
In the mechanics of solids Newton's second law relates the force on a body of known mass to the rate of change in its momentum. In fluid mechanics individual masses are not distinguishable, and it is necessary to deal with a portion of the continuum contained within imaginary fixed boundaries. The resultant external force exerted on the fluid that at any instant lies inside this fixed "control volume" equals the rate of change of momentum of this fluid. Under certain conditions this force simply equals the difference between the rates of outflow and inflow of momentum across the boundary of the control volume.

1.13. Compressible and Incompressible Gases. We may simplify both the static and the dynamic problems of fluid mechanics by the assumption of constant density throughout the continuum. This is exactly what has been done in hydraulics, which deals exclusively with liquids. We know from experience that liquids are practically incompressible. Only when we deal with pressure waves in a liquid are we concerned with its compressibility.

Gases, however, are easily compressed, and substantial volume changes are produced in various types of machinery. For moderate changes in level the atmosphere behaves like an incompressible fluid, and appreciable density change requires an increase in altitude of the order of a mile. Likewise, for ordinary velocities (less than 250 mph) the density change produced by the motion is only a small fraction of the normal density. For velocities approaching the speed of sound (about 750 mph) compressibility is very important.

The facts of experience, therefore, tell us that liquids and atmospheric air may be treated as incompressible fluids in many practical cases. Air need be treated as a compressible fluid only where great density changes are brought about, as by great change in height, by machinery, or by extreme velocity.

For any object moving through a fluid there is some place on its surface where the relative velocity of the fluid is substantially higher than the velocity of translation. Consequently it is possible for this local velocity to reach the velocity of sound while the translatory velocity is subsonic.



a b

FIG. 1.3. — *a*. Flow past an airfoil at a speed well below that of sound. *b*. Flow in which sound velocity is reached locally in a region below the slanting dark lines (shock waves). The flow is upward in both pictures.

The velocity of translation at which local sonic velocity is encountered determines a critical speed for the particular shape of body, marked by the presence of standing compression waves.

Figure 1.3*a* is a photograph of the upward flow of air past an airfoil at a speed of 0.422 sound velocity. Figure 1.3*b* shows the flow pattern for the critical air speed at 0.776 sound velocity. The photographic method used makes visible variations in air density. In Fig. 1.3*a* the airflow is relatively smooth, and no marked discontinuity of density can be observed except close to the tail, where the flow separates to form a turbulent wake. At critical speed, Fig. 1.3*b* shows a great change. The dark lines across the airflow indicate regions of rapid change in density. Behind these lines the flow separates from the airfoil, and a broad wake is formed. The critical speed is marked by a large and abrupt increase in resistance.

The difference in the flow for the two photographs is a consequence of compressibility. At critical speed the airflow next to the thickest part of the model has reached the speed at which a pressure wave is propagated. Hence, pressure changes on the rear of the model cannot influence the flow over the forward part since a pressure change cannot be propagated upstream. To preserve equilibrium a discontinuity of pressure (and density) occurs in the form of standing waves, or "compression shocks," shown by the dark lines. The very sharp increase in pressure at the compression shocks forces the flow to separate from the model, leaving a broad wake. In general, a critical flow is unsteady, and the position of the compression shocks fluctuates with time.

The technical measure of high speed is the Mach number M, defined as the ratio of the general flow velocity to the velocity of sound in the fluid. Subsonic velocity is indicated by $M < 1$ and supersonic velocity by $M > 1$. Airplanes fly at subsonic velocity but may reach a critical speed in a dive with consequent serious effects on control and structural integrity. Propellers, superchargers, and other machinery frequently handle air at critical speeds and higher. Bombs dropped from a great height may attain and pass through a critical velocity. Projectiles are fired with an initial supersonic velocity.

In the discussion of the dynamics of fluid motion, compressibility cannot be neglected when the Mach number approaches unity.

1.14. Steady Motion. The simplest case of flow is a steady motion whose pattern is the same at all times. Such a flow is seen in the steady jet from a nozzle, the steady current in a river, or the steady wind at a good height above the ground. Mathematically we express this statement that the velocity field is independent of time and depends only on the space coordinates x, y, z thus:

$$V = f(x,y,z)$$

Steady motion may be incompressible or not, depending on whether the density is constant or a function of location.

1.15. Boundary Layer. When one looks over the side of a vessel proceeding in smooth water, a belt of so-called "friction eddies" is seen, made visible by day because of air released and often by night by phosphorescence in the water. Just outside this belt the flow seems to be steady and of the order of the velocity of the ship. Since the water wets the ship and is carried along with it, there is, close to the ship's hull, a high velocity gradient that makes viscosity important and creates a strong frictional drag on the hull (see Fig. 1.4).

Outside the boundary layer the velocity gradients are small; hence, for this steady motion, friction or viscosity should be unimportant. The fluid outside the boundary layer seems to flow in a steady and frictionless manner.

Accordingly, to approximate the complicated motion of real fluids, it has been found generally useful to consider the continuum as made up of two distinct and separate portions, the boundary layer and wake, where friction controls the motion, and the region outside, where friction can be neglected. The justification for this concept, due to Prandtl, is its evident resemblance to the observed state of affairs and the useful practical results that can be obtained from analyses based on it.

