INTERNATIONAL MEASUREMENT CONFEDERATION

1st Conference of the Technical Committee (TC15)
on

# MEASUREMENT OF STATIC AND DYNAMIC PARAMETERS OF STRUCTURES AND MATERIALS

Plzen, Czechoslovakia
May 26—28, 1987

## IMEKO TC No. 18

# PROCEEDINGS

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Series editor: Dr. Tamás Kemény
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Inquiries:
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**Transmission photoelasticity**

# INTEGRATED PHOTOELASTICITY OF THE GENERAL THREE-DIMENSIONAL STRESS STATE

Dr.Sc. Hillar Aben - Siim Idnurm, Cand.Sc.
Institute of Cybernetics, Estonian Academy of Sciences
21, Akadeemia tee, 200108 Tallinn, Estonia, USSR

It is shown that integrated photoelasticity permits to determine the general three-dimensional stress state. From six unknown stress components four can be expressed through the remaining two by the aid of equilibrium and compatibility equations. The latter two are determined on the basis of experimental data.

Keywords: photoelasticity, tomography

## 1. Introduction

In integrated photoelasticity [1] stresses in three-dimensional bodies are determined on the basis of optical data, obtained when polarized light passes through the whole body in a usual transmission polariscope. Up to now integrated photoelasticity has been used to determine only comparatively simple stress states (e.g., in shells and bodies of revolution).

In comparison with the frozen-stress and scattered light method integrated photoelasticity has several advantages: experiment can be carried out at room temperature, the method is nondestructive, optical measurements are comparatively simple, in principle it is possible to investigate also non-elastic deformations. Therefore it is of practical interest to investigate how wide is the class of stress states which can be determined by integrated photoelasticity. Since experimental information one obtains with integrated optical measurements is limited, investigators have been discouraged to tackle problems more complicated than the axisymmetric stress state.

In this paper it is shown that in principle integrated photoelasticity can be used to determine arbitrary three-dimensional state of stress. At that we consider only the case when optical measurements are

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made in sections parallel to each other, i.e., the model is to be rotated around only one axis. Although passing of light through the model in arbitrary directions may give more experimental information [2,3], it makes the experimental setup very complicated.

In this respect our approach resembles tomography [4] in which internal structure of three-dimensional objects is also determined by sections. However, while classical tomography considers determination of scalar fields, integrated photoelasticity is to be considered as optical tomography of a tensor field [5].

## 2. Approximation of the Stress Components

Let us assume that the section $z=z_0$ of an arbitrary three-dimensional body is located in a circle with radius R. In cylindrical coordinates r, $\theta$, z we may express the stress components $\sigma_{rr}$, $\sigma_{\theta\theta}$, $\sigma_{zz}$, $\sigma_{r\theta}$, $\sigma_{\theta z}$, and $\sigma_{zr}$ in the following way:

$$\sigma_{ij} = \sum_{mn}\sum (a_{mn}^{ij} r^n \cos m\theta + b_{mn}^{ij} r^n \sin m\theta), \quad i,j=r,\theta,z \tag{2.1}$$

where $a_{mn}^{ij}$ are coefficients we have to determine. Stress components in the auxiliary section situated at a distance $\Delta z$ from the main section will be marked by a hyphen: $\bar{\sigma}_{ij}$, $\bar{a}_{mn}^{ij}$, $\bar{b}_{mn}^{ij}$.

Boundary conditions as well as equilibrium conditions impose on the coefficients $a_{mn}^{ij}$ and $b_{mn}^{ij}$ certain conditions which permit to eliminate part of the coefficients.

## 3. Equations of the Theory of Elasticity

Equations of equilibrium in cylindrical coordinates are as follows:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} + \frac{\partial\sigma_{zr}}{\partial z} = 0, \tag{3.1}$$

$$\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta} + 2\frac{\sigma_{r\theta}}{r} + \frac{\partial\sigma_{r\theta}}{\partial r} + \frac{\partial\sigma_{\theta z}}{\partial z} = 0, \tag{3.2}$$

$$\frac{\partial\sigma_{zz}}{\partial z} + \frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta} + \frac{\sigma_{zr}}{r} + \frac{\partial\sigma_{zr}}{\partial r} = 0. \tag{3.3}$$

Compatibility equation for the components of deformation in the plane of the section, expressed through stresses, is the following

$$\frac{1}{r^2}\frac{\partial^2\sigma_{rr}}{\partial\theta^2} - \frac{1}{r}\frac{\partial\sigma_{rr}}{\partial r} + \frac{2}{r}\frac{\partial\sigma_{\theta\theta}}{\partial r} + \frac{\partial^2\sigma_{\theta\theta}}{\partial r^2} - \frac{2}{r^2}\frac{\partial\sigma_{r\theta}}{\partial\theta} -$$
$$- \frac{2}{r}\frac{\partial^2\sigma_{r\theta}}{\partial r\partial\theta} = \frac{\mu}{1+\mu}\left(\frac{1}{r}\frac{\partial\sigma}{\partial r} + \frac{\partial^2\sigma}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\sigma}{\partial\theta^2}\right), \tag{3.4}$$