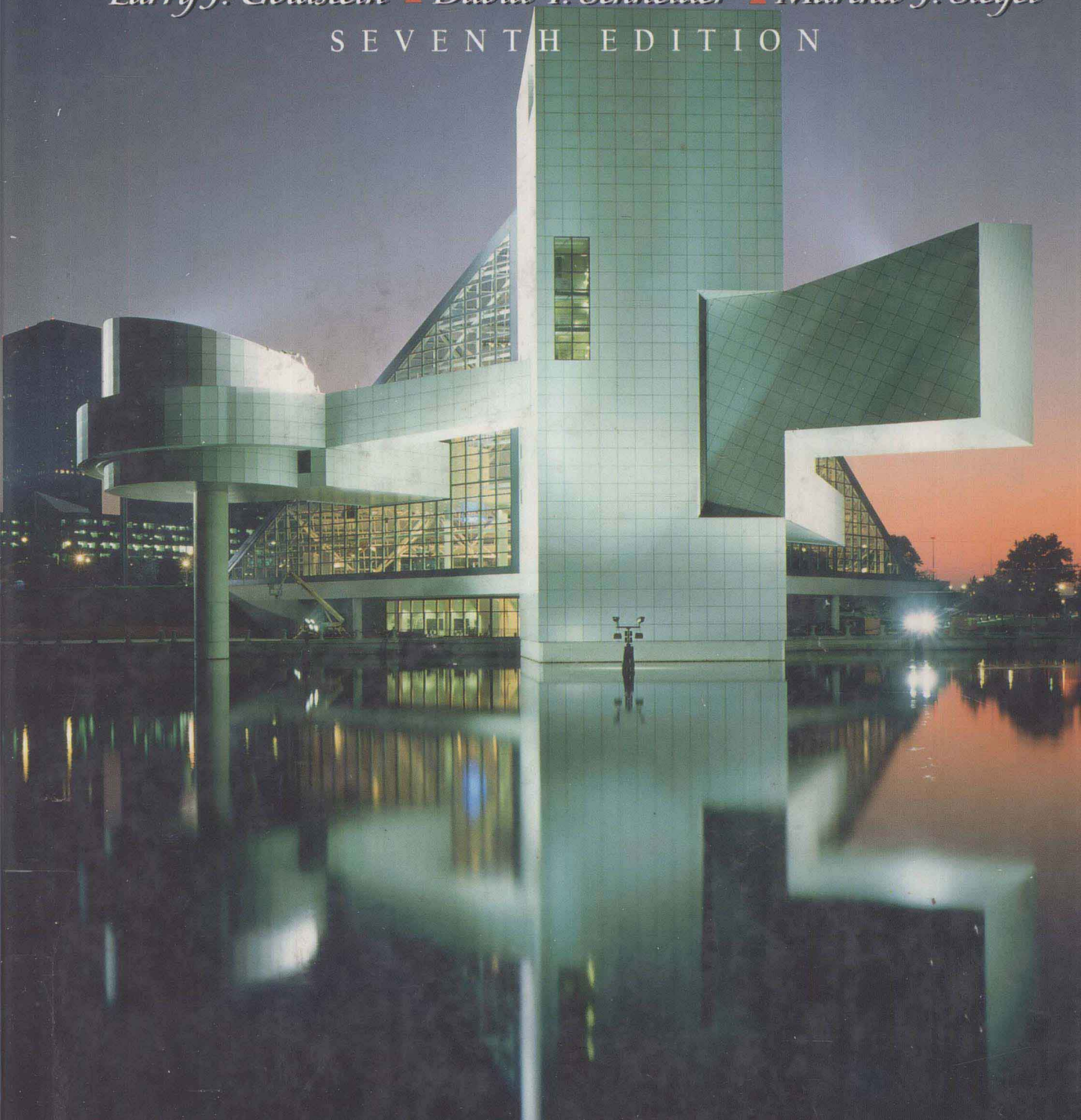


# FINITE MATHEMATICS & ITS APPLICATIONS

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*Larry J. Goldstein ▲ David I. Schneider ▲ Martha J. Siegel*

SEVENTH EDITION



SEVENTH EDITION

# Finite Mathematics & Its Applications

Larry J. Goldstein

Goldstein Educational Technologies

David I. Schneider

University of Maryland

Martha J. Siegel

Towson University



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## Mathematics and Its Applications

This volume is one of a collection of texts for freshman and sophomore college mathematics courses. Included in this collection are the following.