Observation of real fluids shows no discontinuity in velocity through

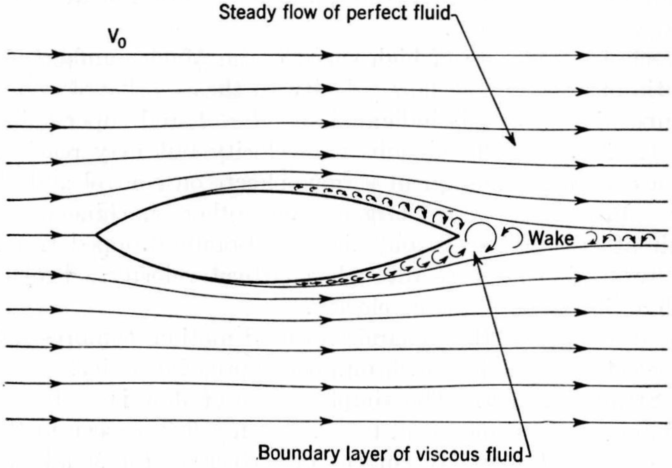


FIG. 1.4. — Fluid particles entering the boundary layer adjacent to a body are set into rotation by the action of viscosity.

the boundary layer. Figure 1.5 indicates a rapid but smooth transition of velocity from zero to V_0 in passing out from the ship's side through a boundary layer such as is represented in Fig. 1.4.

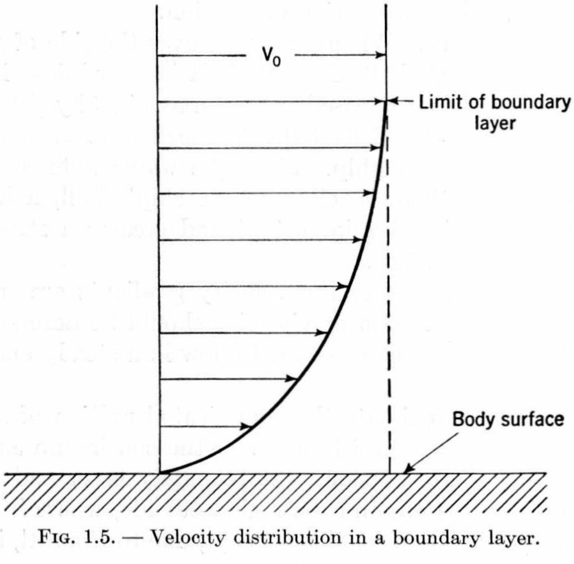


FIG. 1.5. — Velocity distribution in a boundary layer.

1.16. Discontinuity. Where a sharp corner projects into the flow, the steady stream of fluid would be thrown clear, as in Fig. 1.6*a*. On one side of *DD* there is dead water and on the other the full velocity of the stream. There is consequently a discontinuity of velocity, but not of fluid. The

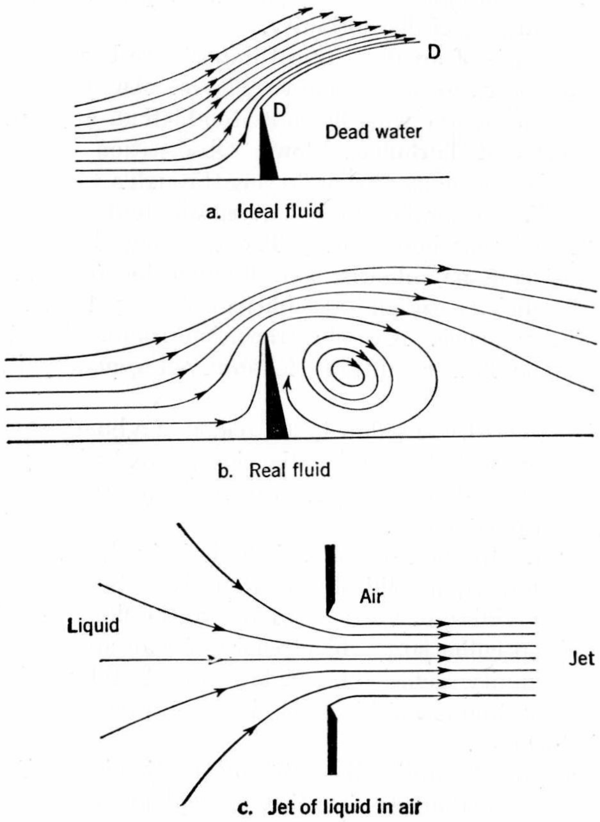


FIG. 1.6. — Surfaces of discontinuity in velocity.

velocity gradient is, for an ideal fluid, infinite at such a "surface of discontinuity."

For real fluids having friction the abrupt change in velocity (high velocity gradient) causes large frictional forces tangential to the direction of motion. The dead water is set into rotation by the stream. The eddy that builds up is indicated in Fig. 1.6*b*.

A liquid jet in air, as from a nozzle, is surrounded by a surface of discontinuity. Here the liquid continuum may be considered to extend throughout the tank and the jet but not to cross the surface of the jet into the air (see Fig. 1.6*c*).