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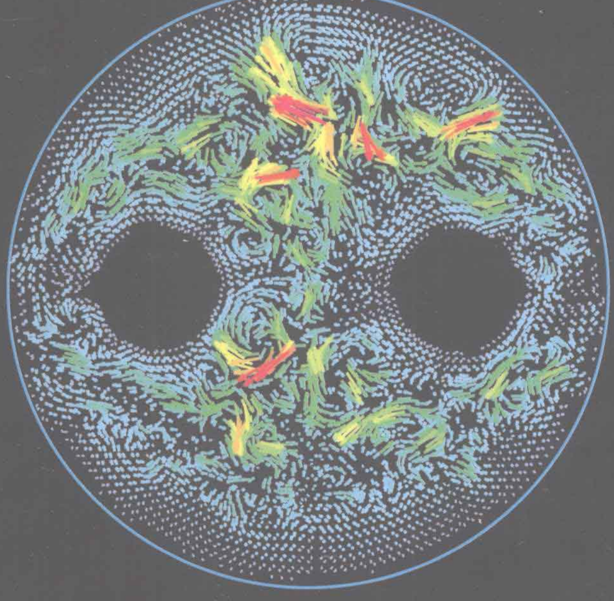
Computational Methods for Engineering Science



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Front cover image: Velocity vector plot of electromagnetic chaotic mixing in a channel, courtesy of ADINA. For more information, please refer to Chapter 1.
Back cover image: Analysis of the complex structure of a honeycomb plate. For more information, please refer to Chapter 10.

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Preface

This volume comprises the Invited Lectures presented at *The Eleventh International Conference on Computational Structures Technology* (CST 2012) and *The Eighth International Conference on Engineering Computational Technology* (ECT 2012) held concurrently in Dubrovnik, Croatia from 4-7 September 2012. I am grateful to the authors and co-authors of the Invited Lectures included in this volume. Their contribution both to these conferences and to this book is greatly appreciated.

Other papers presented at this conference are published as follows:

- *The Invited Review Lectures from CST 2012 and ECT 2012 are published in:* Computational Technology Reviews, Volumes 5 and 6, Saxe-Coburg Publications, Stirlingshire, Scotland, 2012.
- *The Contributed Papers from CST 2012 are published in:* Proceedings of the Eleventh International Conference on Computational Structures Technology, B.H.V. Topping, (Editor), (Book of Summaries with online delivery of full-text papers), Civil-Comp Press, Stirlingshire, Scotland, 2012.
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I would like to thank the members of the CST 2012 and ECT 2012 Conference Editorial Boards for their help before and during the conference.

Finally, I am grateful to Jelle Muylle (Saxe-Coburg Publications) for his help in coordinating the publication of this book and for all his administrative and organisational skills in organising these conferences. I also wish to thank Dawn Sewell (Civil-Comp Press) for her administrative support.

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HERIOT WATT UNIVERSITY

Professor B.H.V. Topping
University of Pécs, Hungary
& Heriot-Watt University, Edinburgh, UK

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Chapter 1

Advances in the Multiphysics Analysis of Structures

K.J. Bathe
Massachusetts Institute of Technology
Cambridge MA
United States of America

Abstract

In this chapter, we survey the advances that we have recently accomplished for the effective analysis of solids and structures, specifically for wave propagations and transient solutions, the analysis of shells, improved stress calculations, the use of interpolation covers, and the solution of the full Maxwell's equations. The structures may be subjected to complex fluid flows and electromagnetic effects. We briefly give the theoretical developments for the formulations, a few illustrative solutions, and conclude by mentioning some further exciting research challenges.

Keywords: finite elements, multiphysics, wave propagations, shells, large strains, improvements of stresses, Maxwell's equations, electromechanics, fluid flows.

