COLLEGE ALGEBRA

FOURTH EDITION

MICHAEL SULLIVAN

College Algebra

Fourth Edition

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For Mary

Preface

To the Student

As you begin your study of college algebra, you might feel overwhelmed by the number of theorems, definitions, procedures and equations that confront you. You may even wonder whether you can learn all this material in a single course. For many of you, this may be your last mathematics course, while for others, just the first in a series of many. Don't worry—either way, this text was written with you in mind. While I write math texts in upper level courses, I also am the father of four college students who came home with frustration and questions—*I know what you are going through!*

This text was designed to help you—the student, master the terminology and basic concepts of college algebra. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of this material easier and more rewarding. This book is meant to be a "machine for learning," one that can help you to focus your efforts and get the most from the time and energy you invest.

Here are some hints I give my students at the beginning of the course:

1. Take advantage of the feature PREPARING FOR THIS CHAPTER. At the beginning of each chapter, I have prepared a list of topics to review. Be sure to take the time to do this. It will help you proceed quicker and more confidently through the chapter.
2. Read the material in the book before the lecture. Knowing what to expect and what is in the book, you can take fewer notes and spend more time listening and understanding the lecture.
3. After each lecture, rewrite your notes as you re-read the book, jotting down any additional facts that seem helpful. Be sure to do the Now Work Problem x as you proceed through a section. After completing a section, be sure to do the assigned problems. Answers to the Odd ones are in the back of the book.
4. If you are confused about something, visit your instructor during office hours immediately, before you fall behind. Bring your attempted solutions to problems with you to show your instructor where you are having trouble.
5. To prepare for an exam, review your notes. Then proceed through the Chapter Review. It contains a capsule summary of all the important material of the chapter. If you are uncertain of any concept, go back into the chapter and study it further. Be sure to do the Review Exercises for practice.

Remember the two "golden rules" of college algebra:

1. DON'T GET BEHIND! The course moves too fast, and it's hard to catch up.
2. WORK LOTS OF PROBLEMS. Everyone needs to practice, and problems show where you need more work. If you can't solve the homework problems without help, you won't be able to do them on exams.

I encourage you to examine the following overview for some hints on how to use this text.

Best Wishes!

Michael Sullivan

OVERVIEW

Chapter 4

PREPARING FOR THIS CHAPTER

Before getting started on this chapter, review the following concepts;
Domain of a variable (p. 14)
Graphs of certain equations (Example 2, p. 173;
Example 3, p. 174; Example 4, p. 174; Example 11, p. 180)
Tests for symmetry of an equation (p. 178)
Procedure for finding intercepts of an equation (p. 176)
Steps for setting up applied problems (p. 98)

FUNCTIONS AND THEIR GRAPHS

4.1 Functions
4.2 More about Functions
4.3 Graphing Techniques
4.4 Operations on Functions; Composite Functions
4.5 One-to-One Functions; Inverse Functions
4.6 Mathematical Models: Constructing Functions
Chapter Review

Preview Getting from an Island to Town

An island is 2 miles from the nearest point P on a straight shoreline. A town is 12 miles down the shore from P.
(a) *If a person can row a boat at an average speed of 5 miles per hour and the same person can walk 2 miles per hour, express the time T it takes to go from the island to town as a function of the distance x from P to where the person lands the boat.*
(b) *How long will it take to travel from the island to town if the person lands the boat 4 miles from P?*
(c) *How long will it take if the person lands the boat 8 miles from P? [Example 9 in Section 4.1]*
(d) *Is there a place to land the boat so that the travel time is least? Do you think this place is closer to town or closer to P? Discuss the possibilities. Give reasons. [Problems 63 and 64 in Exercise 4.1]*

Beginning each chapter, after Chapter One, there is a list of concepts from previous chapters you should understand before moving on to the new chapter. By reviewing this list and making sure you feel comfortable with the material covered, you will begin to notice that learning algebra is a building process. This review makes sure your foundation is laid!

Each chapter opens with a photo and application which illustrates algebra in use around us. The photo is accompanied by a mathematical problem which you will learn how to solve during the course of the chapter. This allows you to understand the kind of issues that will be covered as well as how they might be used in the 'real world'.

Also on the first page of the chapter, you will find an outline of the topics to be covered within. This is a good way to organize your notes for studying.

Perhaps the most central idea in mathematics is the notion of a *function*. This important chapter deals with what a function is, how to graph functions, how to perform operations on functions, and how functions are used in applications.

The word *function* apparently was introduced by René Descartes in 1637. For him, a function simply meant any positive integral power of a variable x. Gottfried Wilhelm von Leibniz (1646–1716), who always emphasized the geometric side of mathematics, used the word function to denote any quantity associated with a curve, such as the coordinates of a point on the curve. Leonhard Euler (1707–1783) employed the word to mean any equation or formula involving variables and constants. His idea of a function is similar to the one most often used today in courses that precede calculus. Later, the use of functions in investigating heat flow equations led to a very broad definition, due to Lejeune Dirichlet (1805–1859), which describes a function as a rule or correspondence between two sets. It is his definition that we use here.

4.1

Functions

In many applications, a correspondence often exists between two sets of numbers. For example, the revenue R resulting from the sale of x items selling for $10 each is $R = 10x$ dollars. If we know how many items have been sold, then we can calculate the revenue by using the rule $R = 10x$. This rule is an example of a *function*.

As another example, if an object is dropped from a height of 64 feet above the ground, the distance s (in feet) of the object from the ground after t seconds is given (approximately) by the formula $s = 64 - 16t^2$. When $t = 0$ seconds, the object is $s = 64$ feet above the ground. After 1 second, the object is $s = 64 - 16(1)^2 = 48$ feet above the ground. After 2 seconds, the object strikes the ground. The formula $s = 64 - 16t^2$ provides a way of finding the distance s when the time t $(0 \le t \le 2)$ is prescribed. There is a correspondence between each time t in the interval $0 \le t \le 2$ and the distance s. We say that the distance s is a *function* of the time t because:

1. There is a correspondence between the set of times and the set of distances.
2. There is exactly one distance s obtained for a prescribed time t in the interval $0 \le t \le 2$.