*Calculus and Its Applications, ninth edition*, by L. Goldstein, D. Lay, and D. Schneider. A text designed for a two-semester course in calculus for students of business and the social and life sciences. Emphasizes an intuitive approach and integrates applications into the development.

*Brief Calculus and Its Applications, ninth edition*, by L. Goldstein, D. Lay, and D. Schneider. Consists of the first eight chapters of the above book with some material from later chapters.

*Finite Mathematics & Its Applications, seventh edition*, by L. Goldstein, D. Schneider, and M. Siegel. A traditional finite mathematics text for students of business and the social and life sciences. Allows courses to begin with either linear mathematics (linear programming, matrices) or probability and statistics. Includes topics in discrete mathematics.

*Applied Calculus: A Graphing Approach* by D. Schneider and D. Lay. A one-semester or two-quarter technology-required reform calculus text for students majoring in business, economics, life sciences, and social sciences.



# Preface

---

This work is the seventh edition of our text for the traditional finite mathematics course taught to first- and second-year college students, especially those majoring in business and the social and biological sciences. Finite mathematics courses exhibit tremendous diversity with respect to both content and approach. Therefore, in revising this book, we incorporated a wide range of topics from which an instructor may design a curriculum, as well as a high degree of flexibility in the order in which the topics may be presented. For the mathematics of finance, we even allow for flexibility in the approach of the presentation.

In this edition we attempt to maintain our popular student-oriented approach throughout and, in particular, through the use of the following features:

## Applications

We provide realistic applications that illustrate the uses of finite mathematics in other disciplines. The reader may survey the variety of applications by referring to the Index of Applications located on the front endpapers. Wherever possible, we attempt to use applications to motivate the mathematics. For example, the concept of linear programming is introduced in Chapter 3 via a discussion of production options for a factory with a labor limitation.

## Examples

We include many more worked examples than is customary in textbooks. Furthermore, we include computational details to enhance comprehension by students whose basic skills are weak.

## Exercises

More than 2200 exercises comprise about one-quarter of the book, the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that homework assignments may be easily made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter amplify the other exercise sets and provide cumulative exercises that require skills acquired from earlier chapters. Answers to the odd-numbered exercises are included at the back of the book.

## Practice Problems

The practice problems are a popular and useful feature of the book. They are carefully selected exercises located at the end of each section, just before the exercise set. Complete solutions follow the exercise set. The practice problems

often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt to do the practice problems and study their solutions before moving on to the exercises.

## Use of Technology

Although the use of technology is optional for this text, many of the topics can be enhanced with graphing calculators and computers. Also, each year more students own graphing calculators that they have used in their high school mathematics courses. Therefore, whenever relevant, we explicitly show the student how to use graphing calculators effectively to assist in understanding the fundamental concepts of the course. In addition, the text contains an appendix on the use of graphing calculators and about 200 specially designated “calculator and computer” exercises. Such exercises are denoted by GC.

In our discussions of graphing calculators, we specifically refer to the TI-82 and TI-83 since these are the two most popular graphing calculators. Therefore, *most* students will have a book customized to their calculator. Students with other graphing calculators can consult their guidebooks to learn how to make adjustments. Had the calculator material been written generically, *every student* would have to make adjustments.

## Examples from Professional Exams

We have included questions similar to those found on CPA and GMAT exams to further illustrate the relevance of the material in the course. These multiple-choice questions are identified with the notation PE.

## Review of Fundamental Concepts

Near the end of each chapter is a set of questions that help the student recall the key ideas of the chapter and focus on the relevance of these concepts.

## New in This Edition

Among the changes in this edition, the following are the most significant.

1. *Visual Representations of Data.* A new optional section has been added to the beginning of Chapter 7 that shows several ways data are represented graphically.
2. *Chapter Summaries.* Each chapter contains a detailed summary of the important definitions and results from the chapter, serving as a handy study tool for the student.
3. *Chapter Tests.* Each chapter has a sample test that can be used by the student to help determine if he or she has mastered the important concepts of the chapter. The answers to the chapter tests are given at the back of the book.
4. *Chapter Projects.* These extended projects can be used as in-class or out-of-class group projects, or special assignments. The projects develop interesting applications or enhance key concepts of the chapters.

## Minimal Prerequisites

Because of the great variation in student preparation, we keep formal prerequisites to a minimum. We assume only a first year of high school algebra. Furthermore, we review, as needed, those topics that are typically weak spots for students.