1 Introduction

The analysis of solids and structures in multiphysics conditions has been given increasing attention during the recent years [1]. A large number of problems considered only a decade ago as very difficult to solve can now be analyzed with relatively little computational effort. However, there are many problem areas where significant advances are still needed for effective simulations. The objective in this chapter is to briefly present some advances that we have recently accomplished. As a result of space limitations, we mention only our books and papers and refer the reader to the many additional references given therein.

When considering research achievements in the field, it is important to realize the philosophy adopted by a research group in its research on computational methods. *Our philosophy – as pursued for about 40 years now – is to focus on the development of methods that are general, reliable and efficient, and advance the current state of the art as practiced in industry and the sciences* [1, 2]. We have not pursued research that is claimed to open up new avenues when it is clear that such research will not lead to an advancement of the current state of the art. New avenues are only of interest if we see the potential for such advancements. In all cases, our final aim – but of course not always reached – is that the methods we propose will ultimately be of use for a large community of engineers and scientists.

Indeed, the ultimate test as to whether a proposed computational scheme is of value is clearly given by whether it is used widely in industry and scientific investigations once published. This extensive use is driven forward by the keen interest in engineering and the sciences to solve ever more complex and difficult physical problems through finite element simulations.

Included in our research are the conception of novel methods, their mathematical analysis, and their testing to establish generality, reliability and efficiency.

A finite element method is 'general' if it is applicable to many varied problems in a certain category of problems; for example, a general shell element can be used for all shell problems described by a general mathematical model like the 'basic shell model' identified in references [3, 4].

A finite element method is 'reliable' and 'efficient' if identified as such; for example, a finite element discretization is reliable and efficient if the ellipticity and inf-sup conditions are satisfied without the use of any artificial factors, and the scheme shows optimal convergence at a low computational cost [1, 2, 4, 5].

To show whether a method is reliable and efficient requires mathematical analysis, as far as such is possible, and well-designed numerical tests [1, 2, 4]. Both, the mathematical analyses and the benchmark tests, frequently cannot 'prove' that a method is always efficient – considering, for example, general nonlinear analysis – but these efforts can give significant insight into numerical schemes.

The objective in this presentation is to briefly summarize our research efforts to advance the state of computational simulations with the above research aims in mind. In the next sections, we present our recent developments regarding the analysis of wave propagation problems, the analysis of shells, the prediction of more accurate stresses, the use of interpolation covers to increase the convergence of finite element discretizations, and the simulation of electromagnetic effects and their coupling to structures and fluid flows. Since each of these developments covers a large field, we can give in this chapter only a brief summary of our developments and need to refer the reader to our papers written on these topics.

2 Some recent developments

In the following sections, we focus briefly on the basic ideas and some results and refer to our papers for details on the research. When we give here solid mechanics solutions, the procedures are also applicable in multiphysics analyses.

2.1 The solution of wave propagation problems

Although much research effort has been expended on the solution of wave propagation problems using finite element methods, the accurate simulation of transient wave propagations and the accurate solution of harmonic problems at high frequencies have remained a significant challenge. Such problems are abundantly encountered, for example, in solid and structural mechanics, seismic engineering, and in electromagnetics. The essential difficulty is that to capture the high frequency response seen in wave propagations, extremely fine meshes of conventional finite elements are needed. However, even with such very fine meshes in transient solutions, spurious oscillations are calculated near the wave fronts, and numerical dispersion and dissipation of waves, arising from the spatial and temporal discretizations, are observed. Hence, spectral methods, spectral element methods, and spectral finite element methods have been proposed but these are not as general and effective as required in engineering practice.

We have developed a finite element method 'enriched for wave propagation analyses' [6]. This method shows considerable promise, in that the standard low-order Lagrangian finite element interpolations are simply enriched with harmonic functions, governed, as usual, by nodal degrees of freedom. An important point is that the usual fundamental theory of finite element methods is applicable.