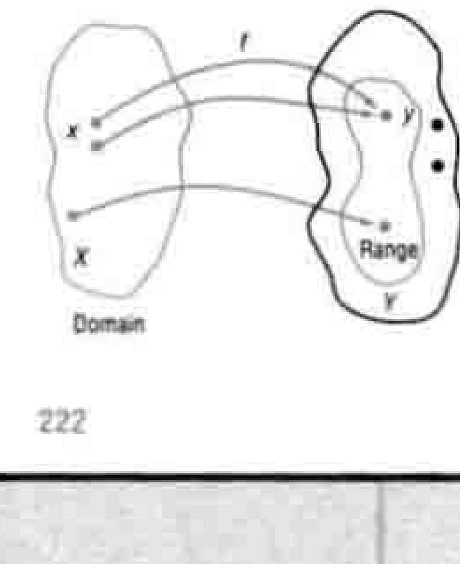
Let's now look at the definition of a function.

Definition of Function

Function

Let X and Y be two nonempty sets of real numbers.* A **function** from X into Y is a rule or a correspondence that associates with each element of X a unique element of Y. The set X is called the **domain** of the function. For each element x in X, the corresponding element y in Y is called the **value** of the function at x, or the **image** of x. The set of all images of the elements of the domain is called the **range** of the function.

FIGURE 1



Refer to Figure 1. Since there may be some elements in Y that are not the image of some x in X, it follows that the range of a function may be a subset of Y.

The rule (or correspondence) referred to in the definition of a function is most often given as an equation in two variables, usually denoted x and y.

*The two sets X and Y can also be sets of complex numbers, and then we have defined a complex function. In the broad definition (due to Lejeune Dirichlet), X and Y can be any two sets.

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Most chapters open with a brief historical discussion of "where this material came from". It is helpful to understand how others created and used these ideas to solve their everyday problems.

New terms appear in boldface type where they are defined.

Major definitions appear in large type enclosed within a color screen. These are important vocabulary items for you to know.

Historical Features place the mathematics you are learning in a historical context. By learning how others have used similar concepts you will understand how it may be used in your own life. Sometimes there are extra historical exercises for you to solve on your own!

bers). To see why, look at this calculation: We know that $\sqrt{100} = 10$. However, it is also true that $100 = (-25)(-4)$, so

$$10 = \sqrt{100} = \sqrt{(-25)(-4)} = \sqrt{-25}\sqrt{-4}$$

Here is the error

$$= (\sqrt{25}i)(\sqrt{4}i) = (5i)(2i) = 10i^2 = -10$$

The Quadratic Formula

Because we have defined the square root of a negative number, we now can restate the quadratic formula without restriction.

Theorem In the complex number system, the solutions of the quadratic equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, are given by the formula

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (1)$$

EXAMPLE 3 *Solving Quadratic Equations in the Complex Number System*

Solve the equation $x^2 - 4x + 8 = 0$ in the complex number system.

Solution Here $a = 1$, $b = -4$, $c = 8$, and $b^2 - 4ac = 16 - 4(8) = -16$. Using equation (1), we find

$$x = \frac{4 \pm \sqrt{-16}}{2} = \frac{4 \pm \sqrt{16}i}{2} = \frac{4 \pm 4i}{2} = 2 \pm 2i$$

The equation has the solution set $\{2 - 2i, 2 + 2i\}$.

Check:

$$2 + 2i: \quad (2 + 2i)^2 - 4(2 + 2i) + 8 = 4 + 8i + 4i^2 - 8 - 8i + 8 = 4 - 4 = 0$$

$$2 - 2i: \quad (2 - 2i)^2 - 4(2 - 2i) + 8 = 4 - 8i + 4i^2 - 8 + 8i + 8 = 4 - 4 = 0$$

Now work Problem 17.

All important formulas are enclosed by a box and shown in color. This is done to alert you to important concepts.

Examples are easy to locate and are titled to tell you what they are about. Although the solution is entirely worked out, you should go through it using pencil and paper as you go. If you get stuck on a homework problem, chances are you can find an example that models the problem. Many solutions contain a Check. It is always a good practice to Check solutions when you can.

The Now Work Problem xx feature asks you to do a particular problem before you go on in the section. This is to ensure that you have mastered the material just presented before going on. By following the practice of doing these Now Work Problems, you will gain confidence and save time.

Theorems are noted with the word "Theorem" in blue. These are especially important to study. When a theorem of the proof is given, the word "Proof" also appears.

A final comment about notation. The notations $C(n, r)$ and $P(n, r)$ are variants of a form of notation developed in England after 1830. The notation $\binom{n}{r}$ for $C(n, r)$ goes back to Leonhard Euler (1707–1783), but is now losing ground because it has no clearly related symbolism of the same type for permutations. The set symbols $\cup$ and $\cap$ were introduced by Giuseppe Peano (1858–1932) in 1888 in a slightly different context. The inclusion symbol $\subset$ was introduced by E. Schroeder (1841–1902) about 1890. The treatment of set theory in the text is due to George Boole (1815–1864), who wrote $A + B$ for $A \cup B$ and AB for $A \cap B$ (statisticians still use AB for $A \cap B$).