## Topics Included

This edition has more material than can be covered in most one-semester courses. Therefore, the instructor can structure the course to the students' needs and interests. The book divides naturally into four parts. The first part consists of linear mathematics: linear equations, matrices, and linear programming (Chapters 1–4); the second part is devoted to probability and statistics (Chapters 5–7); the third part covers topics utilizing the ideas of the other parts (Chapters 8–10); and the fourth part explores key topics from discrete mathematics that are sometimes included in the modern finite mathematics curriculum (Chapters 11–13). We prefer to begin with linear mathematics since it makes for a smooth transition from high school mathematics and leads quickly to interesting applications, especially linear programming. Our preference notwithstanding, the instructor may begin this book with Chapter 5 (Sets and Counting) and then do either the linear mathematics or the probability and statistics.

## Supplements

1. *Instructor's Solutions Manual*: Contains the solutions to every exercise in the text.
2. *Students' Solutions Manual and Explorations in Finite Mathematics Software*: Includes the solution to every odd problem in the text as well as a copy of the premier software package for finite mathematics, developed by David Schneider. "Explorations in Finite Mathematics" includes 28 routines which include an animated solution of geometric linear programming problems, student-directed solutions to Gaussian elimination and simplex method problems, interactive shading of Venn diagrams, and detailed analyses of loans and annuities. Matrix operations use rational arithmetic, and matrices are displayed on-screen with typeset quality. An animated Galton board routine shows in a dynamic fashion how the binomial distribution eventually approaches the normal distribution as  $n$  increases.
3. *Test Item File*: Contains sample test questions, both multiple-choice and standard, for each chapter of the text.
4. *TestGen-EQ* provides nearly 1000 suggested test questions, keyed to chapter and section. *TestGen-Eq* is a test-specific testing program networkable for administering tests and capturing grades online. Edit and add your own questions, or use the new "Function Plotter" to create a nearly unlimited number of tests and drill worksheets.
5. *Prentice Hall Companion Website*: (<http://www.prenhall.com/goldstein>) Created as an extra resource for both students and professors, the site includes the following features:
  - (a) *Excel Tutorials and Projects* written by Revathi Narasimhan at St. Peter's College. Uses Excel to enhance the understanding of many of the topics in the course. Using a combination of specially designed projects and tutorials, students are able to analyze data, draw conclusions, and present their analysis in a professional format.

- (b) *Net Tutor* Real time, on-line tutoring allows students to ask questions and get help on the text material from mathematics instructors.
- (c) *Online Calculator Manuals* for the TI-82, TI-83, TI-85, TI-86, TI-89, TI-92, HP, Sharp and Casio graphing calculators.

## Acknowledgments

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If you have comments or suggestions, we would like to hear from you. We hope that you enjoy using this book as much as we have enjoyed writing it.

Larry J. Goldstein  
larry.goldstein@iln.net

David I. Schneider  
dis@math.umd.edu

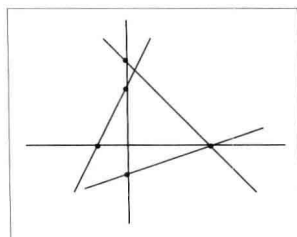
Martha J. Siegel  
siegel@towson.edu

# Finite Mathematics & Its Applications

# Contents

Preface xiii

## 1 Linear Equations and Straight Lines 1



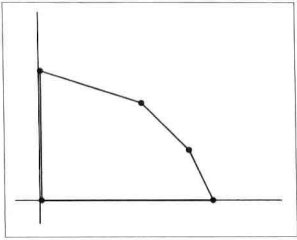
1.1	Coordinate Systems and Graphs	1
1.2	Linear Inequalities	10
1.3	The Intersection Point of a Pair of Lines	20
1.4	The Slope of a Straight Line	25
1.5	The Method of Least Squares	38
	Chapter Test	50
	Chapter Project: Break-Even Analysis	52

## 2 Matrices 53

```
MATRIX[A] 2 x3
[ 5   -2   3 ]
[ 20   .4   1 ]
2, 3 = .6666666666...
```

2.1	Solving Systems of Linear Equations, I	53
2.2	Solving Systems of Linear Equations, II	64
2.3	Arithmetic Operations on Matrices	71
2.4	The Inverse of a Matrix	86
2.5	The Gauss-Jordan Method for Calculating Inverses	95
2.6	Input-Output Analysis	101
	Chapter Test	109
	Chapter Project: Population Dynamics	111

# 3 Linear Programming, A Geometric Approach 113



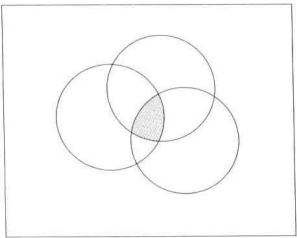
3.1	A Linear Programming Problem	113
3.2	Linear Programming I	120
3.3	Linear Programming II	130
	Chapter Test	143
	Chapter Project: Shadow Prices	145