For two-dimensional solutions, the basic displacement interpolations for a typical solution variable *u(r,s)* are

$$
\begin{aligned}
&u(r,s) = \\
&\sum_{\alpha} h_{\alpha}(r,s) \\
&\left[
\begin{aligned}
&U_{(\alpha,0,0)} + \sum_{k_x=1}^{n}\left\{\cos\left(\frac{2\pi k_x x}{\Lambda_x}\right)U^{C}_{(\alpha,k_x,0)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}\right)U^{S}_{(\alpha,k_x,0)}\right\} \\
&+\sum_{k_y=1}^{m}\left\{\cos\left(\frac{2\pi k_y y}{\Lambda_y}\right)U^{C}_{(\alpha,0,k_y)} + \sin\left(\frac{2\pi k_y y}{\Lambda_y}\right)U^{S}_{(\alpha,0,k_y)}\right\} \\
&+\sum_{k_x=1}^{n}\sum_{k_y=1}^{m}\left\{
\begin{aligned}
&\cos\left(\frac{2\pi k_x x}{\Lambda_x}+\frac{2\pi k_y y}{\Lambda_y}\right)U^{C+}_{(\alpha,k_x,k_y)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}+\frac{2\pi k_y y}{\Lambda_y}\right)U^{S+}_{(\alpha,k_x,k_y)} \\
&+\cos\left(\frac{2\pi k_x x}{\Lambda_x}-\frac{2\pi k_y y}{\Lambda_y}\right)U^{C-}_{(\alpha,k_x,k_y)} + \sin\left(\frac{2\pi k_x x}{\Lambda_x}-\frac{2\pi k_y y}{\Lambda_y}\right)U^{S-}_{(\alpha,k_x,k_y)}
\end{aligned}
\right\}
\end{aligned}
\right]
\end{aligned}
\tag{1}
$$

where the $U_{(\alpha,k_x,k_y)}$ with superscripts are the nodal degrees of freedom, α is the local element node, with h_{α} the conventional finite element interpolation function, and the S, C, and + and – are used in the superscripts to correspond to the harmonic expressions. These interpolation functions can be written using exponentials on the complex plane, but in the analysis of solids using only real arithmetic can be much more effective. Of course, the interpolations for one- and three-dimensional analyses directly follow from equation (1). Here, the two fundamental wavelengths Λ_x and Λ_y, and the wave cut-off numbers n and m with $1 \le k_x \le n$, $1 \le k_y \le m$, and typically $n, m \le 3$, need to be chosen by the analyst as part of the model data.

As example solutions, we consider the field of transient analyses and the field of harmonic problems, each with an illustrative solution.

2.1.1 A transient solution: one-dimensional impact of an elastic bar

This special one-dimensional problem, shown in Figure 1, can be solved accurately using explicit time integration with 2-node linear elements and a lumped mass matrix [2], and also using the Bathe implicit time integration [7, 8] (with a consistent mass matrix and CFL number = 1.0). However, just to test the enriched finite element formulation, we solved the problem using uniform meshes of 2-node linear elements, consistent mass matrices, and the trapezoidal rule with the very small time step $\Delta t = 2.5 \times 10^{-8} s$ (resulting into significant oscillations in the response).

Figure 1 shows the well-known spurious oscillations in the velocity and hence stress predictions. In this case, using the enriched finite elements we can control the large spurious high-frequency oscillations and make them acceptably small. But more studies are needed to identify in how far this solution behavior is applicable in two-dimensional and three-dimensional analyses and how effective the procedure is in practice.

2.1.2 A time harmonic solution: two-dimensional acoustic pressure wave

Here we consider the solution of the Helmholtz problem (see Figure 2)

$$
\begin{aligned}
&\nabla^2 P + k^2 P = 0 && \text{in } V, \\
&\frac{\partial P}{\partial n} = g(x, y) && \text{on } S_f, \\
&\lim_{r \to \infty} \sqrt{r} \left(\frac{\partial P}{\partial r} - ikP \right) = 0. &&
\end{aligned} \tag{2}
$$

where $P(x, y)$ is the unknown harmonic pressure, $k = \omega / c$, r is the distance from the origin in the Cartesian coordinates, and n is the unit normal on S_f.

