



ROUTLEDGE
EXPLORATIONS
IN
ENVIRONMENTAL
ECONOMICS

OPTIMAL CONTROL OF AGE-STRUCTURED POPULATIONS IN ECONOMY, DEMOGRAPHY, AND THE ENVIRONMENT

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ROUTLEDGE

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Preface

The present volume covers a wide range of topics in mathematical modeling and optimization of economic, demographic, technological, and environmental processes. Each chapter represents new advances in modeling theory and practice and is written by recognized experts from 12 countries and four continents. The topics illustrate the fruitful interaction among applied mathematics, economics, demography, environmental sciences, management sciences, and operations research in exploiting global and local change. The interdisciplinary modeling plays a leading role in new prospering sciences such as environmental economics and ecological economics and possesses an enormous potential for discovering new insights in global and regional development. New models often combine several known model blocks and lead to new types of mathematical problems.

The main unifying theme of the book is the use of mathematical and optimization tools to describe age-structured populations in economy, demography, technology, and the environment. The considered objects range from biological populations such as humans, fish, and trees to industrial items such as equipment, capital assets, machines, and new technologies. The authors explicitly deal with the age and size structure of populations and resources under study. The book is a rich and excellent source of the state-of-art modeling expertise and references. Although the majority of presented models and techniques have been developed during the last decades and covered in various scientific journals, the book has no direct analogues in the latest monographic literature.

Starting in the 1980s, a number of books has been written on modeling issues of environmental economics. They include in-depth handbooks, elementary textbooks, research monographs, and edited collections of an interdisciplinary nature, which can be split into mostly environmental books, mostly economic books, and mathematical books. The environmental books usually emphasize practical approaches and do not involve optimization methods. The majority of economic books focuses on sustainable development and integrated assessment tools rather than on optimization and do not consider the heterogeneity of involved agents. Mathematical explorations of economic-environmental models are abundant in scientific journals but are much less covered in monographic literature. Our book is a unique volume that emphasizes a theoretical treatment of specific models using different economic approaches and optimization methods. It analyzes a variety of

economic-environmental issues and mathematical problems with the emphasis on a detailed description of the age structure of involved agents, controlled development, and corresponding management policies.

The book contributes to the theory and practice of optimization modeling of economic-demographic systems (Chapters 1 and 2), optimal management of asset replacement (Chapter 3), vintage capital and embodied technical progress (Chapters 4–6), age-, size-, and stage-structured populations in forestry, fishery, and epidemiology (Chapters 7–11), and aged-structured environmental contamination agents (Chapter 12). The book provides necessary mathematical background for researchers in applied sciences to guide the development of practical models. Alternatively, it surveys the current practice in applied modeling for a general mathematical audience.

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Introduction

Age-structured modeling: past, present, and new perspectives

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Modeling age structures in continuous time typically involves partial or functional differential equations, which sharply departs from the traditional optimal control set-up. While a considerable literature exists on functional differential equations (from the seminal book of Bellman and Cooke, 1963), a relatively much smaller literature has been devoted so far to the optimal control of law of motions governed by such infinitely dimensioned equations. For example, in one of the recent books devoted to functional differential equations, co-authored by Kolmanovski and Myshkis (1999), only one chapter out of 16 actually deals with the optimal control of the latter (with an overwhelming part on the linear-quadratic case). In the optimal control of partial differential equations, numerous theoretical aspects are covered much better than applications to real-life problems, especially on real data. This book is a contribution to the scarce literature on the optimal control of age-structured populations, with applications to economics, demography, and ecology notably. More precisely, we gather a new material accounting for several crucial aspects of age-structured problems, not yet tackled in the related literature. This mainly concerns the following areas.