# 4 The Simplex Method 146

	$x$	$y$	$u$	$v$	$M$	
$y$	0	1	$\frac{3}{16}$	$-\frac{1}{4}$	0	3
$x$	1	0	$-\frac{1}{8}$	$\frac{1}{2}$	0	2
$M$	0	0	$\frac{3}{16}$	$\frac{7}{4}$	1	27

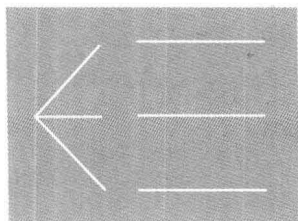
4.1	Slack Variables and the Simplex Tableau	146
4.2	The Simplex Method I: Maximum Problems	155
4.3	The Simplex Method II: Minimum Problems	167
4.4	Marginal Analysis and Matrix Formulations of Linear Programming Problems	175
4.5	Duality	183
	Chapter Test	197
	Chapter Project: Shadow Prices	198

# 5 Sets and Counting 199



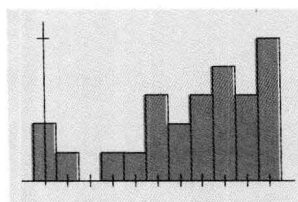
5.1	Sets	199
5.2	A Fundamental Principle of Counting	206
5.3	Venn Diagrams and Counting	212
5.4	The Multiplication Principle	218
5.5	Permutations and Combinations	224
5.6	Further Counting Problems	230
5.7	The Binomial Theorem	236
5.8	Multinomial Coefficients and Partitions	242
	Chapter Test	250
	Chapter Project: Pascal's Triangle	251

## 6 Probability 254



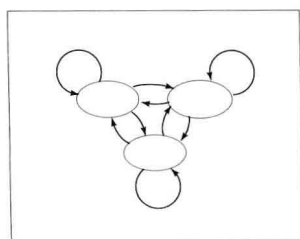
6.1	Introduction	254
6.2	Experiments, Outcomes, and Events	256
6.3	Assignment of Probabilities	264
6.4	Calculating Probabilities of Events	276
6.5	Conditional Probability and Independence	283
6.6	Tree Diagrams	295
6.7	Bayes' Theorem	302
6.8	Simulation	308
	Chapter Test	316
	Chapter Project: Two Paradoxes	318

## 7 Probability and Statistics 319



7.1	Visual Representations of Data	319
7.2	Frequency and Probability Distributions	328
7.3	Binomial Trials	340
7.4	The Mean	346
7.5	The Variance and Standard Deviation	356
7.6	The Normal Distribution	367
7.7	Normal Approximation to the Binomial Distribution	381
	Chapter Test	389
	Chapter Project: An Unexpected Expected Value	391

## 8 Markov Processes 392



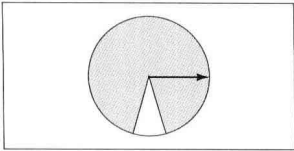
8.1	The Transition Matrix	392
8.2	Regular Stochastic Matrices	402
8.3	Absorbing Stochastic Matrices	411
	Chapter Test	423
	Chapter Project: Doubly Stochastic Matrices	425



9

The Theory of Games

427



9.1	Games and Strategies	427
9.2	Mixed Strategies	434
9.3	Determining Optimal Mixed Strategies	441
	Chapter Test	452
	Chapter Project: Simulating the Outcomes of Mixed-Strategy Games	453

10

The Mathematics of Finance

455

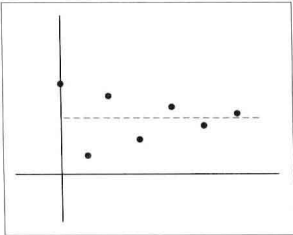
	A	B	C
1	Principal	\$100.00	
2			
3		Compound	Amount
4	Interest Rate	5 Years	10 Years
5	3.00%	\$116.12	\$134.83
6	3.50%	\$119.03	\$141.69
7	4.00%	\$122.02	\$148.89
8	4.50%	\$125.08	\$156.44
9	5.00%	\$128.20	\$164.36
10	5.50%	\$131.41	\$172.68
11	6.00%	\$134.69	\$181.40
12	6.50%	\$138.04	\$190.56
13	7.00%	\$141.48	\$200.16

10.1	Interest	455
10.2	Annuities	466
10.3	Amortization of Loans	476
	Chapter Test	487
	Chapter Project: Individual Retirement Accounts	488