HISTORICAL PROBLEMS

1. *The Problem Discussed by Fermat and Pascal* A game between two equally skilled players, A and B, is interrupted when A needs 2 points to win and B needs 3 points. In what proportion should the stakes be divided? [*Note:* If each play results in 1 point for either player, at most four more plays will decide the game.]
   (a) *Fermat's solution* List all possible outcomes that will end the game to form the sample space (for example, $ABAA$, $ABBB$, etc.). The probabilities for A to win and B to win then determine how the stakes should be divided.
   (b) *Pascal's solution* Use combinations to determine the number of ways the 2 points needed for A to win could occur in four plays. Then use combinations to determine the number of ways the 3 points needed for B to win could occur. This is trickier than it looks, since A can win with 2 points in either two plays, three plays, or four plays. Compute the probabilities and compare with the results in part (a).
2. *Huygens's Mathematical Expectation* In a game with n possible outcomes with probabilities $p_1, p_2, \ldots, p_n$, suppose that the *net* winnings are $w_1, w_2, \ldots, w_n$, respectively. Then the mathematical expectation is

   $$E = p_1w_1 + p_2w_2 + \cdots + p_nw_n$$

   The number E represents the profit or loss per game in the long run. The following problems are a modification of those of Huygens:
   (a) A fair die is tossed. A gambler wins \$3 if he throws a 6 and \$6 if he throws a 5. What is his expectation? [*Note:* $w_1 = w_2 = w_3 = w_4 = 0$]
   (b) A gambler plays the same game as in part (a), but now the gambler must pay \$1 to play. This means $w_5 = \$5$, $w_6 = \$2$, and $w_1 = w_2 = w_3 = w_4 = -\$1$. What is the expectation?

The intercepts of the graph of an equation can be found by using the fact that points on the x-axis have y-coordinates equal to 0, and points on the y-axis have x-coordinates equal to 0.

Procedure for Finding Intercepts

1. To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
2. To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

EXAMPLE 6 *Finding Intercepts from an Equation*

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$.

Solution To find the x-intercept(s), we let $y = 0$ and obtain the equation

$$x^2 - 4 = 0$$

The equation has two solutions, -2 and 2. Thus, the x-intercepts are -2 and 2.

To find the y-intercept(s), we let $x = 0$ and obtain the equation

$$y = -4$$

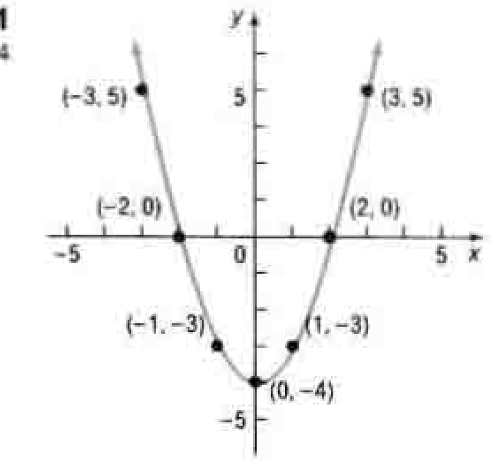
Thus, the y-intercept is -4.

Since $x^2 \geq 0$ for all x, we deduce from the equation $y = x^2 - 4$ that $y \geq -4$ for all x. This information, plus the points from Table 4, provide enough information to graph $y = x^2 - 4$. See Figure 21.

TABLE 4

x	$y = x^2 - 4$	(x, y)
−3	5	(−3, 5)
−1	−3	(−1, −3)
1	−3	(1, −3)
3	5	(3, 5)

FIGURE 21
$y = x^2 - 4$



Now work Problem 13(a).

Comment For many equations, finding intercepts may not be so easy. In such cases, a graphing utility can be used. Read Section 3, The TRACE, ZOOM-IN, and BOX Functions in the Appendix, page 681, to find out how a graphing utility locates intercepts.

Important Procedures and STEPS are noted in the left column and are separated from the body of the text by two horizontal color rules. You will need to know these procedures and steps to do your homework problems and prepare for exams.

Hints or warnings are offered where appropriate. Sometimes there are short cuts or pitfalls that students should know about—I've included them.

The "graphing calculator" icon can be seen throughout the textual material, end of section and review exercises. These indicate that they require the use of a graphing calculator or computer software program.

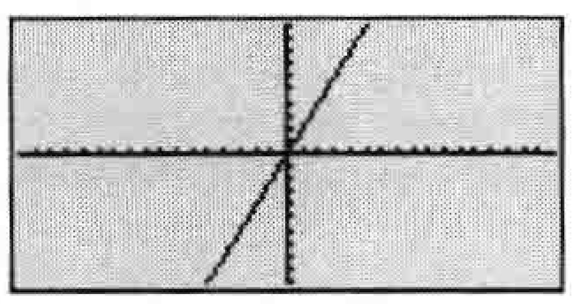
Appendix GRAPHING UTILITIES

1. The Viewing Rectangle
2. Graphing Equations Using a Graphing Utility
3. The TRACE, ZOOM-IN, and BOX Functions
4. Square Screens
5. Approximations

1 The Viewing Rectangle

All graphing utilities, that is, all graphing calculators and all computer software graphing packages, graph equations by plotting points on a screen. The screen itself actually consists of small rectangles, called **pixels.** The more pixels the screen has, the better the resolution. Most graphing calculators have 48 pixels per square inch; most computer screens have 32 to 108 pixels per square inch. When a point to be plotted lies inside a pixel, the pixel is turned on (lights up). Thus, the graph of an equation is a collection of pixels. Figure 1 shows how the graph of $y = 2x$ looks on a T185 graphing calculator.

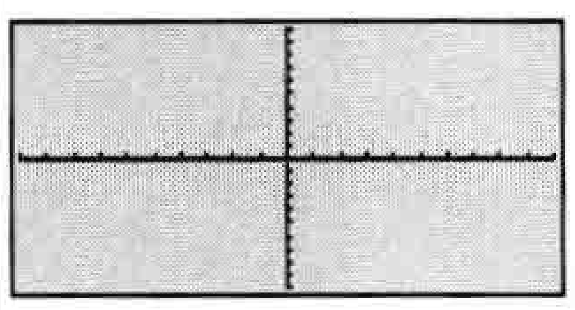
FIGURE 1
$y = 2x$



The screen of a graphing utility will display the coordinate axes of a rectangular coordinate system. However, you must set the scale on each axis. You must also include the smallest and largest values of x and y that you want included in the graph. This is called **setting the RANGE** and it gives the **viewing rectangle** or **windov.**

Figure 2 illustrates a typical viewing rectangle.

