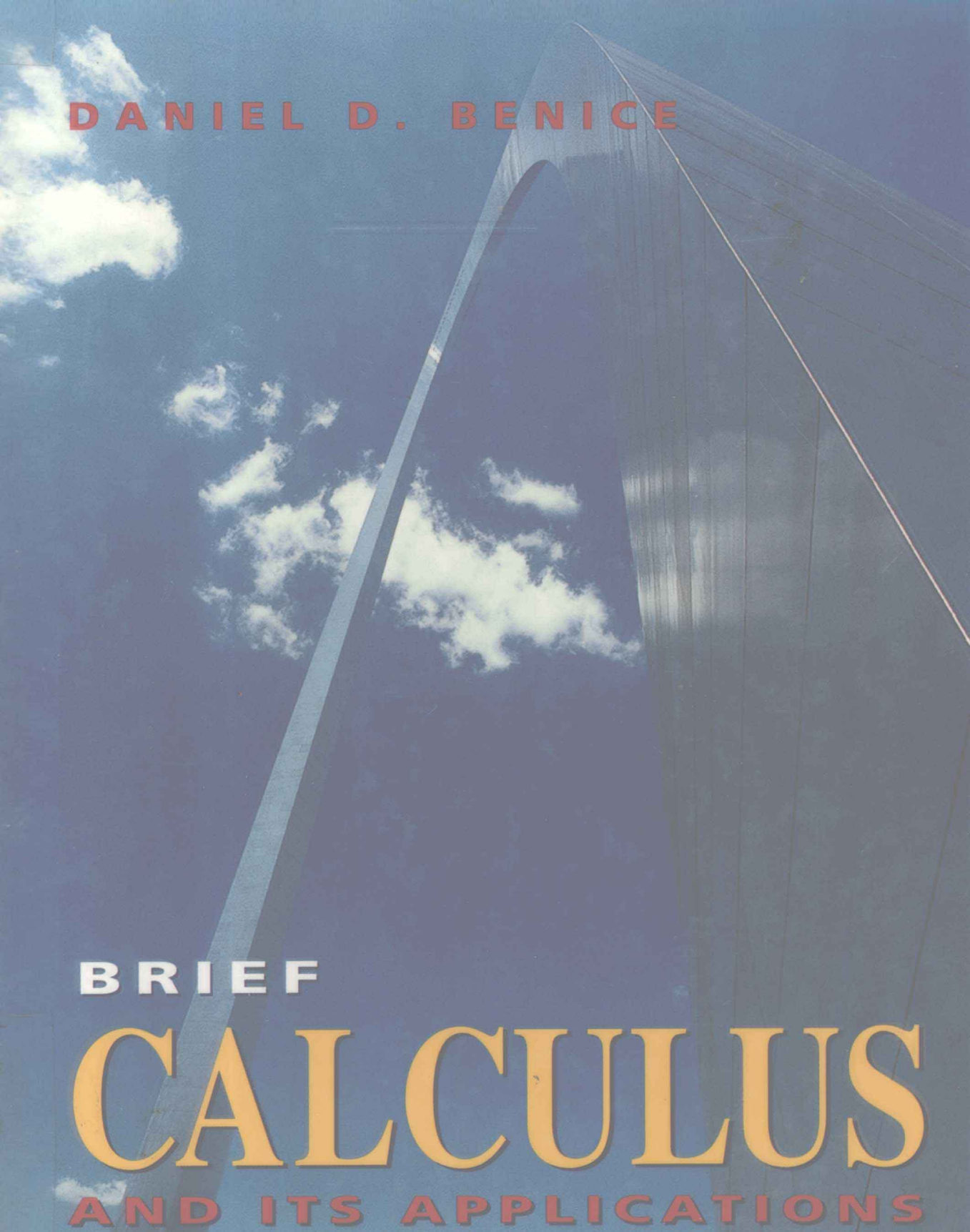

DANIEL D. BENICE

# BRIEF CALCULUS AND ITS APPLICATIONS

# BRIEF CALCULUS
## AND ITS APPLICATIONS

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# PREFACE

*Brief Calculus and Its Applications* provides a one-semester introduction to calculus for students of business, economics, management, social sciences, life sciences, and other fields. The text offers a motivated, comprehensive, applications-oriented approach to the subject. Crafted with care and experience, this book offers instructors and students an ideal combination of content, level, writing style, and special features.