11

Difference Equations and Mathematical Models

489



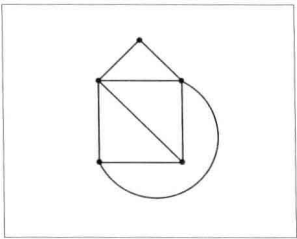
11.1	Introduction to Difference Equations I	489
11.2	Introduction to Difference Equations II	497
11.3	Graphing Difference Equations	503
11.4	Mathematics of Personal Finance	513
11.5	Modeling with Difference Equations	518
	Chapter Test	525
	Chapter Project: Connections to Markov Processes	527

12 Logic528



12.1	Introduction to Logic	528
12.2	Truth Tables	532
12.3	Implication	541
12.4	Logical Implication and Equivalence	548
12.5	Valid Argument	557
12.6	Predicate Calculus	563
	Chapter Test	576
	Chapter Project: A Logic Puzzle	578

13 Graphs579



13.1	Graphs as Models	579
13.2	Paths and Circuits	592
13.3	Hamiltonian Circuits and Spanning Trees	602
13.4	Directed Graphs	613
13.5	Matrices and Graphs	624
13.6	Trees	635
	Chapter Test	645

Appendix A: TablesA1

Table 1	Areas under the standard normal curve	A2
Table 2	$(1 + i)^n$ Compound amount of \$1 invested for $n$ interest periods at interest rate $i$ per period	A3
Table 3	$1/(1 + i)^n$ Present value of \$1. Principal that will accumulate to \$1 in $n$ interest periods at a compound rate of $i$ per period	A4
Table 4	$s_{\overline{n} i}$ Future value of an ordinary annuity of $n$ \$1 payments each, immediately after the last payment at compound interest rate of $i$ per period	A5
Table 5	$1/s_{\overline{n} i}$ Rent per period for an ordinary annuity of $n$ payments, with compounded interest rate $i$ per period, and future value \$1	A6

Table 6	$a_{\overline{n} i}$	Present value of an ordinary annuity of $n$ payments of \$1 one period before the first payment, with interest compounded at $i$ per period	<b>A7</b>
Table 7	$1/a_{\overline{n} i}$	Rent per period for an ordinary annuity of $n$ payments whose present value is \$1, with interest compounded at $i$ per period	<b>A8</b>
<b>Appendix B: Using the TI-82 and TI-83 Graphing Calculators</b>			<b>A9</b>
<b>Answers to Odd-Numbered Exercises and Chapter Tests</b>			<b>A15</b>
<b>Index</b>			<b>I1</b>

# Linear Equations and Straight Lines

► 1.1  
Coordinate  
Systems  
and Graphs

► 1.2  
Linear  
Inequalities

► 1.3  
The Intersection  
Point of a  
Pair of Lines

► 1.4  
The Slope of a  
Straight Line

► 1.5  
The Method of  
Least Squares

Many applications considered later in this text involve linear equations and their geometric counterparts—straight lines. So let us begin by studying the basic facts about these two important notions.

## 1.1 Coordinate Systems and Graphs

Often we can display numerical data by using a *Cartesian coordinate system* on either a line or a plane. We construct a Cartesian coordinate system on a line by choosing an arbitrary point  $O$  (the *origin*) on the line and a unit of distance along the line. We then assign to each point on the line a number that reflects its directed distance from the origin. Positive numbers refer to points on the right of the origin, negative numbers to points on the left. In Fig. 1 we have drawn a Cartesian coordinate system on the line and have labeled a number of points with their corresponding numbers. Each point on the line corresponds to a number (positive, negative, or zero). Conversely, every number corresponds to a point on the line.

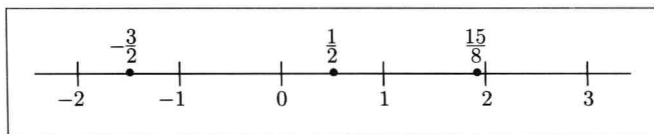


Figure 1.

A Cartesian coordinate system may be used to numerically describe points on a line. In a similar fashion, we can construct a Cartesian coordinate system to numerically locate points on a plane. Such a system consists of two perpendicular lines called the *coordinate axes*. These lines are usually drawn so that one is horizontal and one is vertical. The horizontal line is called the *x-axis*, the vertical line the *y-axis*. Their point of intersection is called the *origin* (Fig. 2). Each point of the plane is identified by a pair of numbers  $(a, b)$ . The first number,  $a$ , tells the number of units from the point to the *y-axis* (Fig. 3). When  $a$  is positive,