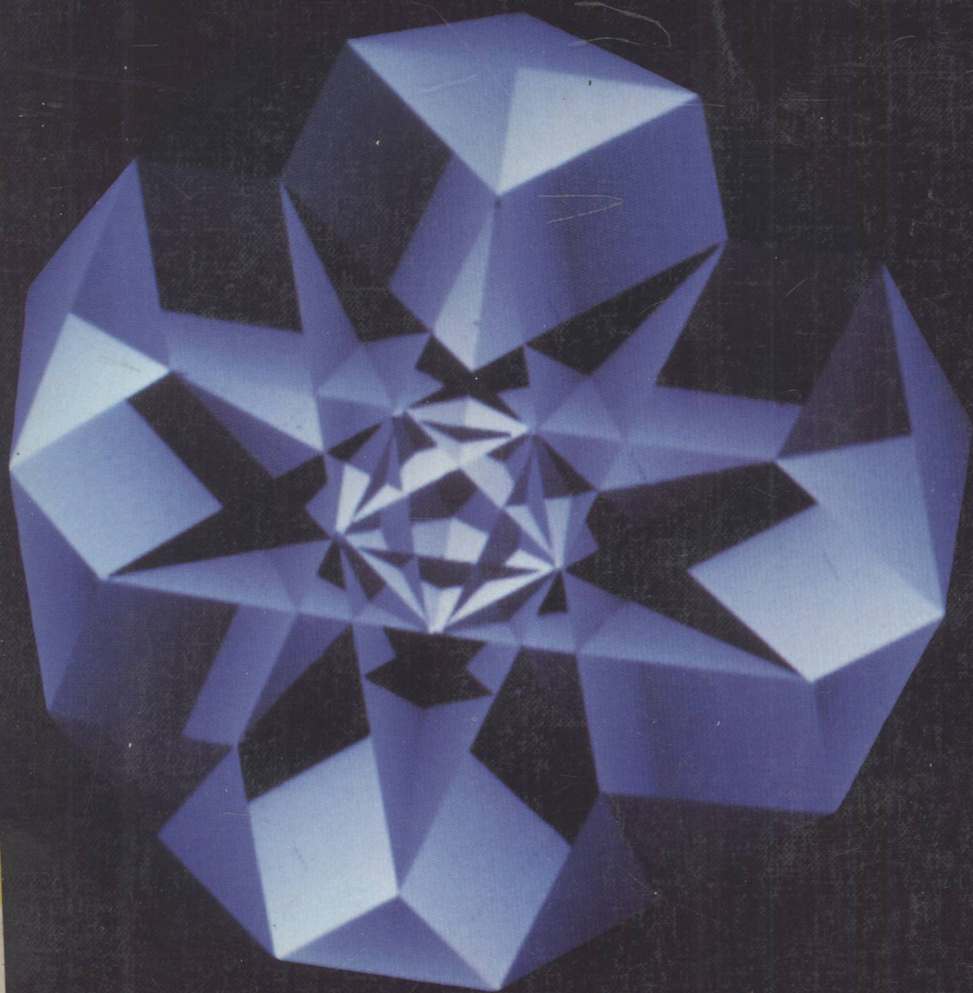


JAY KAPPRAFF

CONNECTIONS

THE GEOMETRIC
BRIDGE BETWEEN
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The Geometric Bridge between Art and Science

Jay Kappraff

*Department of Mathematics
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Preface

The writing of this book has been a personal exploration for me in the widest sense of the word. Its origins can be traced to my friendship with Mary Blade, an engineer, artist, and descriptive geometer who developed a project-oriented course on the relationship between mathematics and design and taught it for many years at the Cooper Union. I am a mathematician and 10 years ago I presented some of Professor Blade's ideas to a number of colleagues from the Mathematics and Computer Science Departments and the School of Architecture at the New Jersey Institute of Technology. These discussions led to the offering of a course for students from the School of Architecture in the Mathematics of Design. Over the past 10 years, I have had the pleasure of observing beautiful works of art and designs created by my students, based on the mathematical ideas that I have presented to them. It was only years after I started that I learned that I was rediscovering a well-established field of inquiry known to some as design science. This book is meant to be an introduction to this field. I have attempted to make it as comprehensive a survey of the field as space and my own involvement in it permits.