- *A textbook that students can read and understand* Explanations are carefully presented. The writing style makes calculus interesting and readily accessible to students. The theory and applications of calculus are explained in ways they can understand.
- *Solid, honest mathematics* Instructors will find the mathematics carefully done and at the right level. Definitions are mathematically accurate. Theorems are presented with convincing geometric or intuitive justification or with a formal proof when appropriate.
- *An abundance of outstanding exercises* The huge number of exercises in the book have been carefully constructed and gently graduated to enable students to gain confidence as they master skills and learn concepts. The variety of exercises will help students to understand and appreciate calculus.

  The number, kind, and arrangement of exercises provides the instructor with an opportunity to teach at the level he or she prefers. Furthermore, the instructor will be able to select exercises even when only part of a section is completed.
- *Realistic applications* A variety of interesting applications are used as examples and exercises to demonstrate real-world use of calculus. The opportunity to apply calculus concepts and skills will create student interest, involvement, and thinking. While the emphasis is on business, economics, and management, many other types of applications are given. Simplifications and assumptions are sometimes made in order to keep the mathematics within the scope of the course. An index of applications can be found on the inside covers of the book.
- *Illustrative examples* Many examples are included to illustrate calculus ideas and techniques. Consistently, you will see that extra step or explanation that helps to avoid confusion and misunderstanding.

- *Motivation and direction* Calculus is exciting mathematics. This book offers students an adventure of discovery, complete with a built-in guide that tells them where they are going, what they will be doing, and why.
- *Reinforcement* Concepts are studied from a variety of perspectives. The initial explanation is followed by examples, usually with accompanying comments or notes. Exercises focus first on small, central ideas and then proceed to combine the ideas into more complex exercises. Applications provide different settings in which to use the calculus concepts. The reinforcement continues in later sections, when concepts are reviewed in a new environment.
- *A valuable reference* The format and organization of the text make it valuable as a reference. It is easy for students to locate concepts, definitions, theorems, and examples when they are studying or doing homework exercises.
- *Integrated problem solving* The text flows naturally into problem-solving examples. Within the examples, great care is taken to discuss the question being asked or the problem being solved. Students are shown how to set up a problem or determine an equation and how to use the appropriate mathematical concepts. The relation of the final solution to the original problem is made clear. This approach will encourage students to become active participants and learn how to solve calculus problems. See Example 1, beginning on page 194. For a less extensive case, see Example 1 on page 160 and the lead-in text preceding it. Briefer still is Example 2 on page 182 and the sentence preceding it.
- *Accuracy* The text is accurate. All exercises have been worked out by four mathematics professors: Joel W. Irish (University of Southern Maine), Robert Levine (Community College of Allegheny County), Michael Schramm (LeMoyne College), and myself. Furthermore, Robert Levine, Michael Schramm, and I have examined the entire book line-by-line for mathematical accuracy. Finally, in order to maximize continuity, I personally have written the answer section and the Exercise Library (mentioned under "Special Features").
- *Calculator use* The use of calculators is incorporated in a natural, helpful way throughout the book. Within exercise sets, the symbol is used to identify problems specifically designed for calculators. (See pages 251 and 263, for example.) Although many classes will use calculators, those instructors who prefer not to use them will not encounter obstacles.

## Special Features

Exercise Library for Graphing Calculators and Computers—

The Exercise Library (blue-edged pages at the back of the book) contains a variety of optional *graphing calculator exercises* that can be used to supplement

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the study of calculus. The exercises can also be done using *computer software*. The symbol appears at the end of exercise sets throughout the book to direct you to the Exercise Library.

Such exercises add a new dimension to the study of calculus. The graphing calculator offers visual perspectives and problem-solving approaches that are unavailable using conventional techniques. You will find that the Exercise Library offers an exciting complement to the extensive exercise sets in the main body of the text.

### Writing Exercises—**W**

Some exercises are marked with a large blue **W**. They are the *writing exercises*, which ask for explanations *in words*. These questions force students to think and to have a better grasp of the subject than might be attained otherwise. If a concept is not really understood, the student is forced to confront the realization. A sample of writing exercises can be seen on page 212.

These innovative questions address proposals from the NCTM, MAA, AMATYC, and others who support writing across the curriculum.