Vintage capital models

Economic growth theory is essentially based on the concept of aggregate production functions with *homogeneous* capital: all capital goods constituting the operating stock of capital are supposed to have the same marginal contribution to output. In other words, one unit of capital generates the same additional output whatever the particular vintage of the capital good involved. Abstracting away from physical depreciation, the age structure of capital is therefore irrelevant in the traditional growth theory (Solow, 1956 and 1957). Accordingly, there is no connection between investment and the rate of technological progress: investment drives the short-term dynamics of the economies under study while their long-term outcomes are dictated by an exogenous rate of *disembodied* technical progress. This crude dichotomy was first criticized by Solow himself (1960, p. 91) in a famous statement:

This conflicts with the casual observation that many if not most innovations need to be embodied in new kinds of durable equipment before they can be

made effective. Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by the replacement of old-fashioned equipment by the latest models.

While this statement has suggested a huge literature in the 1960s on the age structure of capital, building up a quite appealing and comprehensive vintage capital growth theory, it did not threaten at all the predominance of the traditional neoclassical model with homogeneous capital, at least in the 1970s and the 1980s. Three reasons can be put forward to explain such a failure and these are discussed below.

First, until the early 1990s and the fundamental work of Robert Gordon (1990) on the measurement of durable goods prices in the US, there was no empirical argument supporting the so-called vintage effect, that is the relevance of the actual age structure of capital to quantitatively explain economic growth. Indeed, the discussion on the economic growth implications of embodied technical progress was tremendously controversial in the 1960s. In a famous statement, Denison (1964) claimed that “the embodied question” is unimportant. His argument was merely quantitative and starts with the assumption that the embodiment effect should exclusively show up through the age distribution of the stock of capital. Using his own estimates of US growth in the period 1929–57, he argued that if the average age can be changed by one year from 1960 to 1970, this cannot alter the annual growth rate (in the extreme cases) by more than 0.06 percentage points in the period 1960–70. Of course, Denison’s reasoning is specific to a period of time and it uses a quite conventional one-sector-based growth accounting exercise with a restrictive identification of the embodiment channels (exclusively, the average age of capital). In particular, it omits the relative price of capital (in terms of the consumption good) channel: the latter variable must go down under an acceleration in the rate of embodied (or investment-specific) technical progress but the lack of a compelling computation of the relative price of capital series was a clear limit to these claims. In this sense, Robert Gordon’s work (1990) has been very decisive in the resurgence of the vintage capital and embodied technical progress literature in the 1990s.

A second reason is the theoretical work of renowned authors such as Phelps (1962) or Solow himself in his 1960 paper pointing at the possible relevance of vintage capital modeling. Phelps constructed an enlarged growth model allowing for both neutral disembodied technical progress and embodied technical progress (only affecting the new capital goods). He showed that while the composition of technical progress (i.e., the extent to which technological advances are more or less embodied in capital goods) might be relevant in the short run, it is irrelevant in the long run, the rate of economic growth at that term depending only on the rate of total technological progress. Interestingly enough, Phelps’ framework does not use optimal control, it is based on the common assumption at that time according to which investment is a constant fraction of output. In his own paper, Solow

(1960) showed that as long as production factor proportions are not fixed *ex-post* and are freely variable over the lifetime of capital goods (*putty-putty* assumption), the aggregate production function is neoclassical in aggregate capital (defined in efficiency units) and labor. In particular, the optimal capital lifetime is still infinite, as in the standard neoclassical model, and obsolescence of the oldest capital goods only show up in their decreasing labor allocation, which in turn reflects a declining pattern for the value of vintages. Other than this, the model is functionally equivalent to the traditional neoclassical model with homogeneous capital.

Last but not least, it appeared quickly that dealing with vintage capital models is mathematically much more demanding than the simple neoclassical framework. This was already clear in the seminal work of Johansen (1959) (see also Sheshinski, 1967). In contrast to Solow's vintage capital model (1960), Johansen postulated a putty-clay technology. Capital is non-malleable: while substitution between labor and capital is permitted *ex-ante*, it is not allowed once capital is installed. This configuration involves a much more powerful obsolescence mechanism under scarcity of labor resources, leading to finitely lived capital goods at equilibrium, which sharply departs from the typical outcomes of neoclassical theory. The same properties are obtained in the special case where substitution between labor and capital is neither permitted *ex-ante* nor *ex-post*, featuring the so-called Leontief vintage capital model studied by Solow et al. (1966). Even though the associated mathematical set-ups did not make use of optimal control, relying again on the assumption of constant saving rate, the technical difficulties encountered were much beyond the typical ones in neoclassical theory due to the finite life characteristic of capital goods. Moreover when studying the asymptotic behavior of the induced models, Sheshinski (1967) (for the Johansen model) and Solow et al. (1966) (for the Leontief vintage capital model) prove that they converge to a unique stable balanced growth path, just like the standard neoclassical model, while requiring a much larger amount of tricky computations.