What is design science? It is a subject that has advanced from the twin perspectives of the designer and the scientist sometimes in concert with each other and sometimes on their own, and may be considered to be a geometric bridge between art and science. Design science owes its beginnings to the architect, designer, and inventor Buckminster Fuller. In a meeting with Nehru in India in 1958, Fuller said

The problem of a comprehensive design science is to isolate specific instances of the pattern of a general, cosmic energy system and turn these to human use.

The chemical physicist Arthur Loeb is one of the individuals most responsible for recognizing design science as an independent discipline.

He considers it to be the grammar of space and describes it as follows:

Just as the grammar of music consists of harmony, counterpoint, and form which describes the structure of a composition, so spatial structures, whether crystalline, architectural, or choreographic, have their grammar which consists of such parameters as symmetry, proportion, connectivity, stability, etc. Space is not a passive vacuum; it has properties which constrain as well as enhance the structures which inhabit it.

This book is an exploration of this grammar of space, with the objective to show, by way of demonstration, that this grammar can be the basis of a common language that spans the subjects of art, architecture, chemistry, biology, engineering, computer graphics, and mathematics. Perhaps design science's greatest value lies in its potential to reverse the trend toward fragmentation resulting from the overspecialization of our scientific and artistic worlds and to alleviate some of the isolation of discipline from discipline that has been the result of that overspecialization.

Design science is an interdisciplinary endeavor based on the work of mathematicians, scientists, artists, architects, and designers. The early pioneers, some of whom have been influential in its development in varying degrees, include the inventor Alexander Graham Bell, the biologist D'Arcy Thompson, R. Buckminster Fuller, the structural inventor Robert Le Ricolais, Arthur Loeb, the recreational mathematician Martin Gardner, the artist and designer Gyorgy Kepes, the artist M. C. Escher, and several architectural designers who have contributed continually to the field. These include David Emmerich, Stuart Duncan, Janos Baracs, Anne Tyng, Steve Baer, Michael Burt, Peter Pearce, Keith Critchlow, and Hareesh Lalvani. Reference to these people and others is found throughout the chapters and in the bibliography to this book.

Mathematics serves as the foundation of design science, and the mathematicians who have had the most profound influence on my own thinking on this subject are H. S. M. Coxeter, Branko Grünbaum, and Benoit Mandelbrot. Special mention must also be made of the work gathering and disseminating ideas on the part of the structural topology group at the University of Montreal under the leadership of Janos Baracs and the mathematician Henry Crapo. In addition, the chemist Istvan Hargittai has done enormously valuable work editing two large volumes on symmetry as a unifying force behind science and art and starting a new journal entitled *Symmetry*. In addition, I would like to acknowledge another journal, *Space Structures*, which is devoted primarily to structures from an architectural and engineering standpoint.

The unsung heroes of design science also deserve a large share of the credit for its development. These are people who, for a variety of

reasons have labored, often on a single idea, in their studios, laboratories, or studies to discover parts of the thread which binds this discipline together. Today, mathematicians have, for the most part, given up the study of the roots of their subject in two- and three-dimensional geometry in order to delve into greater and greater realms of abstraction. As Branko Grünbaum (1981) has lamented:

It is a rather unfortunate fact (for mathematics) that much of the creative introduction of new geometric ideas is done by nonmathematicians, who encounter geometric problems in the course of their professional activities. Not finding the solution in the mathematical literature, and often not finding even a sympathetic ear among mathematicians, they proceed to develop their solutions as best they can and publish their results in the journals of their own disciplines.

At the same time computer scientists have added their own form of abstraction to the study of geometry by replacing the constructive aspects of this subject with two-dimensional pictures on a computer screen. It is into this dearth of geometrical thinking that artists, architects, designers, crystallographers, chemists, structural biologists, and individuals from other disciplines have come with their extraordinary constructions and discoveries. A large part of this book is devoted to bringing their ideas to light.