### Notes

Many "notes" have been strategically placed throughout the text. Their purpose is to help the reader by anticipating questions or problems, offering reinforcement or reminders, adding information, linking ideas, or making comparisons. To sample a variety of notes, look on pages 53, 135, 262, and 325.

### Applications Examples

A variety of applications examples are given throughout the text. Not only do they illustrate how calculus can be used, but many of them also take the learning process a step further by providing the opportunity for students to be involved in problem solving. See Examples 2–4 on pages 76–78, and Example 3 on page 306.

### A Section on Basic Economics Functions

All of the basic economics functions are presented and carefully explained in one section (Section 1.7). Cost, revenue, and profit functions are introduced. Fixed cost and breakeven quantity are explained. Price functions, demand equations, supply equations, and equilibrium are also covered.

Later, Section 3.4 is dedicated to presenting marginal analysis thoroughly and without interruption.

### Practical Algebra Review

Chapter 1 includes a review of essential basic algebra. Elsewhere, algebra is included when needed so that students can work with the calculus. For example, in Chapter 5 the presentation of exponentials and logarithms provides what is needed, but does not cover everything one might learn about the subject by taking a precalculus course.

## Supplements

The following supplements are available for use with the text:

- *Instructor's Resource Manual* The Instructor's Resource Manual contains a printed test bank of all items in the computerized test generator, two chapter tests for each chapter, materials and exercises to facilitate the introduction and use of graphing calculators, and overhead transparency masters for selected examples from the text.
- *Computerized Test Generator* The Computerized Test Generator contains more than 2000 test questions, organized by section to follow topics in the text. Over one third of these are applications questions. The instructor may choose between multiple choice and free-response answer formats.
- *Solutions Manual* The Solutions Manual contains complete solutions to all exercises.
- *Student Solutions Manual* The Student Solutions Manual contains complete solutions to all odd-numbered exercises.
- *Instructor's Even-numbered Answer Booklet* The Answer Booklet contains answers to all even-numbered exercises, in an easy-to-carry format.
- *PC-81 Emulation Software* PC-81 Emulation Software is available for the IBM PC (and compatibles). This powerful and compact package completely emulates the functionality of the popular TI-81 graphing calculator. The software is offered in cooperation with Texas Instruments.
- *Math Assistant Software* Math Assistant Software is available for the Mac-Intosh, Apple, and IBM PC (and compatibles). This package easily plots algebraic and trigonometric functions.

## Content and Organization

The text offers comprehensive coverage of the calculus topics appropriate for this course. Since there is always the need to tailor a textbook to fit your specific course, we present here a guide that will help you to select the sections and topic sequence that will be best for your class.

The first five sections of Chapter 1 constitute a review of algebra and accordingly can be covered in class or omitted if the students are already familiar with the material. By contrast, Section 1.7 (Functions in Economics) introduces and explains key concepts that will be used in business applications throughout the book. The ideas in optional Section 1.6 are never formally applied elsewhere

in the text; however, those students who become familiar with translation and reflection may choose to use them when they see the opportunity.

Since calculus cannot be understood without some knowledge of limits, Chapter 2 offers a pragmatic study of this important topic. If time constraints or personal preference demand that you get to the derivative quickly, cover only Section 2.1 (Introduction to Limits) and then go directly to Chapter 3 (Derivatives). Other sections of Chapter 2 can be covered as needed and as time permits. Another alternative is to present Sections 2.1 through 2.3 and return later as needed to limits at infinity (2.4) and infinite limits (2.5).

Chapter 3 introduces the derivative. You will want to present Sections 3.1–3.8 in sequence. Notice that Section 3.4 offers thorough coverage of the marginal concepts. Section 3.9 (Related Rates) can be omitted if it is not considered part of your course.

Chapter 4 presents applications of the derivative. The sections should be studied in sequence. Elasticity of demand (Section 4.6) may be omitted, although many business calculus courses consider the topic essential.

Chapter 5 presents the exponential and logarithmic functions. The sections should be covered in sequence. Section 5.5 (Some Additional Business Applications) can be omitted if desired.

Chapter 6 covers integration. Sections 6.1–6.4 should be taught in sequence. The three remaining sections offer applications and can be presented in any order. Note that in Section 6.5 the application of integration to finding the average value of a function and the volume of a solid of revolution are not essential to the continuity of the chapter or the text. Section 6.6 (Surplus) is an important business topic, but it can be omitted or studied later without loss of continuity.