FIGURE 2



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Many students possess a graphing calculator or have access to computer software that does graphing. To help you understand what such devices are capable of, I have placed an Appendix at the end of the book, called Graphing Utilities, that explains some of the capabilities of a grapher. Many examples and Exercises in the book require a grapher. Careful thought has been given to these examples and exercises: most ask for conclusions that cannot be obtained through usual algebra processes. This should help you to see the power of technology in solving problems.

Section 3.4 Parallel and Perpendicular Lines; Circles 205

MISSION POSSIBLE

Chapter 3

PREDICTING THE FUTURE OF OLYMPIC TRACK EVENTS

Some people think that women athletes are beginning to "catch up" to men in the Olympic Track Events. In fact, researchers at UCLA published data in 1992 that seemed to indicate that within 65 years, the top men and women runners would be able to compete on an equal basis. Other researchers disagreed. Here is some data on the 200-meter run with winning times for the Olympics in the year given.

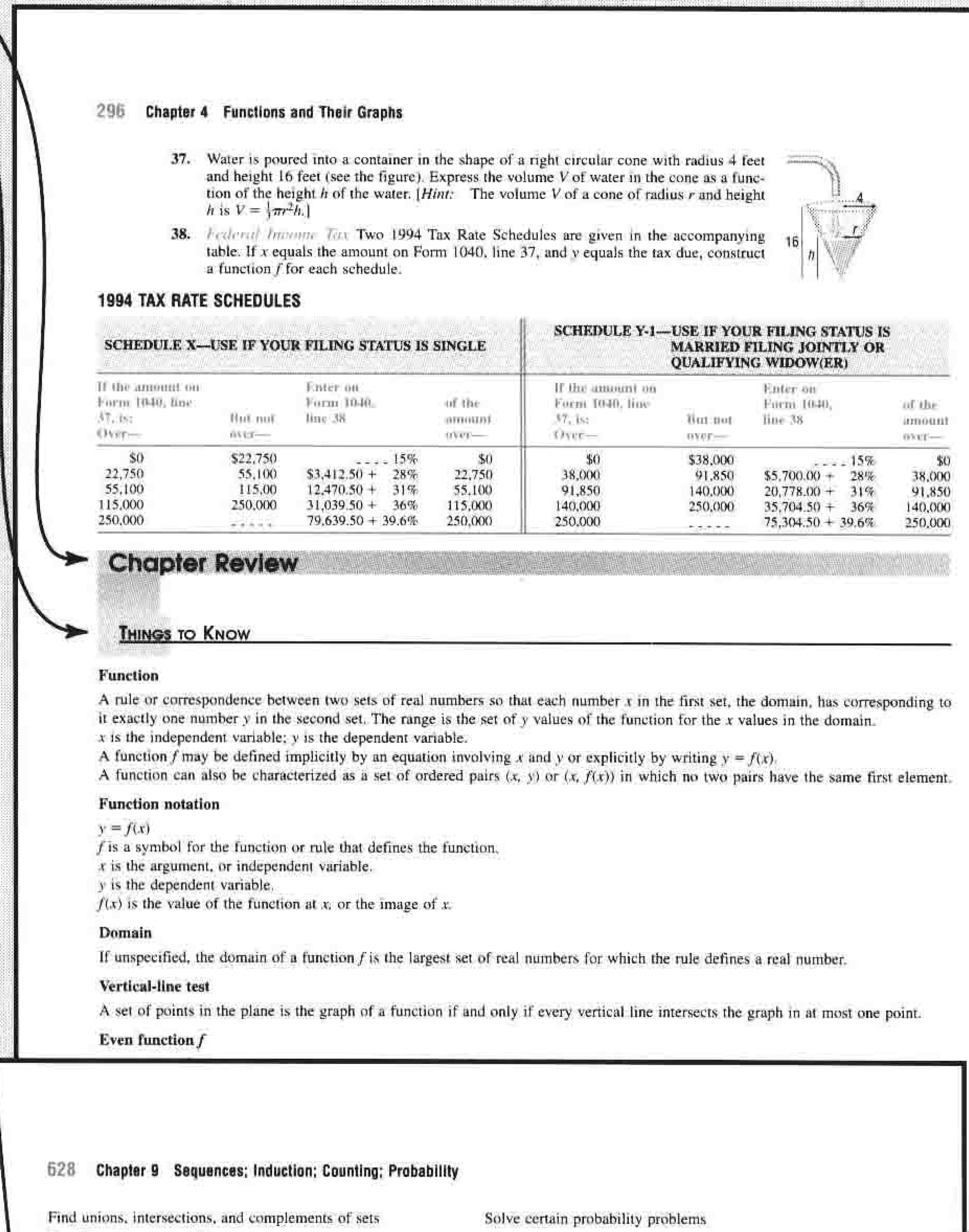
	MEN'S TIME IN SECONDS	WOMEN'S TIME IN SECONDS
1948	21.1	24.4
1952	20.7	23.7
1956	20.6	23.4
1960	20.5	24.0
1964	20.3	23.0
1968	19.83	22.5
1972	20.00	22.40
1976	20.23	22.37
1980	20.19	22.03
1984	19.80	21.81
1988	19.75	21.34
1992	19.73	21.72

(You may notice that the introduction of better timing devices meant more accurate measures starting in 1968 for the men and 1972 for the women.)

1. To begin your investigation of the UCLA conjecture, make a graph of these data. To get the kind of accuracy you need, you should use graph paper. You will need to use only the first quadrant. On your x-axis place the Olympic years from 1948 to 2048, counting by 4's. On your y-axis place the numbers from 15 to 25 which represent the seconds; if you're using graph paper, allow about four squares per one second. Use X's to represent the men's times on the graph and O's to represent the women's times. These should form what we call a "scatter plot," not a neat line or curve.
2. Using a ruler or straightedge, draw a line that represents roughly the slope and direction indicated by the men's scores. Do the same for the women's scores. Write a sentence or two explaining why you think your line is a good representation of the scatter plot.
3. Next find the equation for each line, using your graph to estimate the slope and y-intercept for each. Try to be as accurate as possible, remembering your units and using the points where the line crosses intersections of the grid. The slope will be in seconds per year.
4. Do your two lines appear to cross? In what year do they cross? If you solve the two equations algebraically, do you get the same answer?
5. (Optional) Some graphing calculators enable you do this problem in a statistics mode. They will plot the individual point that you type in and find a linear regression that represents the data. If you have a graphing calculator, find out whether their equations match yours.
6. Make a group decision about whether or not you think that women's times will "catch up" to men's times in the future. Write out 2-3 sentences explaining why you believe they will or will not.