A book such as this must have boundaries and so certain topics were regrettably omitted. For example, Chaps. 7 through 10, devoted to polyhedra, leave off where B. M. Stewart's fascinating toroidal polyhedra begin (Stewart, 1980). Also, most of the topics of this book relate to euclidean geometry, yet projective geometry is a far richer system of geometry as shown in the work of Janos Baracs and Henry Crapo and the many books and monographs on the synthetic approach to projective geometry published by the Rudolf Steiner Institute (Crapo, 1978) (Edwards).

It was only at the conclusion of my work on this book that I discovered what it was about. On one level, this book is a collection of special topics in ancient and modern geometry. On another it introduces the reader to many of the ways that geometry underlies the creation of beautiful designs and structures. At a deeper level, this book shows how geometry serves as an intermediary between the unity and harmony of the natural world and the capability of humans to perceive this order. Le Corbusier has expressed this role of mathematics eloquently (Le Corbusier, 1968b):

The flower, the plant, the tree, the mountain . . . if the true greatness of their aspect draws attention to itself, it is because they seem contained in themselves, yet producing resonances all around. We stop short, conscious of so much natural harmony; and we look, moved by so much unity commanding so much space; and then we measure what we see.

In this book we shall measure and study the consequences of these measurements but try not to lose sight of the spiritual elements which give meaning and life to the study of design science.

The book is written so that the theory is illustrated at each step by either a design or an application. However, no attempt has been made to be exhaustive in either theory or practice. Each chapter of the book is written so that it can be read separately. However, as is characteristic of design science, each chapter is also tightly interwoven with each of the others. As a result, the reader can choose a variety of paths through the book. Design science is a dynamic discipline. It is forever changing as each practitioner brings his or her new perspective to bear on the subject. In this spirit, the reader is invited to actively participate in the discovery of design science by carrying out some of the constructions, experiments, and problems suggested throughout the book and to think about how the ideas arise in the reader's own discipline.

Although this book was not written as a textbook, if supplemented by a manual of additional exercises, problems, projects, and a guide to instructors, it can be used to teach a course like the one I teach at New Jersey Institute of Technology. McGraw-Hill is considering publishing such a supplementary manual.

Jay Kappraff

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I would like to acknowledge support that The Graham Foundation offered to make the writing of this book possible. In addition to the people already mentioned, I would like to acknowledge the invaluable help of Alan Stewart, who taught the mathematics of design with me for several years and made many contributions to its development, and to Denis Blackmore, Bill Strauss, and Steve Zdepski, who also worked with me on the early development of the ideas found in this book. A special thanks goes to the generations of students who have taken my course and who, through their creations, have inspired me to develop the ideas found in this book. I wish to acknowledge the help of Branko Grünbaum and Denis Blackmore who read and commented on the manuscript in its early stages and for the help and encouragement of Istvan Hargittai. I am indebted to Haresh Lalvani who made the results of his advanced research in design science generously available to me. He helped me to see how the many parts of this subject fit together, and you will see much of his work displayed throughout this book. N. Rivier and Janos Baracs were also generous in sharing the results of their work with me. I am also grateful for the help of Eytan Carmel, Hyung Lee, and David Henig-Elona, who created many of the drawings, and Richard McNally, Rebeca Daniels, and Vedder Wright, who contributed their comments, ideas, and encouragement. A special thanks goes to Bruce Brattstrom who played a major role in creating drawings and models and in offering a calming influence as final deadlines approached. My patient family also deserves thanks since without their encouragement the completion of this task would have been more difficult and less enjoyable. Finally, McGraw-Hill has been an ideal partner in the creation of this manuscript. I have enormous appreciation for their venturesome spirit in the production of this unusual book. I could not have had two finer editors to work with than Joel Stein and Nancy Young. I, of course, take full responsibility for any errors of content found within these covers.

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ABOUT THE AUTHOR

Jay Kappraff is an associate professor of mathematics at the New Jersey Institute of Technology where he has developed a course in the mathematics of design for architects and computer scientists. Prior to that, he taught at Cooper Union College and held the position of aerospace engineer at NASA. He has published numerous articles on such diverse subjects as fractals, phyllotaxy, design science, plasma physics, passive solar heating, and aerospace engineering. He has also lectured widely on the relationship between art and science. Dr. Kappraff holds a Ph.D. from the Courant Institute of Mathematical Science at New York University.

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