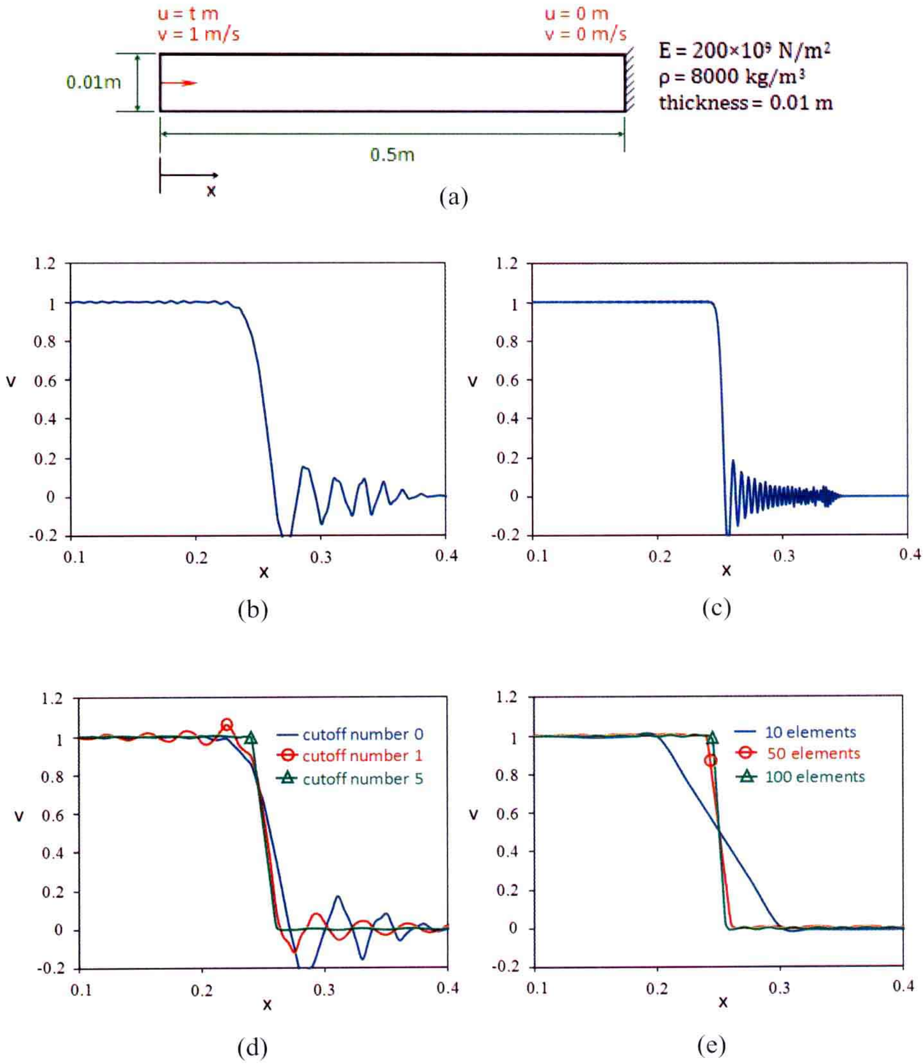
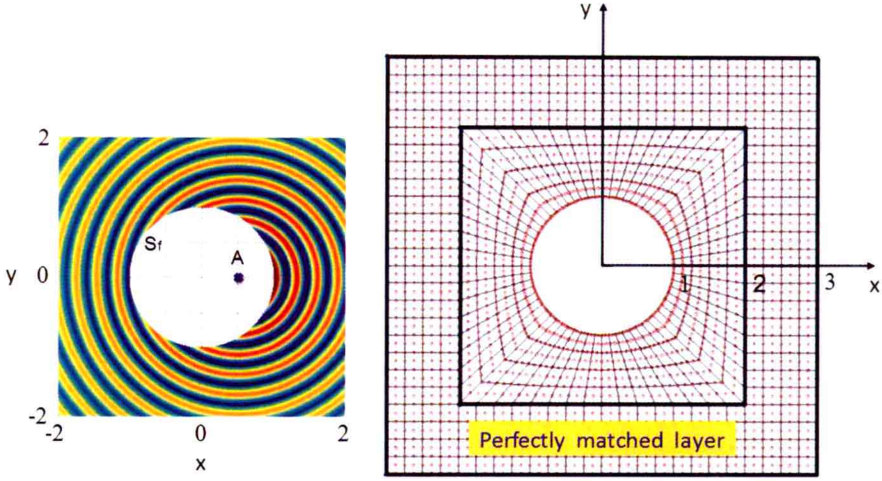