Mission Possible: In the "real world" colleagues often collaborate to solve more difficult problems—or problems that may have more than one answer. Every chapter has a "Mission Possible" for you and your classmates to collaborate on. All of these projects will require you to verbally communicate or write up your answers. Good communication skills are very important to becoming successful—no matter what your future holds.

The Chapter Review is for your use in checking your understanding of the chapter materials. "Things to Know" is the best place to start. Check your understanding of the concepts listed there. Then demonstrate to yourself that you know "How to" solve the items contained within that section. "Fill in the Blanks" will determine your comfort with vocabulary. "True/False" is a stickler for knowing definitions! If you are uncertain of any concept, go back into the chapter and study it further. Be sure to do the Review Exercises for practice. These reviews are for your success in this course—Make good use of them.



296 **Chapter 4 Functions and Their Graphs**

37. Water is poured into a container in the shape of a right circular cone with radius 4 feet and height 16 feet (see the figure). Express the volume V of water in the cone as a function of the height h of the water. [*Hint:* The volume V of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

38. *Federal Income Tax* Two 1994 Tax Rate Schedules are given in the accompanying table. If x equals the amount on Form 1040, line 37, and y equals the tax due, construct a function f for each schedule.

1994 TAX RATE SCHEDULES

SCHEDULE X—USE IF YOUR FILING STATUS IS SINGLE				SCHEDULE Y-1—USE IF YOUR FILING STATUS IS MARRIED FILING JOINTLY OR QUALIFYING WIDOW(ER)			
If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—	If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—
$0	$22,750	----- 15%	$0	$0	$38,000	----- 15%	$0
22,750	55,100	$3,412.50 + 28%	22,750	38,000	91,850	$5,700.00 + 28%	38,000
55,100	115,00	12,470.50 + 31%	55,100	91,850	140,000	20,778.00 + 31%	91,850
115,000	250,000	31,039.50 + 36%	115,000	140,000	250,000	35,704.50 + 36%	140,000
250,000	-----	79,639.50 + 39.6%	250,000	250,000	-----	75,304.50 + 39.6%	250,000

Chapter Review

THINGS TO KNOW

Function

A rule or correspondence between two sets of real numbers so that each number x in the first set, the domain, has corresponding to it exactly one number y in the second set. The range is the set of y values of the function for the x values in the domain.
x is the independent variable; y is the dependent variable.
A function f may be defined implicitly by an equation involving x and y or explicitly by writing $y = f(x)$.
A function can also be characterized as a set of ordered pairs (x, y) or $(x, f(x))$ in which no two pairs have the same first element.

Function notation

$y = f(x)$
f is a symbol for the function or rule that defines the function.
x is the argument, or independent variable.
y is the dependent variable.
$f(x)$ is the value of the function at x, or the image of x.