Integration by substitution (Section 7.1) is an important extension of the integration ideas presented in Chapter 6. The remainder of the chapter includes standard topics that can be covered or omitted to fit the nature of your course.

Section 7.6 (Differential Equations) can be taught after 7.1, if desired.

Section 7.7 (Probability and Calculus) can also be covered after 7.1 or even without 7.1 if the integration in Example 2 is done by using an earlier formula rather than by substitution.

The sections of Chapter 8 (Multivariable Calculus) are arranged in a fairly standard order. However, you can rearrange the sequence or omit sections. The only restrictions are that Sections 8.1 and 8.2 must be presented first and that Section 8.3 must be covered before 8.5.

Many students taking this course have never studied trigonometry. Consequently, what seems like a simple review (Sections 9.1 and 9.2) may in fact serve as an introduction to trigonometry for some students. The sections of Chapter 9 should be covered in sequence.

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Daniel D. Benice

# CONTENTS

*PREFACE* **xi**

**CHAPTER 1**

## *FUNCTIONS 1*

**1.1** *Real Numbers* 2
**1.2** *Some Algebra Review* 5
**1.3** *Introduction to Functions* 15
**1.4** *Linear Functions* 21
**1.5** *Graphs of Functions* 33
**1.6** *Translations and Reflections (optional)* 45
**1.7** *Functions in Economics* 49

**CHAPTER LIST** 60

**REVIEW EXERCISES** 61

**CHAPTER 2**

## *AN INTRODUCTION TO LIMITS 63*

**2.1** *Introduction to Limits* 64
**2.2** *Continuity* 72
**2.3** *One-Sided Limits* 80
**2.4** *Limits at Infinity* 88
**2.5** *Infinite Limits* 96

**CHAPTER LIST** 102

**REVIEW EXERCISES** 102

CHAPTER 3

## DERIVATIVES 105

**3.1** *Introduction to the Derivative* 106
**3.2** *Basic Rules for Differentiation* 117
**3.3** *Rates of Change* 125
**3.4** *Marginal Analysis* 133
**3.5** *The Product and Quotient Rules* 140
**3.6** *The Chain Rule* 147
**3.7** *Higher-Order Derivatives* 153
**3.8** *Implicit Differentiation* 158
**3.9** *Related Rates* 164
**3.10** *Differentials* 172

**CHAPTER LIST** 176

**REVIEW EXERCISES** 177

CHAPTER 4

## ADDITIONAL APPLICATIONS OF THE DERIVATIVE 179

**4.1** *Increasing and Decreasing, Graphs, and Critical Numbers* 180
**4.2** *Relative Extrema and Curve Sketching* 193
**4.3** *Concavity, the Second Derivative Test, and Curve Sketching* 201
**4.4** *Absolute Extrema* 213
**4.5** *Additional Applications. Applied Maximum/Minimum* 216
**4.6** *Elasticity of Demand* 228

**CHAPTER LIST** 233

**REVIEW EXERCISES** 234

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CHAPTER 5

## EXPONENTIAL AND LOGARITHMIC FUNCTIONS 237

**5.1** *Exponential Functions* 238
**5.2** *Logarithmic Functions* 251
**5.3** *Differentiation of Exponential Functions* 267
**5.4** *Differentiation of Logarithmic Functions* 275
**5.5** *Some Additional Business Applications* 287

**CHAPTER LIST** 293

**REVIEW EXERCISES** 293

CHAPTER 6

## INTEGRATION 295

**6.1** *Antidifferentiation* 296
**6.2** *Some Applications of Antidifferentiation* 304
**6.3** *The Definite Integral as the Area under a Curve* 313
**6.4** *The Fundamental Theorem of Calculus* 321
**6.5** *Some Applications of the Definite Integral* 330
**6.6** *Surplus* 340
**6.7** *Area in the Plane* 344

**CHAPTER LIST** 353

**REVIEW EXERCISES** 354

CHAPTER 7

## TECHNIQUES OF INTEGRATION 355

**7.1** *Integration by Substitution* 356
**7.2** *Integration by Parts* 365
**7.3** *Integration by Tables* 372
**7.4** *Numerical Methods of Approximation* 376
**7.5** *Improper Integrals* 384
**7.6** *Differential Equations* 393
**7.7** *Probability and Calculus* 400