Figure 1: Solution of impact of a bar, at time 0.00005s; (a) elastic bar considered; (b) with 100 traditional linear elements; (c) with 700 traditional linear elements; (d) with 50 linear enriched elements; (e) with cutoff number 5

For the numerical test solution, we prescribed $g(x,y)$ given by the analytical solution and used $k=22.06$. Figure 2 shows the analytical solution and the mesh for our finite element solution. We should note the rather coarse mesh used.

Included in the mesh is a 'perfectly matched layer' to model the infinity of the physical domain.



(a) (b)

Figure 2: Solution of pressure wave; (a) the analytical pressure, A is the pole at $(x_0,y_0)=(0.5,0)$; (b) mesh of 9-node elements

Figure 3 gives contour plots of the pressure numerical solutions using the cutoff numbers from 0 to 2 and the convergence in the L^2 norm (although a better norm might be used). We note that the result obtained using the cutoff numbers $(n,m)=(2,2)$ is in good agreement with the analytical solution.

While, as mentioned earlier, we employ for the most part only real arithmetic, in this example solution we used complex arithmetic for the perfectly matched layer in the discretized domain.

2.2 The analysis of shells

Significant research efforts over some decades have been spent on the analysis of shells, but there are still many outstanding challenges in the field of shell analysis [4]. In the following, we focus on two important items, namely the proper benchmark testing of shell elements, and the analysis of large strain conditions in shells.

2.2.1 The testing of shell elements

Commonly, shell elements have been tested in linear analysis by solving plate and some well-known shell problems, like the pinched cylinder, Scordelis-Lo, and hemispherical shell problems, but not including a shell of negative Gaussian curvature. In these solutions, some displacements at certain points are measured. It should, however, be recognized that such solutions do not constitute a thorough assessment of the capabilities of a shell solution scheme. Instead, it is important to measure the scheme on the following criteria:

– As basic requirements, the shell element used should not be based on artificial factors, not contain any spurious modes and be geometrically isotropic.