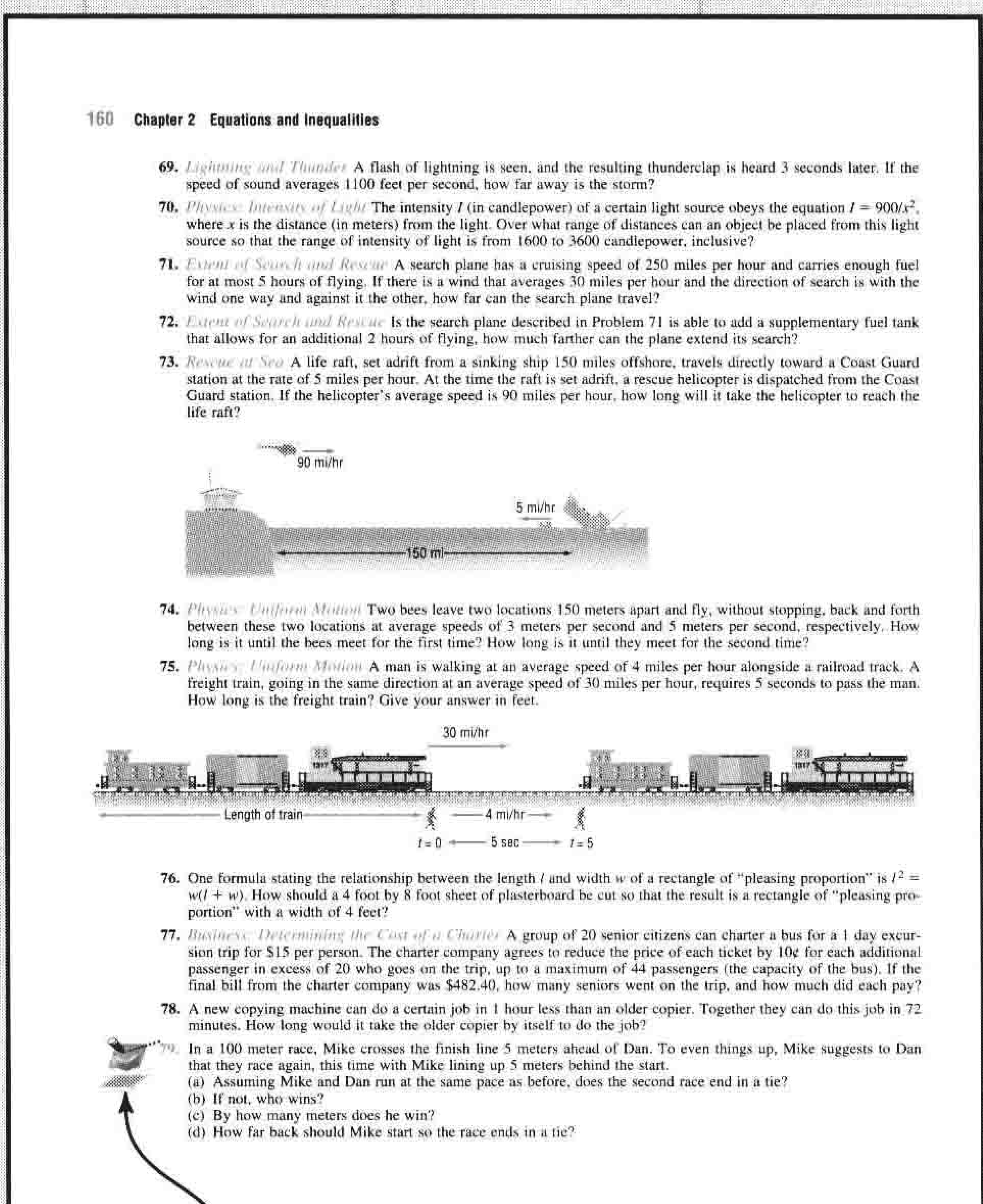
Domain

If unspecified, the domain of a function f is the largest set of real numbers for which the rule defines a real number.

Vertical-line test

A set of points in the plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

Even function f



160 **Chapter 2 Equations and Inequalities**

69. *Lightning and Thunder* A flash of lightning is seen, and the resulting thunderclap is heard 3 seconds later. If the speed of sound averages 1100 feet per second, how far away is the storm?

70. *Physics: Intensity of Light* The intensity I (in candlepower) of a certain light source obeys the equation $I = 900/x^2$, where x is the distance (in meters) from the light. Over what range of distances can an object be placed from this light source so that the range of intensity of light is from 1600 to 3600 candlepower, inclusive?

71. *Extent of Search and Rescue* A search plane has a cruising speed of 250 miles per hour and carries enough fuel for at most 5 hours of flying. If there is a wind that averages 30 miles per hour and the direction of search is with the wind one way and against it the other, how far can the search plane travel?

72. *Extent of Search and Rescue* Is the search plane described in Problem 71 is able to add a supplementary fuel tank that allows for an additional 2 hours of flying, how much farther can the plane extend its search?

73. *Rescue at Sea* A life raft, set adrift from a sinking ship 150 miles offshore, travels directly toward a Coast Guard station at the rate of 5 miles per hour. At the time the raft is set adrift, a rescue helicopter is dispatched from the Coast Guard station. If the helicopter's average speed is 90 miles per hour, how long will it take the helicopter to reach the life raft?

90 mi/hr
5 mi/hr
150 mi

74. *Physics: Uniform Motion* Two bees leave two locations 150 meters apart and fly, without stopping, back and forth between these two locations at average speeds of 3 meters per second and 5 meters per second, respectively. How long is it until the bees meet for the first time? How long is it until they meet for the second time?

75. *Physics: Uniform Motion* A man is walking at an average speed of 4 miles per hour alongside a railroad track. A freight train, going in the same direction at an average speed of 30 miles per hour, requires 5 seconds to pass the man. How long is the freight train? Give your answer in feet.

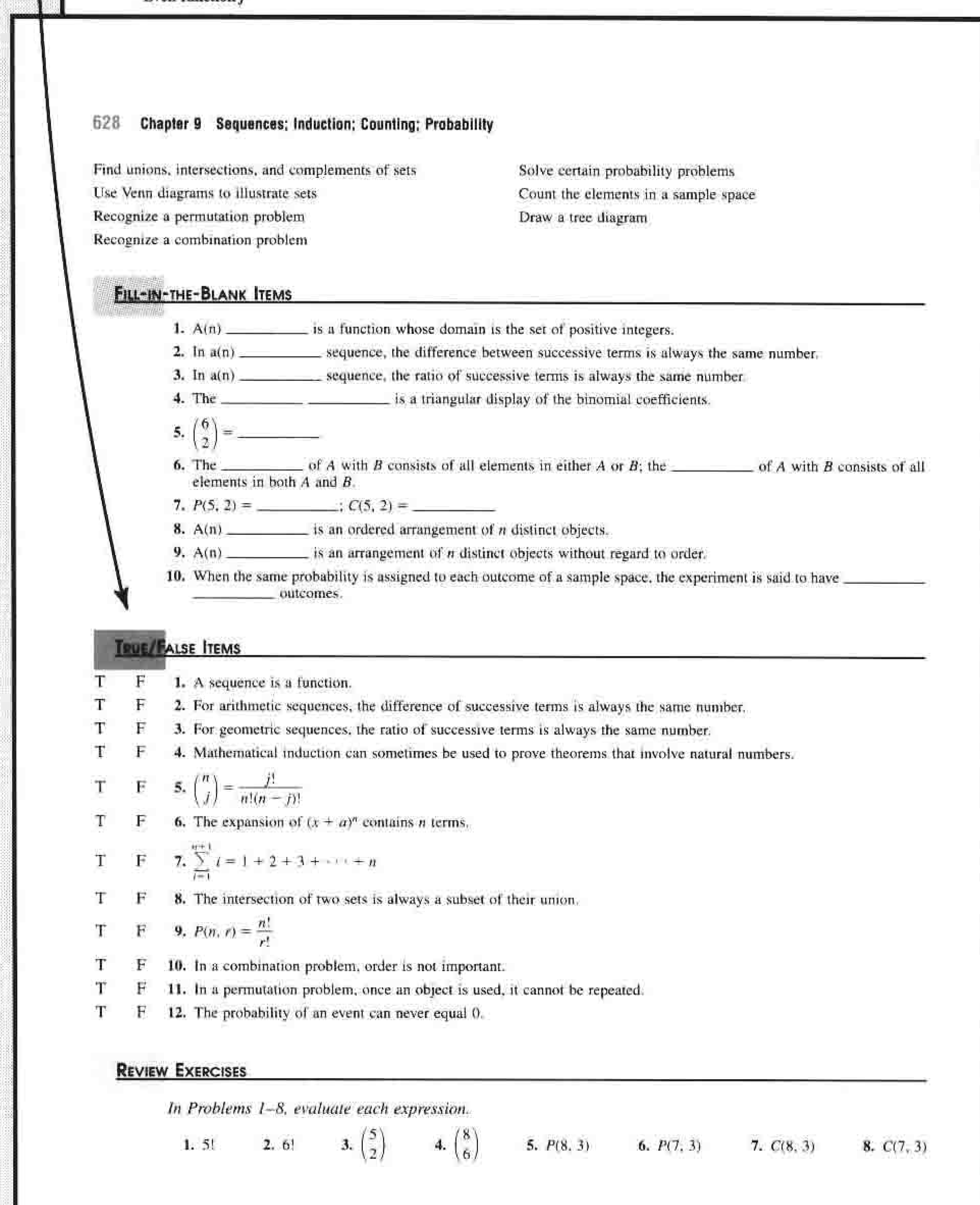
30 mi/hr
Length of train
4 mi/hr
$t = 0$ 5 sec $t = 5$