Optimal control made the difference 30 years later! Indeed, Boucekkine et al. (1997) relaxed the assumption of constant saving rate in the Leontief vintage capital growth model and consider instead a problem of a central planner maximizing the intertemporal discounted stream of utility from consumption over time. Within this optimal growth setting, they proved that the optimal lifetime of capital goods is constant (mimicking a well-known property in operation research, the so-called Terborgh–Smith property), inducing oscillatory dynamics, in sharp contrast with the neoclassical model and the early vintage capital literature with constant saving rates. The obtained everlasting fluctuations in investment, output, and consumption follow the so-called replacement echoes mechanism, inherent to the periodic replacement of obsolete capital.

The same structural properties were established in the parallel work by Hritonenko and Yatsenko (1996), which was essentially based on earlier investigation of vintage models by Glushkov et al. (1980, 1982). Interestingly, while Boucekkine et al. (1997) have relied on ad hoc Lagrangean-like technique first

suggested by Malcomson (1975), Hritonenko and Yatsenko (1996) have developed a systematic calculus-of-variation technique conveniently treating the main peculiarity of the induced optimal control problems, that is a control (i.e., scrapping age) showing up as an integration bound. This technique is general and applicable to a wide range of optimization problems in vintage modeling (e.g., Hritonenko and Yatsenko 2005, 2007a). In further work, Boucekkine et al. (2004, 2005) transformed the integral constraints of the optimal control problems into differential-difference equations, which allowed them to take advantage of the relatively scarce literature on the optimal control of functional differential equations, as mentioned above. Last but not least, explicit dynamic programming methods have been recently developed to deal with vintage capital models in the Leontief case (Fabbri and Gozzi, 2008).

In this book, three original contributions to the literature of vintage capital and embodied technical progress are gathered.

Vintage capital models under uncertainty

The literature on vintage capital surveyed above is purely deterministic. It goes without saying that as for any investment problem, incorporating uncertainty is a desirable step to take. The difficulty of doing so in the vintage capital framework comes from the specific scrapping time control variable. In the standard optimal control vintage capital model, optimal scrapping is derived under the explicit condition that a machine that is scrapped cannot be re-used in the future, which describes pretty well the irreversibility of technological obsolescence. When the environment is stochastic like in the model developed by Bruno Cruz and Aude Pommeret in their contribution (Chapter 4), a resource or commodity price is subject to stochastic shocks (say oil shocks), and the irreversibility property outlined above is much more difficult to ensure. The authors, therefore, choose to first study the case where this irreversibility is not imposed. In this sense, capital scrapping is analogous to the capacity utilization rate decision in more traditional investment problems. This shortcut allows the authors to provide a highly interesting preliminary investigation of the stochastic vintage capital model using the real option approach advocated by Pindyck (1988). The authors do not only identify the so-called cleansing effect of recessions, they also show that the economy responds asymmetrically to good versus bad resource price draws, which can be connected quite straightforwardly to the macroeconomic literature on oil shocks (starting from Bailly, 1982).

Viability and sustainability in vintage capital models

Noël Bonneuil in Chapter 5 provides a first study of vintage capital models using the viability approach (see Aubin, 1991 and Bonneuil and Saint-Pierre, 2008 for an earlier application to economics). One result in viability theory is the existence and the computation of the largest (possibly empty) set, called the viability kernel, containing all initial states from which there exists at least one trajectory along which