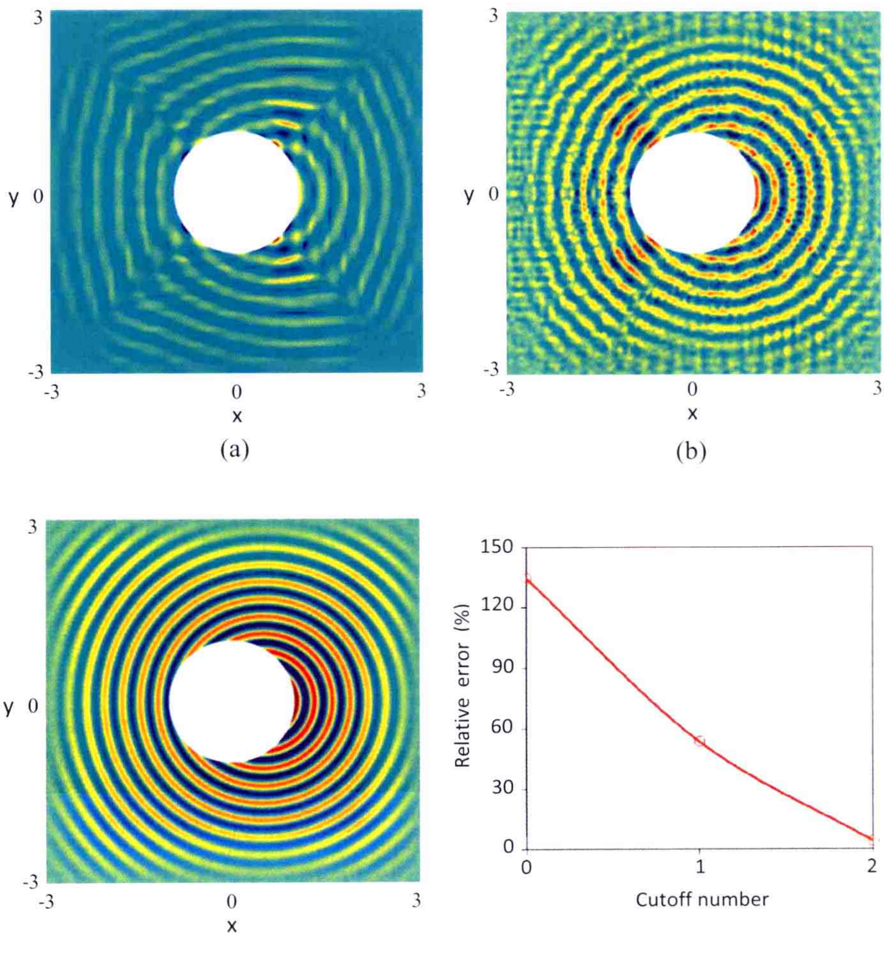


Figure 3: Numerical solutions: (a) (n,m)=(0,0), (b) (n,m)=(1,1), (c) (n,m)=(2,2) and (d) relative error in L^2 norm not including the perfectly matched layer

– The element should be tested in the solution of the hyperboloid shell problems shown in Figure 4, or similar problems of shells with negative Gaussian curvature, and in these solutions proper norms should be used.

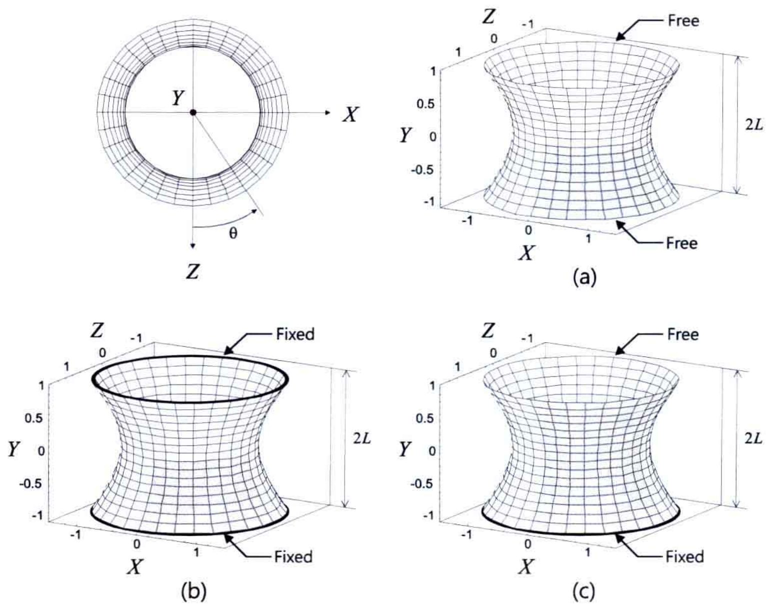


Figure 4: Three shell test problems; $L = 1.0$, $E = 1.0\times10^{11}$, $\nu = 1/3$

The shell surfaces in Figure 4 are given by $X^2 + Z^2 = 1 + Y^2$ and the loading is the pressure $p(\theta) = \cos(2\theta)$. Only the shaded regions in the figure are modeled.