76. One formula stating the relationship between the length l and width w of a rectangle of "pleasing proportion" is $l^2 = w(l + w)$. How should a 4 foot by 8 foot sheet of plasterboard be cut so that the result is a rectangle of "pleasing proportion" with a width of 4 feet?

77. *Business: Determining the Cost of a Charter* A group of 20 senior citizens can charter a bus for a 1 day excursion trip for $15 per person. The charter company agrees to reduce the price of each ticket by 10¢ for each additional passenger in excess of 20 who goes on the trip, up to a maximum of 44 passengers (the capacity of the bus). If the final bill from the charter company was $482.40, how many seniors went on the trip, and how much did each pay?

78. A new copying machine can do a certain job in 1 hour less than an older copier. Together they can do this job in 72 minutes. How long would it take the older copier by itself to do the job?

79. In a 100 meter race, Mike crosses the finish line 5 meters ahead of Dan. To even things up, Mike suggests to Dan that they race again, this time with Mike lining up 5 meters behind the start.
(a) Assuming Mike and Dan run at the same pace as before, does the second race end in a tie?
(b) If not, who wins?
(c) By how many meters does he win?
(d) How far back should Mike start so the race ends in a tie?



628 **Chapter 9 Sequences; Induction; Counting; Probability**

Find unions, intersections, and complements of sets
Use Venn diagrams to illustrate sets
Recognize a permutation problem
Recognize a combination problem
Solve certain probability problems
Count the elements in a sample space
Draw a tree diagram

FILL-IN-THE-BLANK ITEMS

1. A(n) __________ is a function whose domain is the set of positive integers.
2. In a(n) __________ sequence, the difference between successive terms is always the same number.
3. In a(n) __________ sequence, the ratio of successive terms is always the same number.
4. The __________ __________ is a triangular display of the binomial coefficients.
5. $\binom{6}{2} =$ __________
6. The __________ of A with B consists of all elements in either A or B; the __________ of A with B consists of all elements in both A and B.
7. $P(5, 2) =$ __________; $C(5, 2) =$ __________
8. A(n) __________ is an ordered arrangement of n distinct objects.
9. A(n) __________ is an arrangement of n distinct objects without regard to order.
10. When the same probability is assigned to each outcome of a sample space, the experiment is said to have __________ __________ outcomes.

TRUE/FALSE ITEMS

T F 1. A sequence is a function.
T F 2. For arithmetic sequences, the difference of successive terms is always the same number.
T F 3. For geometric sequences, the ratio of successive terms is always the same number.
T F 4. Mathematical induction can sometimes be used to prove theorems that involve natural numbers.
T F 5. $\binom{n}{j} = \frac{j!}{n!(n-j)!}$
T F 6. The expansion of $(x + a)^n$ contains n terms.
T F 7. $\sum_{i=1}^{n+1} i = 1 + 2 + 3 + \cdots + n$
T F 8. The intersection of two sets is always a subset of their union.
T F 9. $P(n, r) = \frac{n!}{r!}$
T F 10. In a combination problem, order is not important.
T F 11. In a permutation problem, once an object is used, it cannot be repeated.
T F 12. The probability of an event can never equal 0.

REVIEW EXERCISES

In Problems 1–8, evaluate each expression.

1. 5! 2. 6! 3. $\binom{5}{2}$ 4. $\binom{8}{6}$ 5. $P(8, 3)$ 6. $P(7, 3)$ 7. $C(8, 3)$ 8. $C(7, 3)$

The "pencil and book" icon is used to indicate open-ended questions for discussion, writing, group or research projects.

Student's Supplementary Aids

You may find yourself seeking out extra help with this course. Many students have found the following items to be useful in becoming successful in college algebra. Your college bookstore should have these items available, but if not, they can order them for you.

Student's Solutions Manual Contains complete step-by-step worked out solutions to all the odd numbered exercises in the textbook. This is terrific for getting instant feedback on whether you are proceeding correctly while solving problems. ISBN: 013-370164-6