**CHAPTER LIST** 408

**REVIEW EXERCISES** 408

CHAPTER 8

## MULTIVARIABLE CALCULUS 411

**8.1** *Functions of Several Variables* 412
**8.2** *Partial Derivatives* 422
**8.3** *Maximum and Minimum* 431
**8.4** *Lagrange Multipliers* 441
**8.5** *The Method of Least Squares* 447
**8.6** *Total Differentials* 453
**8.7** *Double Integrals* 457

**CHAPTER LIST** 466

**REVIEW EXERCISES** 466

**CHAPTER**

## *TRIGONOMETRIC FUNCTIONS 469*

**9.1** *Right Triangles* 470
**9.2** *Radians and the Trigonometry for Calculus* 475
**9.3** *Differentiation of Trigonometric Functions* 487
**9.4** *Integration of Trigonometric Functions* 498
**9.5** *Integration by Parts Revisited* 505

**CHAPTER LIST** 507

**REVIEW EXERCISES** 507

*TABLES 509*

*EXERCISE LIBRARY 529*

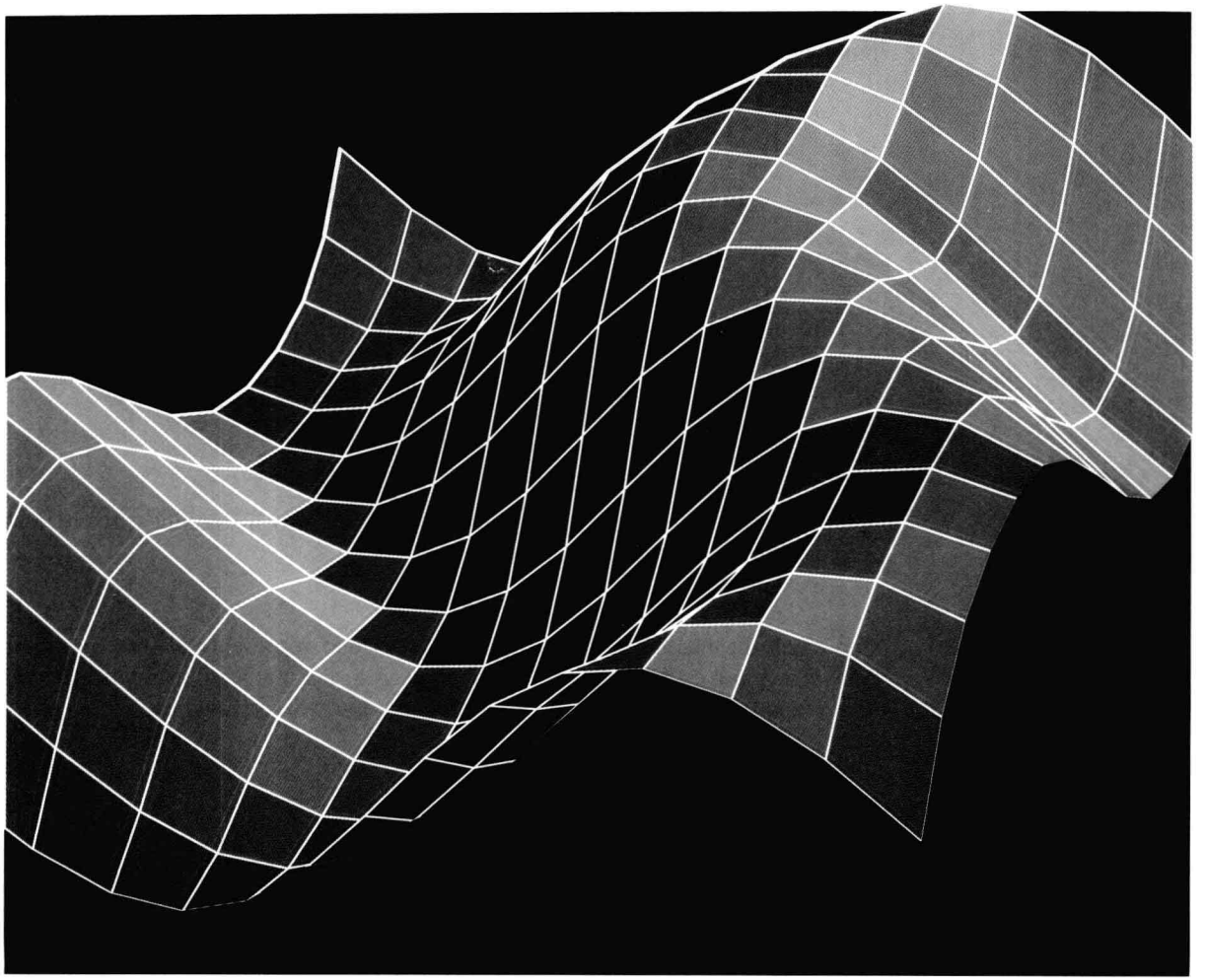
*ANSWERS TO ODD-NUMBERED EXERCISES 545*

*INDEX 589*

# 1

# *FUNCTIONS*


**1.1** *REAL NUMBERS*
**1.2** *SOME ALGEBRA REVIEW*
**1.3** *INTRODUCTION TO FUNCTIONS*
**1.4** *LINEAR FUNCTIONS*
**1.5** *GRAPHS OF FUNCTIONS*
**1.6** *TRANSLATIONS AND REFLECTIONS (OPTIONAL)*
**1.7** *FUNCTIONS IN ECONOMICS*

This chapter is intended to prepare you for a successful exploration of the ideas and applications of elementary calculus. We have included some algebra review, an introduction to functions and graphs, and a presentation of the functions used in business and economics. Since the material presented here will be used throughout the book, it is important that you become familiar with it.

## 1.1 REAL NUMBERS

The study of elementary calculus requires a knowledge of the real number system. The real numbers can be considered points on a line. To every real number there corresponds one point. To every point there corresponds one real number. (See Figure 1.)

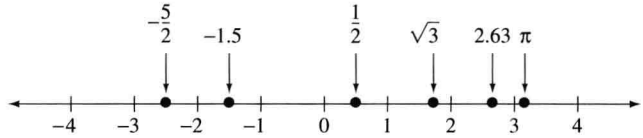



**Figure 1** The real number line

Inequalities can be used to compare real numbers. The symbols used are $>$ (greater than), $<$ (less than), $\geq$ (greater than or equal to), and $\leq$ (less than or equal to). For example,

$x > 3$ ($x$ is greater than 3)

$y \leq -2$ ($y$ is less than or equal to $-2$)