As discussed in references [4, 9], the use of an appropriate norm in the error measure is very important. Using the s-norm defined in these references, we present the test results obtained using the MITC4 shell element in Figure 5. This element satisfies, of course, the basic requirements mentioned above and, as seen in Figure 5, performs very well in the analysis of the shell problems. For details on the testing of plate and shell elements we refer to references [4, 9-11].

2.2.2 The large strain analysis of shells

The large strain analysis of shells is pursued in many applications of science and engineering. Examples occur in biomechanical situations and in the crush and crash analyses of structures.

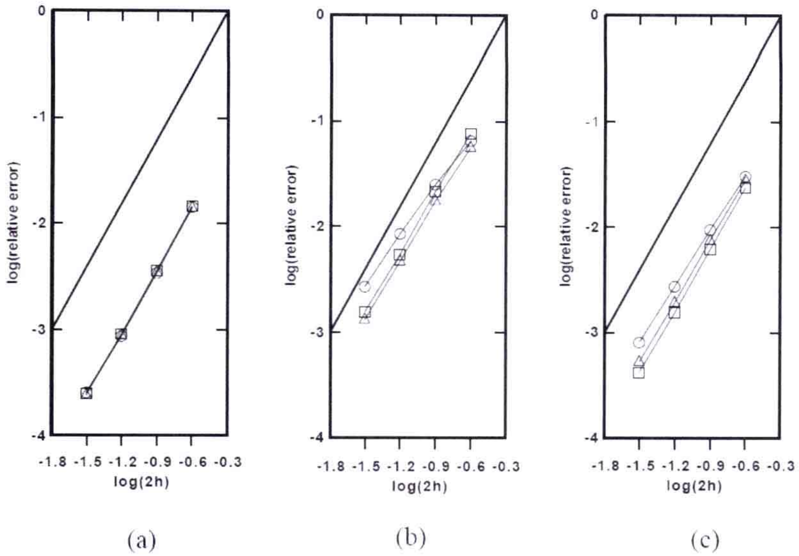


Figure 5: Convergence curves for the MITC4 shell element used in the three problems of Figure 2.2-1, for decreasing shell thickness, using the s-norm; (a) free-free shell; (b) fixed-fixed shell; (c) fixed-free shell; see reference [9]

We have developed 3-node and 4-node three-dimensional-shell elements that build upon the classical MITC shell elements but that include important three-dimensional effects [12]. The elements can be used to model very large deformations with large plastic strains using the Updated Lagrangian Hencky total strain formulation [2, 13]. An important point is that the three-dimensional-shell elements can be employed in explicit and implicit dynamic solutions and in static analyses, since no reduced integration with hourglass control is used and there are no artificial stabilization factors in the formulation. The three-dimensional-shell elements can be employed like the conventional MITC shell elements with 5 or 6 degrees of freedom at each node, but, when invoked by additional nodal degrees of freedom at the shell mid-surface, the elements represent through-the-thickness straining (2 extra degrees of freedom) and warping of the transverse fibers (2 or 3 extra degrees of freedom). Thus, while all degrees of freedom are defined at the shell mid-surface nodes in accordance with a shell theory, from a displacement interpolation point of view, the elements can be thought of as higher-order three-dimensional solid elements, with assumptions, when the additional degrees of freedom are invoked.

In the formulations, MITC interpolations are used to prevent shear locking, and in incompressible analysis the u/p formulation is employed [2, 14]. A particular aspect addressed in reference [12] is to give benchmark solutions for large strain analyses.

Figure 6 shows an application in a large strain solution that represents a good benchmark test [12]. Here the large strains in the structure result in a significant downward shifting of the mid-surface nodes during the response.