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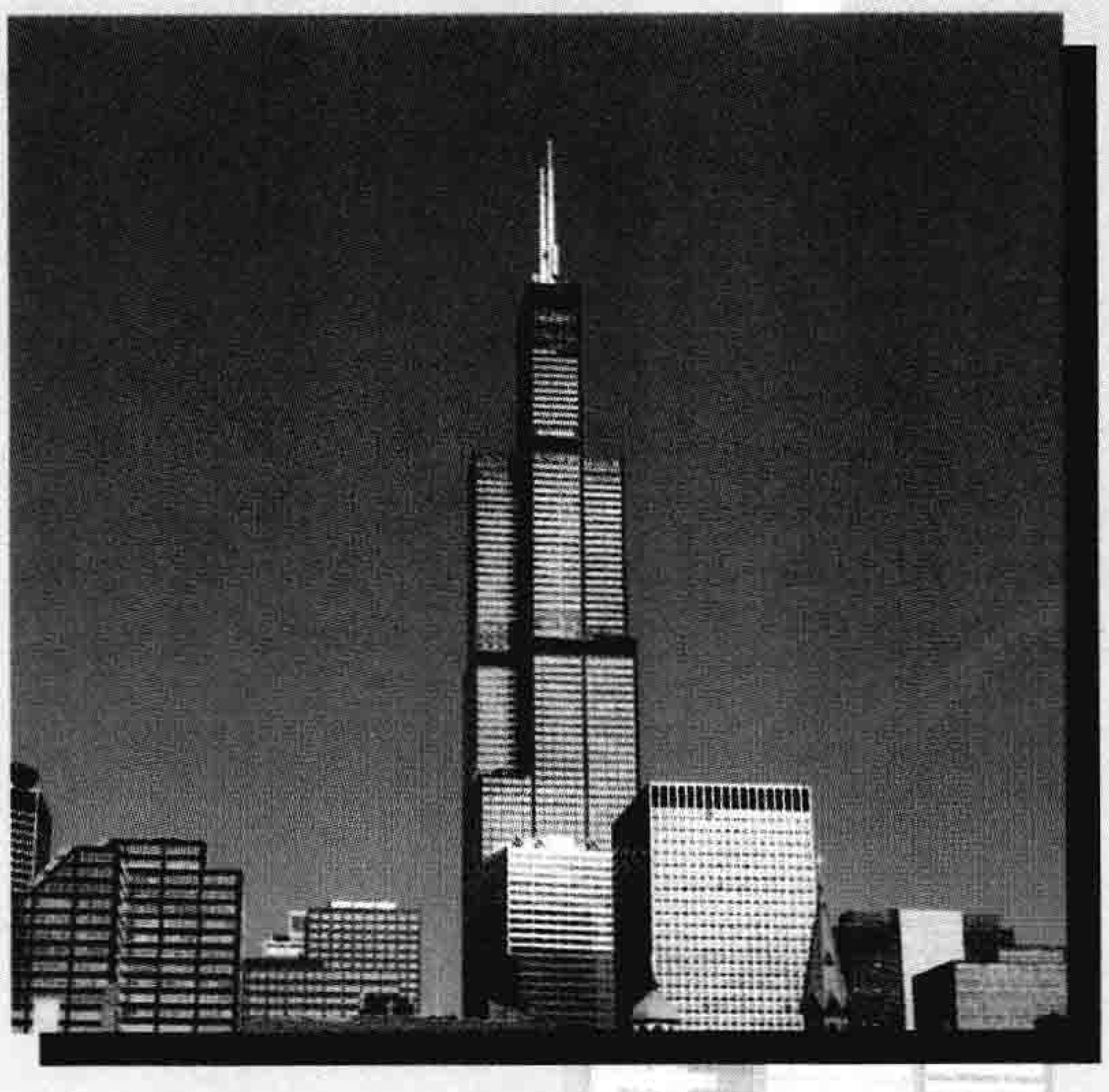
Chapter 9	Stained Glass Window	Wolfgang Koohler
Chapter 10	Corporate Jet	Joe Towers/The Stock Market

College Algebra

Contents

Chapter 1

Preliminaries



Preview How Far Can You See?

The tallest inhabited building in North America is the Sears Tower in Chicago. If the observation tower is 1454 feet above ground level, how far can a person standing in the observation tower see? [Problem 35 in Exercise 1.10]* ■

**Guinness Book of World Records.*

The word *algebra* is derived from the Arabic word *al-jabr.* This word is a part of the title of a ninth century work, "Hisâb al-jabr w'al-muqâbalah," written by Mohammed ibn Mûsâ al-Khowârizmî. The word *al-jabr* means "a restoration," a reference to the fact that, if a number is added to one side of an equation, then it must also be added to the other side in order to "restore" the equality. The title of the work, freely translated, means "The science of reduction and cancellation." Of course, today, algebra has come to mean a great deal more.

1.1 Real Numbers

Algebra can be thought of as a generalization of arithmetic in which letters may be used to represent numbers.

Operations

We will use letters such as *x, y, a, b,* and *c* to represent numbers. The symbols used in algebra for the operations of addition, subtraction, multiplication, and division are $+$, $-$, $\cdot$, and /. The words used to describe the results of these operations are **sum, difference, product,** and **quotient.** Table 1 summarizes these ideas.

TABLE 1

OPERATION	SYMBOL	WORDS
Addition	$a + b$	Sum: a plus b
Subtraction	$a - b$	Difference: a less b
Multiplication	$a \cdot b$, $(a) \cdot b$, $a \cdot (b)$, $(a) \cdot (b)$, ab, $(a)b$, $a(b)$, $(a)(b)$	Product: a times b
Division	a/b or $\frac{a}{b}$	Quotient: a divided by b

In algebra, we generally avoid using the multiplication sign $\times$ and the division sign $\div$ so familiar in arithmetic. Notice also that when two expressions are placed next to each other without an operation symbol, as in ab, or in parentheses, as in $(a)(b)$, it is understood that the expressions, called **factors,** are to be multiplied.

The four operations of addition, subtraction, multiplication, and division are called **binary operations,** since each is performed on a pair of numbers.

The symbol $=$, called an **equal sign** and read as "equals" or "is," is used to express the idea that the number or expression on the left of the equal sign is equivalent to the number or expression on the right.

EXAMPLE 1 *Writing Statements Using Symbols*

(a) The sum of 2 and 7 equals 9. In symbols, this statement is written as $2 + 7 = 9$.

(b) The product of 3 and 5 is 15. In symbols, this statement is written as $3 \cdot 5 = 15$. ■

Equality

We have used the equal sign to convey the idea that one expression is equivalent to another. Three important properties of equality are listed next. In this list, a and b represent numbers.

Reflexive Property

1. The **reflexive property** states that a number always equals itself; that is, $a = a$. Although this result seems obvious, it forms the basis for much of what we do in algebra.