In some applications it is useful to combine two inequalities in order to express an **interval**. For example,

$$2 < x < 5$$

combines the inequalities $2 < x$ and $x < 5$ and represents all real numbers between 2 and 5. The notation (2, 5) is used to denote such an **open interval** that excludes the endpoints. Graphically, the interval is shown as

The inequality $2 \leq x \leq 5$ expresses a **closed interval**, one in which the endpoints are included. The interval is denoted [2, 5] and is shown graphically as

The two intervals (2, 5) and [2, 5] and others are shown in Figure 2. A parenthesis is used to indicate that an endpoint is not included. A bracket is used to indicate that an endpoint is included. Intervals such as (2, 5] and [2, 5) are called **half-open intervals**. The symbol $\infty$ (infinity) is used to specify that the interval extends infinitely far to the right. Similarly, $-\infty$ (minus infinity) is used to specify that an interval extends infinitely far to the left.

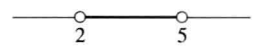
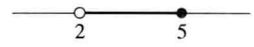
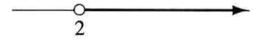
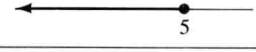
| inequality | interval notation | graph |
|---|---|---|
| $2 < x < 5$ | $(2, 5)$ |  |
| $2 \le x \le 5$ | $[2, 5]$ | |
| $2 < x \le 5$ | $(2, 5]$ |  |
| $2 \le x < 5$ | $[2, 5)$ | |
| $x > 2$ | $(2, \infty)$ |  |
| $x \le 5$ | $(-\infty, 5]$ |  |

**Figure 2** Intervals

This section concludes with a brief review of *linear inequalities*, which you will see have intervals for solutions. Some calculus problems require the solution of inequalities.

You should recall that linear inequalities are solved in much the same way as linear equations. But there is one key difference.

> If both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality is reversed: $>$ becomes $<$, and $<$ becomes $>$.

***EXAMPLE 1*** Solve the inequality $3x + 4 > 15$.

***SOLUTION*** Begin by adding $-4$ to both sides of the inequality $3x + 4 > 15$. The result is

$$3x > 11$$

Now, divide both sides by 3.

$$x > \frac{11}{3}$$

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