CALCULUS
ONE AND SEVERAL VARIABLES

THIRD EDITION

S.L. SALAS

EINAR HILLE

CALCULUS:

One and Several Variables

with Analytic Geometry

Third Edition

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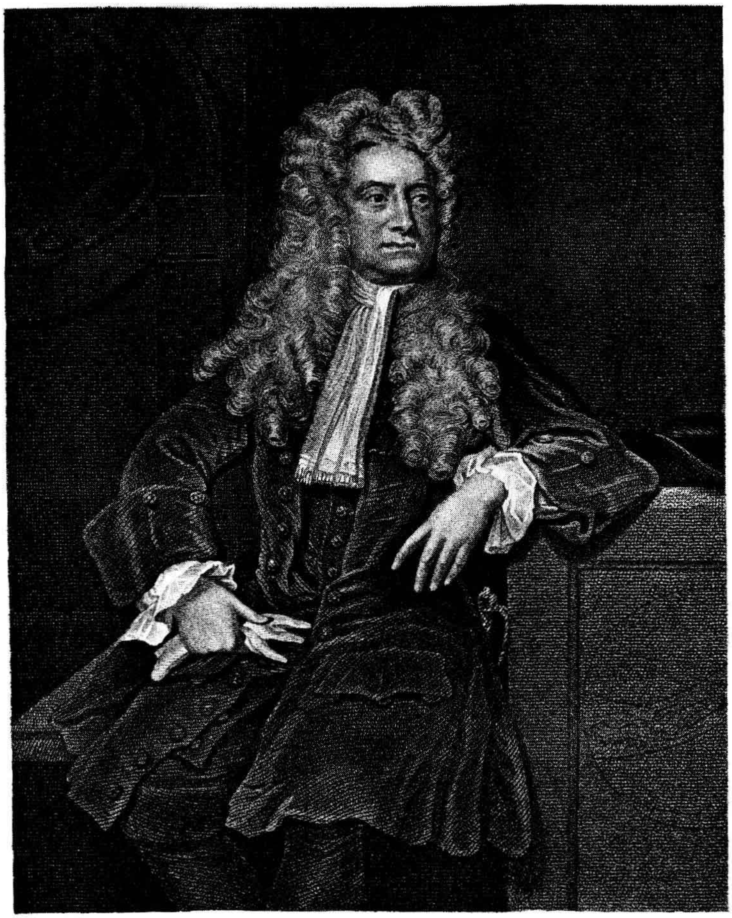

Sir Isaac Newton

Preface

Calculus: One and Several Variables is available both as a complete volume and in two parts. In the two-part version, Part 1 considers functions of one variable, analytic geometry, and sequences and series (Chapters 1 to 13 of the complete volume). Part 2 repeats the material on sequences and series and goes on to discuss functions of several variables and vector calculus (Chapters 12 to 19 of the complete volume).

Both in content and in spirit this edition is very similar to the previous ones. There are, however, some noticeable differences:

1. All the figures have been redrawn.
2. Chapters which were rather long in previous editions have been broken up, resulting in a marked increase in the number of chapters.
3. For the benefit of students who come weakly prepared, we have added a review section on lines and slowed the pace of our discussion of polar coordinates.
4. Although we continue to define the definite integral in terms of upper and lower sums, we also introduce Riemann sums and use them in several applications.
5. Indeterminate forms, formerly discussed after infinite series, now appear before the chapter on infinite series. This makes L'Hospital's rule available for radius of convergence arguments.
6. Line integrals, formerly appearing at the end of our chapter on functions of several variables, now form part of a new final chapter. In this new chapter we take up curl and divergence and give an elementary view of Green's Theorem, the Divergence Theorem, and Stokes's Theorem.

Here and there you may find other changes—a paragraph rewritten, an additional example, a new exercise—but most of these changes are minor and will probably pass unnoticed.

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Through the efforts of Mr. Ostedt we received valuable critiques from many distinguished colleagues. In this regard we wish to acknowledge our special debt to Professors John T. Anderson (Hamilton College), Harvey B. Keynes (University of Minnesota), Clifford Kottman (Simpson College), John W. Lee (Oregon State University), David Lovelock (University of Arizona), Stanley M. Lukawecki (Clemson University), Gordon D. Prichett (Hamilton College), and Donald R. Sherbert (University of Illinois at Urbana-Champaign).

Finally a word about an assistant who, to the point of exhaustion, read and reread and reread the manuscript, questioning this, questioning that, suggesting this, suggesting that. For all this, and more, we offer our thanks to Charles G. Salas.

S. L. S.

A Note From The Publisher

A *Student Supplement* is available for use either as a self-study guide or in conjunction with any calculus course based on this text. Preliminary editions of this supplement have been classroom tested since 1972.

The *Supplement* provides alternate explanations and perspectives on the material in the text. Additional examples are worked out in detail to help focus attention on the central concept(s) of each section.

The Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ϵ	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	o	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Gottfried Leibniz

Contents

Introduction

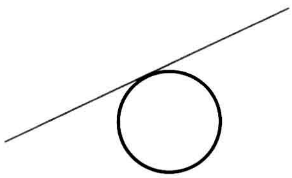
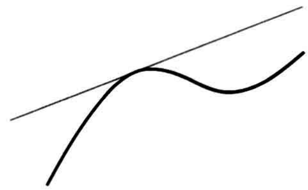
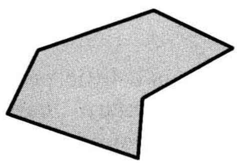
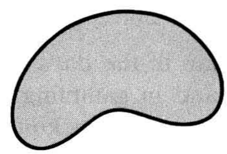
1

1.1 What is Calculus?

To a Roman in the days of the empire a "calculus" was a little pebble used in counting and in gambling. Centuries later the verb "calculare" came to mean "to compute," "to reckon," "to figure out." To the engineer and mathematician of today calculus is the branch of mathematics that takes in elementary algebra and geometry and adds one more ingredient: *the limit process*.

Calculus begins where elementary mathematics leaves off. It takes ideas from elementary mathematics and extends them to a much more general situation. Here are some examples. On the left-hand side you will find an idea from elementary mathematics; on the right, this same idea as enriched by calculus.

Elementary Mathematics	*Calculus*
	
slope of a line $y = mx + b$	slope of a curve $y = f(x)$

 tangent line to a circle	 tangent line to a more general curve
average velocity, average acceleration	instantaneous velocity, instantaneous acceleration
distance moved under a constant velocity	distance moved under varying velocity
 area of a region bounded by line segments	 area of a region bounded by curves
sum of a finite collection of numbers $a_1 + a_2 + \cdots + a_n$	sum on an infinite series $a_1 + a_2 + \cdots + a_n + \cdots$
average of a finite collection of numbers	average value of a function on an interval
 length of a line segment	 length of a curve