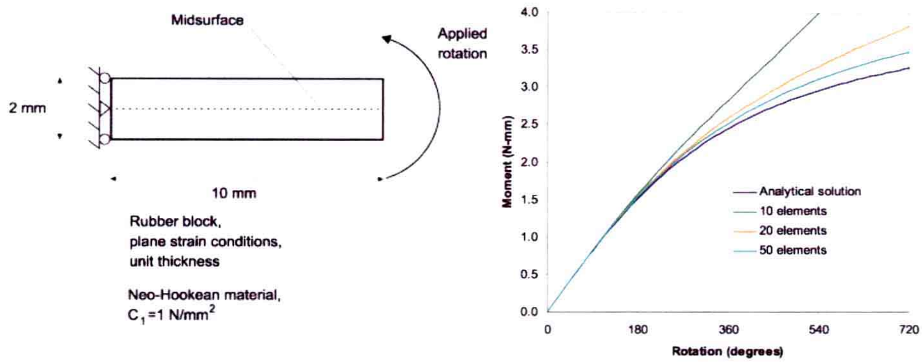


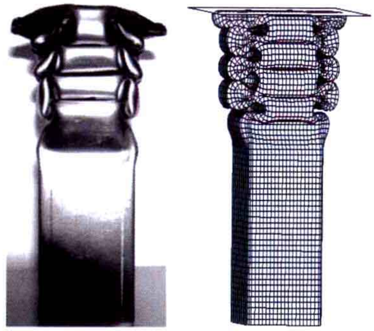
Figure 6: Large strain analysis of thick cantilever; incompressible Mooney-Rivlin material; results using the MITC4 3D-shell element

Figure 7 shows some analysis results using the MITC4 three-dimensional-shell element in a slow crush analysis and Figure 8 shows a crash solution result, all obtained using the Bathe implicit time integration scheme, for details see reference [15].

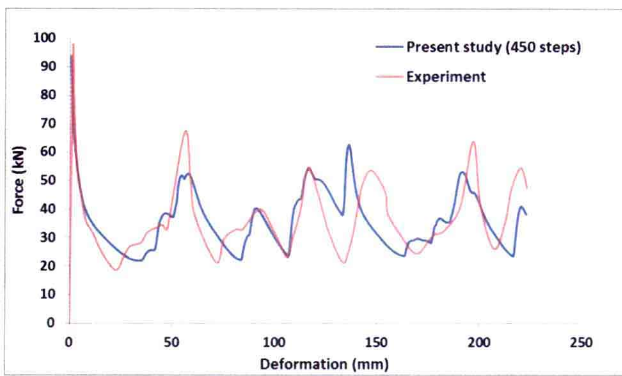
Here, the important point is that the same elements are employed to solve static and dynamic problems. Of course, since full numerical integration is used, solutions with explicit time integration are computationally quite expensive compared to those using elements based on reduced integration and hourglass control.

2.3 A procedure for stress improvements

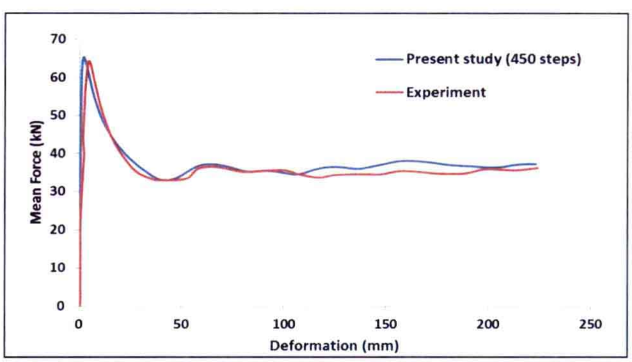
It is well known that the low-order displacement-based finite elements (3-node triangular and 4-node quadrilateral elements in two-dimensional solutions, and 4-node tetrahedral and 8-node brick elements in three-dimensional solutions) are not effective in predicting stresses accurately. Very fine meshes are needed in practice. On the other hand, the elements are quite robust, and the bandwidth of the resulting finite element equations is relatively small. Hence, if the order of stress convergence could be increased, the elements would be quite attractive in various analyses.



(a)



(b)



(c)

Figure 7: Quasi-static crushing of a square-section tube, length of tube is 310 mm; (a) experimental and computed results in final configuration; (b) force – displacement curves; (c) mean crushing force – displacement curves